To the Journal “GEOSCIENTIFIC MODEL DEVELOPMENT”

Attached is the revised manuscript of our paper by Gavrilov N. M., S. P. Kshevetskii, A. V Koval “Verifications of the high-resolution numerical model and polarization relations of atmospheric acoustic-gravity waves”, which we prepared for publishing in the GMD journal. We made corrections according to all reviewers’ comments (see attached detailed answers to the reviewers).

We also want to add a new co-author – Andrej V. Koval. He participated in computer simulations and preparation of revised text of the paper. Please, consider the possibility of inclusion Koval A. V. into the list of authors of the paper.

Best regards.
N. M. Gavrilov, S. P. Kshevetskii, A. V. Koval.

Reply to the Interactive comment on “Verifications of the high-resolution nonlinear numerical model and polarization relations of atmospheric acoustic-gravity waves” by N. M. Gavrilov and S. P. Kshevetskii
Anonymous Referee #1
Received and published: 19 January 2015

We would like to thank the reviewer #1 for useful comments, which help to improve our paper. Our replies to his comments are given in bold font below.

The main motivation of the discussion paper “Verifications of the nonlinear numerical model and polarization relations of atmospheric acoustic-gravity waves” by N. M. Gavrilov and S. P. Kshevetskii is to perform comparisons of amplitudes of atmospheric gravity waves (AGWs) and related characteristics of direct numerical simulations (DNS) with linear polarization relationships (LPR) given by the steady-state theory of nonrotational non-dissipative AGWs. Authors considered to use their DNS results for monochromatic sources of AGWs as a potential tool for testing and verifications of LPR used in the simplified parameterizations of AGWs. The abstract provides a concise and complete summary of the paper. As suggested, DNS of AGWs may be useful tools for testing of simplified GW parameterizations employed in the climate and weather models. The nonlinear breaking of AGWs by DNS and limitations for applications of LPR during wave breaking, however, are not examined in the article.

Wave breaking violates applicability of LPR due to two main reasons. First, wave breaking leads to generation of a spectrum of secondary wave modes instead of one main mode in stable region. Parameters of these secondary modes change in time, which make difficult estimations of their LPR. Second, breaking AGW generate wave induced mean flow very fast growing in time. This also makes difficult LPR estimations, as far as all analytical AGW theories assume background fields slowly varying in time. We added such statements into the revised version of the paper.

As discussed in the paper, comparisons between numerical simulations and analytical expressions for AGW parameters, “reveal atmospheric regions, where analytical theories give substantial errors”. Modeling protocol of results is summarized in two tables for two selected AGW modes. It is suitable approach for addressing difference between the DNS and analytical solutions of the linear theory of AGWs. Authors emphasized that the DNS are required to accurately represent the transient wave fields after "switching on" the monochromatic wave source at the surface. However, for the steady state wave regimes, the representation of the DNS and analytical solutions by illustrations for vertical profiles of the energy, heat and momentum fluxes can enhance the presentation of results and conclusions, when and where LPRs are not valid for variable intensity of the AGW amplitude at the source level.

From analytical AGW theory, we can get local linear polarization relationships, but not vertical profiles of wave characteristics for dissipative atmosphere and realistic temperature profiles.
Therefore, direct comparisons of DNS numerical solutions with “analytical” profiles in wide altitude regions are problematic. Many vertical profiles of AGW characteristics at different times and their changes at different altitudes calculated with the DNS model readers may find in our paper by N. M. Gavrilov and S. P. Kshevetskii. “Dynamical and thermal effects of nonsteady nonlinear acoustic-gravity waves propagating from tropospheric sources to the upper atmosphere”, which is already published in Advances in Space Research (2015).

Authors provided proper credit to related work and reference on their previous modeling studies. The title of the technical report reflects content of the paper. However, stress on the nonlinear aspects of numerical model results (that present in the title) is not fully discussed in the paper. The text and tables are more solicited on the initial transience of the quasi-linear wave packets.

We replaced the word “nonlinear” by “high-resolution” in the title of revised manuscript.

There are no discussions on the impact of strength of wave sources on the development of the nonlinearity and transience of simulated AGWs.

We added some discussion of the impact of strength of wave sources.

Perhaps authors can make explicit statements that they considered (a) the quasi-linear DNS of monochromatic dissipative waves to verify their model by the LPR valid for non-dissipative AGWs, and (b) explain their motivation to evaluate the spin-up of transient (for t < te) model results by the steady-state LPR that can be applied for t > te. It appears, the analytical transient linear wave solutions in the windless isothermal non-dissipative background atmosphere can be more appropriate analytical approach to verify transient propagation of the broad spectra of linear AGW forced in the DNS by localized sources.

