Efficient modeling of sun/shade canopy radiation dynamics explicitly accounting for scattering

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Abstract

The separation of global radiation \((R_g)\) into its direct \((R_b)\) and diffuse constituents \((R_d)\) is important when modeling plant photosynthesis because a high \(R_d:R_g\) ratio has been shown to enhance Gross Primary Production (GPP). To include this effect in vegetation models, the plant canopy must be separated into sunlit and shaded leaves, for example using an explicit 3-dimensional ray tracing model. However, because such models are often too intractable and computationally expensive for theoretical or large scale studies simpler sun-shade approaches are often preferred. A widely used and computationally efficient sun-shade model is a model originally developed by Goudriaan (1977) (GOU), which however does not explicitly account for radiation scattering.

Here we present a new model based on the GOU model, but which in contrast explicitly simulates radiation scattering by sunlit leaves and the absorption of this radiation by the canopy layers above and below (2-stream approach). Compared to the GOU model our model predicts significantly different profiles of scattered radiation that are in better agreement with measured profiles of downwelling diffuse radiation. With respect to these data our model’s performance is equal to a more complex and much slower iterative radiation model while maintaining the simplicity and computational efficiency of the GOU model.

1 Introduction

Realistic estimation of radiation profiles in plant canopies is important in order to correctly simulate processes occurring at the leaf-level, such as photosynthesis and evaporation. The inclusion of both diffuse and direct radiation transfer is important when modeling photosynthesis in plants. For example, an increased ratio of incoming diffuse \((R_{0,d})\) to global radiation \((R_{0,g})\) increases photosynthesis during clear-sky conditions (Spitters, 1986; de Pury and Farquhar, 1997; Alton et al., 2005, 2007; Cai et al., 2009; Mercado et al., 2009). This increase is caused by shaded leaves, which are normally
not light saturated, receiving an increased photon flux thus increasing photosynthesis (Roderick, 2001). Because the sunlit leaves are light saturated the simultaneous decrease in direct radiation on the sunlit leaves will not cause a large enough offset to decrease total canopy photosynthesis.

Existing canopy radiation models differ greatly in the level of detail used to simulate radiation profiles. More detailed models assume non-homogeneous canopies and/or calculate radiation interception by individual trees (e.g. Charles-Edwards and Thornley, 1973; Mann et al., 1979; Chen et al., 1994; North, 1996; Bartelink, 1998). These models are however complex and cannot be solved analytically and are therefore not sufficiently tractable for use in theoretical studies (e.g. optimization studies; Franklin, 2007). Perhaps the most important limitation of the complex models are their computational demands. For example, the new generation of large scale vegetation models include height structured competition for light as one of the most important processes shaping plant communities (Albani et al., 2006; Scheiter and Higgins, 2009). Because this process results in dynamic interactions between many plants within the canopy, the resulting computational demands are too high to allow the use of complex iterative or other computationally slow radiation interception models.

A relatively simple sun/shade model which can be used in global vegetation models was originally developed by Goudriaan (1977; p. 13–40) (GOU) and later implemented by Spitters (1986). This model has been used in several studies (e.g. Anten, 1997; dePury and Farquhar, 1997; Wang and Leuning, 1998; Wang, 2003). However, a potential disadvantage of this model compared to similar but more computationally demanding models (e.g. Norman, 1979, 1980; Sellers, 1985) is that it does not explicitly account for the scattering of direct radiation, which leads to an erroneous distribution of scattered radiation in the canopy.

In this study our aim is to derive a canopy radiation model with the simplicity and calculation speed of the GOU model combined with a more realistic representation of the scattering process. Taking the GOU model as a starting point we extend it by explicitly accounting for upstream and downstream fluxes of scattered radiation. Importantly, in
contrast to previous comparable models (e.g. Norman, 1979, 1980; Sellers, 1985) we do this without sacrificing the analytical solvability. The model is then tested against a measured profile of downwelling scattered radiation previously used to validate the model by Norman (1979, 1980) in a study by Baldocchi et al. (1985).

2 Materials and methods

In this study we used the sun/shade model originally developed by Goudriaan (1977, p. 13–40) (GOU) as our starting point. This model has previously been described in detail (Spitters, 1986; Anten, 1997; de Pury and Farquhar, 1997; Wang and Leuning, 1998; Wang, 2003). The difference between our model (BF) and the GOU model lie only in the treatment of scattered radiation but for clarity a short description of the full GOU model is given below. The fluxes included in both models are explained in Fig. 1.

