

## ***Interactive comment on “Semi-Lagrangian methods in air pollution models” by A. B. Hansen et al.***

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Answer to referee #1

Thank you for your very thorough review of our work. Below the comments are answered one by one.

Referee: The scope of this paper is impressive and it undoubtedly stems from a considerable effort behind the implementation of the numerical schemes that the authors compare. The content is appropriate for publication in GMD. As the ordering of the numerical schemes is based on a rank, I would expect some discussion of its relevance. For the rotating cone results with rural chemistry (Table 1) one generally notices (and the authors state it 5.3.1) that the higher the spatial resolution, the higher

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the rank. For a given spatial resolution, the larger the time step, the higher the rank. For a given spatio-temporal resolution, filtered SL schemes perform better than the non-filtered ones. Generally, with a couple of exceptions, all three norms associate similar ranks to the schemes. For the slotted cylinder (Table 2) this resolution-wise and filter presence-wise ordering is less of the case. But if you take a closer look, it turns out that it is  $\text{rank}(l_\infty)$  which considerably lowers the rank of the test cases with the spatial resolution  $\Delta x = 0.5$ , for the filtered schemes. To a lesser degree this is also the case in Table 4 where it lifts the position of the non-filtered schemes. It is clear that  $l_\infty$  is crucial from some applications of the air pollution modeling. I would therefore suggest the authors go beyond a simple description which scheme is ranked higher according to each of the norms and discuss the appropriateness of the numerical schemes for some applications rather than for others. On a technical side, I would recommend that the authors have the article read with a focus on the logical coherence of the sentences, on matching the nouns and verbs and put them either in singular or plural, on the correct prepositions and on the usage of articles. This suggestion refers in particular to section 5 of the paper, 5.3 (and especially 5.3.1 and 5.3.2) being really difficult to follow.

Answer: We thank the reviewer for many constructive comments and items for discussion. First of all, we have rewritten Section 5.3.1 and 5.3.2.

We thank the reviewer for this discussion. The reviewer is completely right in the general comments and conclusions about resolutions, time steps, and the norms. In general the Eulerian methods perform better with smaller time steps while the opposite is true for the semi-Lagrangian methods. The time step for the SL methods is, however, not limited due to stability criterion but limited for physical reasons (the Courant number should not exceed 1). The discussion about appropriateness or relevance is also very important, however, it cannot be answered with results from this study. When developing and testing new advection schemes, the schemes should first of all be tested for accuracy and whether they are giving the expected results, which we have carried out in this paper using the rotation test. If a scheme is not sufficiently accurate, or it is

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inconsistent with expected performance, it will be discovered by these extensive tests where the combination of advection and non-linear chemistry will reveal any faults, errors or inconsistencies. Since the performance of an advection scheme when used in a real model simulation depends on its accuracy versus computing time, the next step will be to test exactly this. In the end the ultimate test is to couple the advection schemes to a comprehensive chemistry-transport model and the relevance of the individual schemes will be revealed in the real use and test of the different schemes. The final goal of this work is to develop a sufficiently accurate scheme that is significantly faster than the already used scheme in the Danish Eulerian Hemispheric Model (DEHM). The accuracy versus computing time will be very important when running the model with very resolution, which is the final goal of this work. The results of these tests and the relevance of the different schemes used for real simulation will be the focus of a future paper.

Referee: It is very valuable that the authors present the numerical schemes they implement and compare. The task of finding a balance between a concise description of a numerical scheme (which avoids rewriting the original paper) on the one hand and an understandable presentation on the other hand is very difficult. I would, therefore, suggest the authors take a refreshed look on the presentation of the numerical schemes addressing the following items: 1. Section 3.2 I am disturbed by the way consistency is used here. I am aware that the authors follow Machenhauer et al. (2008) but 'consistency' in the studies of the numerical schemes refers to a property that a discrepancy between an operator and its discretized version, applied to a reasonable solution, tends to 0 when  $\Delta x$  and  $\Delta t$  tend to zero. I would only suggest considering using consistency enriched with some descriptive addition like, for example, consistency of the discretization or something of this sort.

Answer: Consistency has been replaced with consistency of discretization.

Referee: 2. Section 3.3.3 The description of the cascade interpolation is somewhat difficult to follow even if I have Fig.1 of Nair et al. in front of me. It is obvious that

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understanding how the scheme works requires an effort from a reader. I would suggest, nevertheless, chopping the description into a larger number of small paragraphs. Also the sentence on p 2380, l 10 on piecewise parabolic profiles of the vertical columns leaves me perplexed. I suggest skipping 'vertical' at least and, even better, talk about  $i$ -th and  $j$ -th or  $\lambda$  and  $\mu$  directions instead of horizontal and vertical as it is very confusing, especially without Nair et al. Fig.1.