We agree, “analytical transient linear wave solutions” can be more appropriate analytical approach to verify transient AGW propagation”. However, recently we do not know such analytical solutions. May be the reviewer can help to find them. Our results show that such transient linear wave solutions are strongly needed. We hope, such AGW theories will be developed in the future. We added respective statement into the text of paper.

The upper part of tables that summarizes comparisons between DNS and the analytical steady-state (t > te) LPRs is the most appropriate for verification of DNS results for the quasi-linear monochromatic waves, while for DNS verifications at t < te (second part of tables) it would be difficult to rely on the validity of LPR for single steady-state wave without considerations of the analytical solutions for transient waves.

Our results are not only for DNS verification, but also for verification of LPRs themselves. One of the results of our paper is poor validity of steady state LPR in transient conditions. However, in many cases such steady-state LPRs are used for parameterizations of transient AGW effects. We hope, our results will help in developing more realistic parameterizations.

The discussions paper of N. M. Gavrilov and S. P. Kshevetskii represents a substantial contribution to modeling science of AGWs, and it is definitely within the scope of Geoscientific Model Development Journal.

Specific comments.
(a) Abstract. “Reasonable agreements between simulated and analytical wave parameters satisfying the scope the limitations of the AGW theory proof adequacy of the used nonlinear numerical model”. Sentence needs additional clarification.

We changed the word “nonlinear’ to “wave”.
The modeling was performed beginning from the MSIS initial state (zero wave fields)... It is worthy to mention the windless background flow.

The phrase is modified.

Therefore, waves with longer vertical wavelengths can better penetrate to the upper atmosphere, where they can produce larger dynamical and thermal effects than those with shorter vertical wavelengths (see Gavrilov and Kshevetskii, 2014b). This sentence requires more clarification, because indeed AGWs with larger vertical wavelength can faster propagate from the surface to the upper layers but they also subject less effective dissipation and nonlinear breaking in the thermosphere. DNS results can also depend on the non-zero background flow.

The phrase is modified in the revised manuscript.

Yours Sincerely.
N. M. Gavrilov, S. P. Kshevetskii, A. V. Koval

Reply to the “Interactive comment on “Verifications of the nonlinear numerical model and polarization relations of atmospheric acoustic-gravity waves” by N. M. Gavrilov and S. P. Kshevetskii, Anonymous Referee #2”

We would like to thank the Referee #2 for useful comments helping us to improve the paper. Our replies are given below with bold font.

The paper is devoted to the comparisons of atmospheric wave parameters calculated using Direct Numerical Simulation (DNS) model with polarization relations (PR) given by analytical theory of linear acoustic-gravity waves (AGWs). Such comparisons can be used to test as the numerical models as the PR themselves. There are no such direct comparisons of analytical PRs with wave DNS models in the scientific literature. Therefore, the paper presents new and important results, which are to be useful for verifications of atmospheric wave DNS models and to improve the parameterizations of AGW effects in atmospheric dynamical models. The paper is within the scope of the GMD journal. The title and abstract reflect the contents of the paper. The overall presentation is clear and adequately structured. Therefore, the paper can be recommended for publication in the GMD after some minor revision.

Specific comments:

P7809, L7 “… of the continuity, momentum, and heat balance”.
**The revised text is corrected.**
P7809, L20-21. “… laws of the momentum, mass, and energy”.
**Corrected.**
P7809, L20-25. It would be helpful to extend the description of the numerical algorithm and give some explanations concerning the difference between the standard Lax and Wendroff scheme and suggested modifications.
**The extended description is added to the revised text.**
P7810, L2. Please explain, why you need so small vertical grid spacing (12 m) near the ground.
**Small vertical grid spacing in the boundary layer are needed because of high gradients of velocity their. We added this description into the revised text.**
P7810, L2. “… at altitudes of about 500 km…”
**Corrected.**
P7811, L6. “… sound speed…”
Corrected.
P7811, two last equations (5). What denotes alpha?

“\( \alpha = 1/(2H) \);” is added

P7812, equation (7). “… \((UW^*)/2\), where “*” denotes the complex conjugate value.

Corrected.
P7812, L10. Reformulate the phrase in the middle “To make simulations matching to the linear AGW theory (see Eq. (2)), …”

The phrase is reformulated.
P7813, L3. “very high altitudes”.

Corrected.
P7814, First sentence. “These intervals grow …”.

Corrected.
P7814, L5. “The Table contains simulated SDs at each altitude averaged over n model outputs during …”

The phrase is modified in the revised paper.
P7815, L15. Why you did not compare DNS model and linear PR above 100 km? If such comparisons were made, it would be helpful to give respective description.

Above altitude 100 km we found big disagreement? Because linear AGW theory does not involve molecular dissipation. We added this description into the revised paper.

We added a short description and the citation.
P7815, Last para. “… momentum fluxes \( F_{mz} \) given by Eq. (7) and…”

The phrase is modified in the revised text.
P7821. Tables 1, 2. The authors used steady-state analytical PR for comparisons during transience time intervals. Why the authors suppose validity of the steady state PR in this case?