2.1 The GOU model

The radiation profile of diffuse radiation \( R_d \) is calculated as:

\[
R_d(L) = R_{0,d}(1 - \rho) e^{-k_d L}
\]  

In Eq. (1) \( R_{0,d} \) is incoming diffuse radiation, \( \rho \) is canopy reflectance, \( k_d \) is the extinction for diffuse radiation and \( L \) is cumulative (single sided) Leaf Area Index (LAI). Canopy reflectance is a function of solar elevation \( \beta \) and leaf scattering \( \sigma \) (see Spitters, 1986). Leaf scattering \( \sigma \) includes transmittance \( t \) and reflectance \( r \), which in the GOU model are assumed to be equal. The extinction coefficient for diffuse radiation \( (k_d) \) can be calculated as (Spitters, 1986):

\[
k_d = 0.8 \sqrt{1 - \sigma}
\]  

The direct radiation on sunlit leaves is assumed to be equal at all canopy depths but with the fraction of sunlit leaves \( (A_{sl}) \) decreasing with canopy depth:

\[
A_{sl}(L) = e^{-k_b L}
\]
In this study we assume a spherical leaf angle distribution, which leads to a radiation extinction coefficient for black leaves ($k_b$):

$$k_b = \frac{0.5}{\sin \beta}$$  \hspace{1cm} (4)

In the GOU model, the profile of scattered radiation ($R_{sc}$) is calculated as the difference between “total direct radiation” ($R_{b,\text{tot}}$) and beam radiation. $R_{b,\text{tot}}$ includes beam radiation and scattered radiation generated by the reflectance and transmittance of beam radiation.

At each canopy depth, $R_{sc}$ is calculated as the difference between “total direct radiation” ($R_{b,\text{tot}}$) and beam radiation ($R_{b,b}$) (Spitters, 1986):

$$R_{sc}(L) = R_{b,\text{tot}}(L) - R_{b,b}(L) = R_{0,b}(1 - \rho)e^{-\sqrt{1 - \sigma}k_bL} - R_{0,b}(1 - \sigma)e^{-k_bL}$$  \hspace{1cm} (5)

### 2.2 The BF model

In more complex radiative transfer models, scattered radiation is treated as two separate streams: one upward stream generated by direct radiation being reflected by leaves, and one downward stream generated by transmittance of beam radiation through leaves. The BF model is formed by replacing the original implicit treatment of scattering in the GOU by an explicit two stream approach for scattered beam radiation.

Scattering of direct (beam) radiation gives rise to upstream (reflection) and downstream (transmission) fluxes of diffuse radiation. The fraction of incoming direct radiation that is transmitted at canopy depth $\xi$ can be calculated by multiplying Eq. (3) with transmittance. This transmitted radiation is then treated as downwelling diffuse radiation. The fraction of transmitted radiation formed at $\xi$ remaining at canopy depth $L$ (which is below $\xi$) is:

$$f_t(L) = e^{-k_d(L-\xi)}$$  \hspace{1cm} (6)
Using Eqs. (3) and (6) the fraction of incoming direct radiation that has been transmitted at canopy depth $\xi$ and that is available at canopy depth $L$ can be calculated as:

$$R_t(L) = tA_{sl}(L)f_t(L) = te^{-k_b\xi}e^{-k_d(L-\xi)}$$

(7)

The downwelling scattered radiation at canopy depth $L$ can be calculated by integrating Eq. (7) over all canopy depths between the canopy top and canopy depth $L$:

$$R_{sc\downarrow}(L) = R_{0,b}t \int_{0}^{L} e^{-k_b\xi}e^{-k_d(L-\xi)}d\xi = \frac{te^{-k_bL} - e^{-k_dL}}{k_d - k_b}$$

(8)

Following the same rationale the upward stream of scattered radiation becomes:

$$R_{sc\uparrow}(L) = R_{0,b}r \int_{L}^{L_{tot}} e^{-k_b\xi}e^{-k_d(\xi-L)}d\xi = \frac{re^{-k_bL} - e^{k_dL-(k_b+k_d)L_{tot}}}{k_d + k_b}$$

(9)

The GOU model can now be converted into the BF model by replacing the equation for scattered radiation (Eq. 5) in the GOU model by the sum of Eqs. (8) and (9):

$$R_{sc}(L) = R_{0,b}\left[te^{-k_bL} - e^{-k_dL} + \frac{re^{-k_bL} - e^{k_dL-(k_b+k_d)L_{tot}}}{k_d + k_b}\right]$$

(10)

In addition to the upstream flux generated by leaf reflectance, an upward flux generated by surface reflectance can be added.

$$R_{st}(L) = W[R_{0,b}A_{sl}(L_{tot}) + R_d(L_{tot}) + R_{sc\downarrow}(L_{tot})]e^{-k_d(L_{tot}-L)}$$

(11)

where $W$ is surface albedo.