Answer: The section has been modified according to the reviewer.

Referee: 3. Section 3.3.5 There is a sentence in the second paragraph of this section which gives a general idea behind this scheme. Maybe the authors could, nevertheless, intertwine the sequence of formulas which follow with some explanations making them more digestible.

Answer: General plain word description of the filter The filter operates as a localized re-organization of the mass in the original unfiltered high-order mass-conserving forecast, i.e. at time step  $n+1$ . The mass is re-organized in such a way that the resulting filtered, mass conserving forecast approaches certain target values while not – if possible – exceeding a certain monotonicity interval (also ensuring positive definiteness) relative to the original field values at the previous time step  $n$ . The target values are calculated by first applying a non-linear, i.e. scale dependent, anti-diffusion to the original non-filtered forecast (still keeping this forecast). The anti-diffusion acts to reduce the inherent diffusion of the semi-Lagrangian high order interpolation. Once anti-diffused, the target values are truncated to the monotonicity interval.

After (22): ,  $k$  as above represents a general grid point index, i.e.  $k = 1, \dots, K$ , where  $K$  is the total number of grid points.

Comments reg . 23. (replace sentence after (23)) with:  $\alpha_k$  determines the amount of anti-diffusion - or diffusion in the case where  $\alpha_k < 0$ . In this study  $\alpha_k$  is determined from: (24) In (24)  $\sigma$  is a parameter determining the local scale of  $\rho$  with small scales corresponding to high values of  $\sigma$ . The

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coefficients in (24) have been obtained empirically, see details in Kaas and Nielsen 2010.

After (26): Change “, given by” to . In one dimension this quantity is defined as”

After eq (28) In the two dimensional case  $\$D\$$  is determined as the average scale in each of the two directions

After (29): Replace “rg is proportional . . . are ignored” with: Ignoring truncation errors  $\$r_g\$$  is proportional to the value  $\$D^4\$$  would take for a wave with wave number 1

Delete sentence after (30): “The resulting field of Eq. . . .”

Some words about the mass-re-organisation: Details of the re-organisation of mass can be found in Kaas and Nielsen 2010. In brief, it consists of a small series of local mass-redistributions which gradually brings the mass conserved forecast closer to the target values. It can be thought of as a type of nudging under the strong constraint of local mass conservation.

Referee: 4. Section 5.3.1 I suggest rewriting this section using full sentences and structuring the presentation. In your description, do you initially analyze the spatial resolution of  $\hat{U}_x = 1$  and only afterwards at  $\hat{U}_x = 0.5$ ? If so, please state clearly. If not, please also state clearly according to which criteria you structure the analysis of the results. You mention 'traditional', 'fine' and 'finest' resolution which suggest there are three resolutions at least, while I can only see two in Table 1.

Answer: Section rewritten according to the reviewer.

Referee: 5. Section 5.3.1 If I understand correctly you consider LMCSL with filter to be the main challenger of the ASD. Did you actually check if they perform better than the ASD if you increased the time step for the spatial resolution of 0.5?

Answer: The ASD scheme is unstable for Courant numbers over 0.5. The semi-Lagrangian schemes has been run with increased time step, resolution 1\_05.

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Referee: 6. Section 5.3.2 I also suggest rewriting this section.

Answer: Section rewritten

Referee: 1. Section 2.2.1 I find it valuable to remind the readers the reasons behind the choice of a particular advection scheme for DEHM. But the reader's appetite is not satiated as the authors remind four schemes of Brandt et al. (1996a) but briefly discuss only two of them. I would suggest adding a three-sentence paragraph on the Holm's algorithm at least. Also, a link between those schemes and the scheme finally selected for the current implementation of DEHM is not very clear to the reader.

Answer: "The upwind method is computationally very cheap, but it produces a lot of artificial numerical diffusion, which makes it unsuitable for use in air pollution models.

The Bott and Holm schemes produce smooth concentration fields with no negative concentrations, but they are in general more expensive than the upwind or finite element schemes. The Holm's scheme is the best one among these methods. However, it is also the most the most time-consuming (Brandt et al., 1996a)." Will be added.

Referee: 2. Section 2.3.1 Could the authors remind the reader what  $\sigma$  and  $\dot{\sigma}$  stand for?

Answer: "The  $\sigma$  coordinate is a terrain following coordinate, where the pressure is normalized by the surface pressure."  $\dot{\sigma}$  is the vertical wind speed in this coordinate system.

Referee: 3. I would suggest adding an ordering column to the tables analyzed in the text. In this way you could actually state in the text which entry in a table you mean while describing it. It is not always necessary but in some circumstances it could help the readers.

Answer: We think that the tables are already very extensive, therefore an additional column is not added.

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Referee: 4. Table 1 p 2414 has four identical bottom entries on this page. Is this correct? This also occurs in Table 5 in both p 2422 and 2423 for non-filtered SL schemes.