We do not suppose, but we checked the validity of steady state PRs during transience time intervals, because such PRs are applied to non-steady AGWs in many studies. Our comparisons show bad agreement between steady-state PRs and non-stationary numerical simulations, which show need for developments of theories of nonstationary AGWs. We added this statement to the revised text of the paper.

English in the manuscript needs improvements.

English in the revised text was corrected by a professional expert.

Yours sincerely,

Nikolai M. Gavrilov, Sergey P. Kshevetskii, Andrej V. Koval
Verifications of the nonlinear high-resolution numerical model and polarization relations of atmospheric acoustic-gravity waves

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Abstract

Comparisons of amplitudes of wave variations of atmospheric characteristics obtained using direct numerical simulation models with polarization relations given by conventional theories of linear acoustic-gravity waves (AGWs) could be helpful for testing these numerical models. In this study, we performed high-resolution numerical simulations of nonlinear AGW propagation at altitudes 0 – 500 km from a plane wave forcing at the Earth’s surface and compared them with analytical polarization relations of linear AGW theory. After some transition time \( t_e \) (increasing with altitude) subsequent to triggering the wave source, initial wave pulse disappear and the main spectral components of the wave source dominate. The numbers of numerically simulated and analytical pairs of AGW parameters, which are equal with confidence 95%, are largest at altitudes 30 - 60 km at \( t > t_e \). At low and high altitudes and at \( t < t_e \), numbers of equal pairs are smaller, because of influence of the lower boundary conditions, strong dissipation and AGW transience making substantial inclinations from conditions, assumed in conventional theories of linear nondissipative stationary AGWs in the free atmosphere. Reasonable agreements between simulated and analytical wave parameters satisfying the scope the limitations of the AGW theory proof adequacy of the used nonlinear wave numerical model. Significant differences between numerical and analytical AGW parameters reveal circumstances, when analytical theories give substantial errors and
numerical simulations of wave fields are required. In addition, direct numerical AGW
simulations may be useful tools for testing simplified parameterizations of wave effects in the
atmosphere.

1 Introduction

Observations show frequent presence of acoustic-gravity waves (AGWs) generating at
tropospheric heights and propagating to the middle and upper atmosphere (e.g., Fritts and
Alexander, 2003). These AGWs can break and produce turbulence and perturbations in the
atmosphere. For example, sources of AGWs could be mesoscale turbulence and convection in
the troposphere (e.g., Fritts and Alexander, 2003; Fritts et al., 2006). Turbulent AGW
generation may have maxima at altitudes 9–12 km in the regions of tropospheric jet streams
(Medvedev and Gavrilov, 1995; Gavrilov and Fukao, 1999; Gavrilov, 2007).

Non-hydrostatic models are useful for direct numerical simulations of wave and turbulence in
the atmosphere. For example, Baker and Schubert (2000) simulated nonlinear AGWs in the
atmosphere of Venus. They modeled waves in the atmospheric region having horizontal and
vertical dimensions of 120 and 48 km, respectively. Fritts and Garten (1996), also Andreassen
et al. (1998) and Fritts et al. (2009, 2011) simulated instabilities of Kelvin-Helmholtz and
turbulence produced by breaking atmospheric waves. These models simulate turbulence and
waves in atmospheric regions with limited vertical and horizontal dimensions. The models
exploited spectral methods and Galerkin-type series for converting partial differential
equations (versus time) into the ordinary differential equations for the spectral series
components. Yu and Hickey (2007) and Liu et al. (2008) developed two-dimensional
numerical models of atmospheric AGWs.

Gavrilov and Kshevetskii (2013) developed a two-dimensional model for high-resolution
numerical simulating nonlinear AGWs using a finite-difference scheme taking into account
hydrodynamic conservation laws as described by Kshevetskii and Gavrilov (2005). This
scheme increases the stability of numerical scheme and allows us obtaining non-smooth
solutions of nonlinear wave equations. This permitted getting generalized physically
acceptable solutions to the equations (Lax, 1957; Richtmayer and Morton, 1967). Gavrilov
and Kshevetskii (2014a) created a three-dimensional version of this algorithm for simulating
nonlinear AGWs in the atmosphere. They modeled waves produced by sinusoidal horizontally
homogeneous wave forcing at the Earth’s surface.
Karpov and Kshevetskii (2014) used similar numerical three-dimensional model to study AGW propagation from local non-stationary wave excitation at the Earth’s surface. They showed that infrasound going from tropospheric sources could provide substantial mean heating in the upper atmosphere. Dissipating nonlinear AGWs can also create accelerations of the mean flows in the middle atmosphere (e.g., Fritts and Alexander, 2003). However, details of the mean heating and mean flows created by non-stationary nonlinear AGWs in the atmosphere need further studies.