### 2.3 Model testing

To test a radiation scattering model against measured data requires measured radiation profiles where scattered and direct components are separable. Because such data are scarce modelers often use more detailed and more complex models as a benchmark.
instead of measured data. In this study we use a measured profile of downwelling scattered radiation (Baldocchi et al., 1985) which has previously been used to validate the model by Norman (1979, 1980). Using the same parameter values as reported in the study by Baldocchi (1985) we compare our model results against both a measured profile and a more complex iterative model by Norman (1979, 1980) as well as the GOU model.

In the study by Baldocchi et al. (1985) radiation was measured within an oak-hickory stand located near Oak Ridge, TN, USA (35°57′ N 84°17′ W) using sensors mounted on trams that traversed 30 m transects. They calculated a daily mean profile of downwelling diffuse radiation based on hourly-averaged data between 08:00 and 17:00 EST for Julian day 273, 1981.

Hourly downwelling diffuse radiation was modeled (on the half-hour) as the sum of Eqs. (1) and (8) for the BF model and Eqs. (1) and (5) for the GOU model using the same value for $t$ and the fraction of diffuse to global radiation ($f_{Dif}$) as reported in Baldocchi et al. (1985) (0.22 and 0.17 respectively) with the value of $\beta$ calculated using a three-dimensional geometry (Appendix A). In the GOU model $\sigma$ was set to equal $t$ in order to only account for downwelling scattered radiation.

The individual profiles of scattered radiation for the two models was compared (Eq. 5 for GOU and Eqs. 8 and 9 for BF) using the same values for $\beta$, $f_{Dif}$ and $t$ as above. For the BF model leaf reflectance ($r$) was set to 0.30 (Fig. 5: Baldocchi et al., 1985) and in the GOU model $\sigma$ to the sum of $r$ and $t$.

The modeled radiation profiles were then compared against both the measured (Baldocchi et al., 1985) and the modeled profile using the Norman (1979, 1980) model.

Modeling Efficiency (ME: Appendix B) of the simulated profiles compared to measurements was also calculated.
3 Results and discussion

3.1 Model differences in the radiation profile

The difference between the GOU and BF models lies in the treatment of $R_{sc}$, where the GOU model treats $R_{sc}$ implicitly as the difference between $R_{b,\text{tot}}$ and $R_{b,b}$. Contrastingly, the BF model calculates $R_{sc}$ explicitly and thus accounts for all processes included in the radiation interception model (Fig. 1). The profiles of total scattered radiation ($R_{sc}$) also differ markedly between the models (Fig. 2) at the top of the canopy. The shapes of the curves for upward and downward scattered radiation within the BF model also differ. Scattered radiation in the GOU model follows an exponential extinction in line with Lambert-Beer’s law. In the BF model the shape of the curve for upward scattered radiation ($R_{sc}^{\uparrow}$) follows a similar shape whereas downwelling scattered radiation ($R_{sc}^{\downarrow}$) is 0 at the canopy top and increases to a maximum at $L \sim 1.5$ followed by a shallow decline. The hump shaped profile for scattered radiation in the BF model follows from the competing effects of the accumulation of generated scattered radiation and the cumulative absorption of this radiation down the canopy.

3.2 Comparison with a measured radiation profile

Tested against measurements of downwelling diffuse radiation our model (BF model) performs significantly better than the original GOU model (Fig. 3) with ME = 0.46 compared to −2.45 for the GOU model (excluding the trivial point $L = 0$). Notably, the radiation profile for downwelling diffuse radiation using the BF model is almost identical to the one using the more complex Norman model (Norman, 1979, 1980). The overestimation of downwelling radiation near the top of the canopy found in the Norman model thus can also be seen in the BF model.
3.3 Implications for canopy radiation modelling

Given the prominent role of light in shaping vegetation through responses at scales from leaf physiology to community dynamics, an accurate representation of canopy light absorption is important in vegetation modelling. Not only does light absorption limit photosynthesis but it also controls leaf N concentration (Franklin and Ågren 2002) and water use (Buckley et al., 2002) with implications for ecosystem resource balances. It is therefore not surprising that much effort has been put into construction of realistic canopy radiation models. Because realism often comes at a computational cost and loss of tractability there is also a large demand for simple and computationally efficient models, such as the widely used Goudriaan model (Spitters, 1986; Anten, 1997; dePury and Farquhar, 1997; Wang and Leuning, 1998; Wang, 2003). However, it is notable that this model still represents state of the art in this field despite its shortcomings.