Answer: Yes

Referee: âĀĀ p 2364 | 24 The authors mean 'implement' and not 'develop', I believe. To my view 'develop' suggests the schemes are proposed in the paper. As mentioned at the bottom of p 2363 it is the case for one of them but not all.

Answer: Implement has been added

Referee: âĀĀ p 2366 | 20 Just 'mixing ratios' and not 'mixing ratios concentrations'

Answer: OK

Referee: âĀĀ p 2367 | 5 Skip 'the' before both

Answer: OK

Referee: p 2370 | 7-8 'longer alkenes lump' are mentioned twice

Answer: Intentionally

Referee: p 2371 | 9 Should be 'is' instead of 'are'

Answer: OK

Referee: âĀĀ p 2379 | 19 I believe the authors mean 'Eulerian' latitudes

Answer: Text altered.

Referee: âĀĀ p 2380 | 25 'should sum to one' appears twice

Answer: removed

Referee: âĀĀ p 2381 | 7 'k-th'

Answer: OK

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Referee: ĀĀĀ p 2381 second line of the formula (21) - skip the hat above  $w_{k,l}$

Answer: OK

Referee: ĀĀĀ p 2382 and 2383 Could you please state clearly what k and K stand for?

Answer: Grid point of interest and total number of grid points, respectively

Referee: ĀĀĀ p 2383 In the formulas (29) and (30) you could maybe replace '[' with " and the first interval with a colon or semi-colon

Answer: The square brackets have been replaced by parentheses.

Referee: ĀĀĀ p 2386 formula (37) Similarly, I would suggest replacing '[' with "

Answer: As above

Referee: ĀĀĀ p 2387 | 7 Looks like you are back to one dimension here. Please, state it. ĀĀĀ

Answer: "In one spatial dimension" is added

Referee: p 2387 - 2388 formula (44) Do you mean here upper- or lower-case c?

Answer: Lower case c

Referee: ĀĀĀ p2389|9 Skip 'In'

Answer: OK

Referee: ĀĀĀ p 2390 | 22-23 grid cells

Answer: OK

Referee: ĀĀĀ p 2391 | 20 Split the sentence after 'Sect.5.1'.

Answer: OK

Referee: ĀĀĀ p 2391 | 23 A comma after 'and last', please.

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Answer: OK

Referee: âĀĀ p 2391 | 26 Remove one of the 'only's.

Answer: OK

Referee: âĀĀ p 2392 | 7-9 Please, rewrite the sentence as: 'semi-Lagrangian schemes are compared to ASD'. IC being common to all of the schemes, there is no need to mention it.

Answer: OK

Referee: âĀĀ p 2392 | 24 Make up your mind id you want to use 'plots' or 'shows'

Answer: shows

Referee: âĀĀ p 2392 | 25 'as a cone', I believe

Answer: yes

Referee: âĀĀ p 2394 | 1 I believe it should be 'what' instead of 'this' and 'In' instead of 'On'. Check with a native speaker.

Answer: On has been replaced by "in"

Referee: âĀĀ p 2393 | 2 'axes' or 'is'; 'in the top plots'

Answer: axes

Referee: âĀĀ p 2393 | 4 Consider splitting into three sentences instead of commas.

Answer: Split into two sentences

Referee: âĀĀ p 2394 paragraph 5.1.3 Could you, please, state clearly the name of the analyzed scheme in agreement with Section 4.1 (and Fig.2).

Answer: LMC cascade is added

Referee: âĀĀ p 2394 paragraph 5.1.4 Same remark as above. Also the way the first

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sentence of the paragraph is written suggests that each of the plots in Fig.3 shows a cone for each of the numerical schemes. In reality it is one numerical scheme combining all those characteristics and, therefore, it would be better to just use the name introduced in Section 4.1.

Answer: LMCSL

Referee: 2394 | 23 'species'

Answer: OK

Referee: 2394 | 24 'From a comparison', I believe Answer: By comparing

Referee: 2394 | 25 I believe the authors want to say 'bottom right plot of Figs. 1-3' Answer: Yes

Referee: 2395 | 2 But the bulk occurred for a different scheme, didn't it.

Answer: Filtered added to sentence and sect. 5.1.4

Referee: 2395 | 5 'than for the other'

Answer: OK

Referee: 2395 | 5 'worse', please

Answer: OK

Referee: 2395 | 8-9 I would propose: 'The maximum value is again closer to that of the initial condition than for the semi-Lagrangian schemes using 1\_1 resolution but not as close as for the ASD', or something of the sort.

Answer: OK

Referee: 2395 | 15-16 'solutions' or 'performs'

Answer: Performs

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Referee: âĀĀ p 2396 | 4 'throughout'

Answer: OK

Referee: âĀĀ p 2396 | 12 Do you mean 'cylinder' or 'cone'?

Answer: Cone

Referee: âĀĀ p 2397 | 5 'smaller' instead of 'less'

Answer: OK

Referee: âĀĀ p 2397 | 13 'scheme' or 'smooth'

Answer: Smooth

Referee: âĀĀ p 2398 | 4 I believe it should be 'The scales in the four plots vary'

Answer: The scale on the z-axis in the four plots varies

Referee: âĀĀ p 2398 | 6 I am not sure how to understand 'and the lowest value of the shown plots as well'

Answer: Lowest minimum value

Referee: âĀĀ p 2399 | 6 I think it should read 'and giving points to the methods relative to their results' or something of that sort

Answer: OK

Referee: âĀĀ p 2399 | 20 Please check it with a native speaker but I think it should be 'the better the performance'

Answer: Absolutely

Referee: âĀĀ p 2399 | 21 I tend to think that an error could be 'smaller' but I would not use 'better' in this context

Answer: OK

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Referee: âĀĀ p 2399 | 22 'than for the'

Answer: OK

Referee: âĀĀ p 2399 | 24 Remove 'The semi-Lagrangian schemes the'

Answer: OK

Referee: âĀĀ p 2399 | 26 Do you really mean '3\_1'? Or '10\_1'?

Answer: 3\_1

Referee: âĀĀ p 2399 | 26 - p 2400 | 1 | simply do not understand how do the results from Table 1 support this statement.

Answer: Courant number less than 1, res 3\_1, or the traditional DEHM resolution,  $\Delta t = 90$ ,  $\Delta x = 1.0$

Referee: âĀĀ p 2400 | 9 To me, 'the second resolution is 10\_1' does not really mean much. It would be enough to insert 'The second best performing resolution' and the sentence would be so much smoother.

Answer: OK

Referee: âĀĀ p 2400 | 7 - 11 You mix the spatial and temporal resolution here which is perturbing. In 5.3.1 you talk about 'coarser time step' which is more informative.

Answer: In the section, references are only made to specific resolutions.

Referee: âĀĀ p 2400 | 18 Maybe 'The second class of the filtered'

Answer: OK

Referee: âĀĀ p 2400 | 20 - 21 Could you please reformulate this sentence and avoid the phrase 'worst resolution'. Also how do you see that the resolution currently used in DEHM gives inferior results to all the other tested spatial and temporal resolutions. Are you only comparing the performance of the ASP schemes? I mean, it is hard to realize

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which entries in Table 2 you actually analyze.

Answer: The results using the resolution used in DEHM give the lowest score, even worse than the non-filtered resolution 1\_05 for most cases. Almost exactly as is the case for the distribution of non-filtered schemes.

Referee: âĀĀ p 2400 | 23 Skip the comma after l1 Answer: OK

Referee: âĀĀ p 2401 | 21 It is not easily understandable what you mean starting from the words 'as well, the filtered ...'

Answer: As well, namely the filtered. . .

Referee: âĀĀ p 2401 | 22 Maybe you could insert 'resolution 05\_05, third according to the l2 error, is the best ...'

Answer: OK

Referee: âĀĀ p 2401 | 25 - 26 Please reformulate the last sentence on this page. Something of the sort: After the filtered high resolution ASD, LMC cascade scores second and the (pure? bare?) cascade comes third.

Answer: After the filtered high resolution ASD, LMC cascade scores second and pure cascade comes third, both of the latter with resolution 1\_05.

Referee: âĀĀ p 2402 | 5 'overall'

Answer: OK

Referee âĀĀ p 2402 | 10-11 It is not clear to me what 'with the filtered schemes first and hefiltered resolution 3\_1'. By the way is it 3\_1 or 10\_1?

Answer: With the filtered schemes first, followed by the unfiltered solutions and the filtered resolution 3\_1.

Referee: âĀĀ p 2408 | 8 Should be 'compares'. I would also start a sentence with an 'A ranking'

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Answer: OK

Referee: âĀĀ p 2408 l 14 I would merge it with the previous paragraph.

Answer: OK

Referee: âĀĀ p 2409 l 16 Should be 'improves'.

Answer: OK

Referee: âĀĀ p 2409 l 25 - 29 There is something missing in this sentence. I would also suggest splitting it into two sentences after 'steps'.

Answer: Added "they are" after namely, and "," after steps.

Referee: âĀĀ p 2410 l 7 'straightforward' âĀĀ p 2410 l 14 Should be 'computational' âĀĀ p 2412 l 23 'Meteorology'

Answer: OK

Please also note the supplement to this comment:

<http://www.geosci-model-dev-discuss.net/3/C892/2011/gmdd-3-C892-2011-supplement.pdf>

Interactive comment on Geosci. Model Dev. Discuss., 3, 2361, 2010.

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