Our addition of an explicit treatment of radiation scattering to the Goudriaan approach significantly and qualitatively changed the shape of the modelled scattered radiation profile compared to the original model. The predictions of the new BF model were in better agreement with measured canopy radiation profiles of diffuse downwelling radiation than the GOU model. Furthermore, our model compares to the more complex and computationally intensive Norman model (Norman, 1979, 1980) while maintaining the high computational efficiency of the GOU model.

Given the importance of canopy radiation modeling, our qualitative as well as quantitative improvement of a tractable analytical canopy radiation model can be used to improve a wide range of plant, vegetation and ecosystem models. For example, the new generation of large scale vegetation models include height structured competition for light as one of the most important processes shaping plant communities but do not explicitly account for radiation scattering (e.g. Smith et al., 2001; Albani et al., 2006; Scheiter and Higgins, 2009). Such vegetation models could readily be improved at very low computational cost by adding a simple canopy scattering approach, such as the BF model.
Appendix A

Solar elevation

Solar elevation changes diurnally and seasonally as well as with latitude. It was calculated as (cf. de Pury and Farquhar, 1997):

\[
\sin \beta = \sin \psi \sin \delta + \cos \psi \cos \delta \cosh
\]

where \( \psi \) is latitude, \( \delta \) is the declination angle, and \( h \) is hour angle (all in radians). Declination angle was calculated as:

\[
\delta = \frac{\pi}{180}23.45 \cdot \sin \left[ \frac{2\pi}{365} (284 + d) \right]
\]

where \( d \) is day of the year.

Appendix B

Modeling efficiency

Modeling Efficiency (ME) was calculated as (Janssen and Heuberger, 1995):

\[
ME = \frac{\sum_{i=1}^{n} (O_i - \bar{O})^2 - \sum_{i=1}^{n} (P_i - O_i)^2}{\sum_{i=1}^{n} (O_i - \bar{O})^2}
\]

where \( n \) is number of data, \( O_i \) is observed value, \( \bar{O} \) is average observed value, and \( P_i \) is predicted value.
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References


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Fig. 1. Diagram of the radiation interception in the two different models: GOU (Goudriaan, 1977) and our new model (BF). Both the GOU (a) and BF model (b) simulate the fraction of sunlit leaves (white areas) to shaded leaves (grey areas) in the same way according to Lambert-Beer’s law. The absorption of incoming diffuse radiation (broken grey arrows) is also simulated the same way with incoming diffuse radiation being absorbed by both sunlit and shaded leaves also following Lambert-Beer’s law. The difference between the models lies in the fluxes of scattered radiation, which in the BF model are generated by the transmittance and reflectance of beam radiation (black arrows) at all levels of the canopy. The GOU model calculates the total interception of incoming direct radiation (broken black arrow) as an aggregated flux that includes both the interception of beam radiation and the scattered radiation generated by the reflectance and transmittance of beam radiation. Scattered radiation is then calculated at each level as the difference between total direct radiation (black arrow) and beam radiation (based on the area of sunlit leaves). In the BF model, scattered radiation is generated by beam radiation (black arrow) being intercepted by sunlit leaves and scattered into one up- and one downstream (solid grey arrows) of scattered (diffuse) radiation.
Fig. 2. The modeled profiles of scattered radiation ($R_{sc}$) relative incoming beam radiation ($R_{0,b}$) for the respective models. For the BF model scattering is divided into upwelling ($BF_{sc}^{↑}$: dashed line) and downwelling scattered radiation ($BF_{sc}^{↓}$: thin solid line). For the GOU model total scattered radiation is simulated ($GOU_{sc}$: dotted/dashed line) using scattering equal to the sum of transmittance ($t=0.22$) and reflectance ($r=0.30$). For the BF model scattered radiation ($BF_{sc}$: thick solid line) is the sum of $BF_{sc}^{↓}$ and $BF_{sc}^{↑}$.
Fig. 3. Modeled downwelling diffuse radiation simulated using the BF model (solid line) and GOU model (dashed line). Leaf transmittance $t = 0.22$ and fraction of diffuse to global radiation $f_{Dif} = 0.17$. Measured data (open triangles) and model results using the Norman model (dotted) are extracted from Baldocchi et al. (Fig. 5, 1985). Model efficiency, $ME = 0.46$ and $-2.45$ for the BF and GOU model, respectively.