Numerical models of atmospheric AGWs require verifications. For plane stationary wave components with small amplitudes conventional linear theories (e.g., Gossard and Hooke, 1975) give the dispersion equation and polarization relations, which connect wave frequency, vertical and horizontal wave numbers and ratios of amplitudes of different wave field variations. One can expect that such relations could exist between corresponding parameters of the numerical model solutions. Therefore, theoretical polarization relations could be useful for verifications of direct simulation models of atmospheric AGWs.

In this paper, using the high-resolution numerical three-dimensional model by Gavrilov and Kshevetskii (2014a,b), we made comparisons of calculated ratios of amplitudes of different wave fields with polarization relations given by the conventional linear AGW theory. We considered simple AGW forcing by plane wave oscillations of vertical velocity at the surface, which is similar to the assumptions made in analytical wave theory. We found height regions of the atmosphere, where numerical results agree with analytical ones, and regions of their substantial disagreement.

Theoretical dispersion equation and polarization relations are widely used for developing simplified parameterizations of AGW dynamical and thermal effects in the general circulation models of the middle atmosphere. Therefore, comparisons of numerically modeled and analytical polarization relations are useful for both verifications of numerical models, and obtaining limits of analytical relation applicability and for verifications of AGW parameterizations.

### 2 Numerical model.

The three-dimensional numerical AGW model calculates velocity components $u$, $v$, and $w$ along horizontal ($x$, $y$) and vertical, $z$, axes, respectively. The model also calculates departures of pressure $p'$, temperature $T'$, and density $\rho'$ from background hydrostatic stationary fields.
\[ \rho_0, T_0 \text{ and } \rho_s, \text{ respectively. Gavrilov and Kshevetskii (2014a) described the set of} \]

hydrodynamic nonlinear equations used in the model. The set includes equations of
continuity, momentum and heat balance. At the upper boundary \( z = 500 \text{ km} \), the
conditions involve zero vertical gradients of perturbations of temperature, pressure, density
and horizontal velocity, also zero vertical velocity. At the Earth’s surface, the lower boundary
conditions consist of zero perturbations of temperature, pressure, density and horizontal
velocity (see Gavrilov and Kshevetskii, 2013; 2014a,b). In this study, we assume horizontal
periodicity of wave solutions:

\[ r(x, y, z, t) = r(x + L_x, y + L_y, z, t), \quad (1) \]

where \( r \) denotes any of the calculated variables, and \( L_x = m \lambda_x, L_y = n \lambda_y \) are the horizontal
dimensions of the considered atmospheric region, \( m \) and \( n \) are integer constants, \( \lambda_x \) and \( \lambda_y \)
are wavelengths along horizontal axes \( x \) and \( y \), respectively. Variations of vertical velocity \( w_0 \)
= \( w(x, y) \) at the ground \( z = 0 \) generate AGWs in the model.

The used numerical scheme is analogous to the two-dimensional algorithm described by
Kshevetskii and Gavrilov (2005). It is a modification of the method by Lax and Wendroff
(1960). This algorithm involves the conservation laws of momentum, density, mass
and energy. The main difference of our scheme from the classical Lax and Wendroff (1960)
algorithm is the implicit approximating equations of hydrodynamic at first half step in time,
which diminish errors of description of acoustic waves (Kshevetskii, 2001a, b, c). In the
model we utilize a staggered grid, in which temperature, density and pressure are specified at
the same nodes, but mesh points for the components of velocity \( u, v, w \) are displaced half grid
step along axes \( x, y, z \), respectively.

We use numerical scheme similar to the two-dimensional algorithm developed by
Kshevetskii and Gavrilov (2005). Used hydrodynamic equations (see Gavrilov and
Kshevetskii, 2013b, 2014a) can be presented in the conservation law forms

\[ \frac{\partial s}{\partial t} + \frac{\partial X(s)}{\partial x} + \frac{\partial Y(s)}{\partial y} + \frac{\partial Z(s)}{\partial z} = 0, \quad (2) \]

where \( s \) represents any of momentum, energy or mass per unit volume, \( X, Y, Z \) denote
components of respective quantity fluxes along axes \( x, y, z \). Additionally to the model by
Kshevetskii and Gavrilov (2005), the current energy balance equation contains terms
describing heating caused by viscosity. Used numerical method exploits the Lax and
Wendroff (1960) scheme, which approximates Eq. (2) with the second-order finite-difference analog

\[ \frac{s_{n+1}^{x} - s_{n}^{x}}{\Delta t} + \frac{X_{n+1/2}^{x} - X_{n-1/2}^{x}}{\Delta x} + \frac{Y_{n+1/2}^{y} - Y_{n-1/2}^{y}}{\Delta y} + \frac{Z_{n+1/2}^{z} - Z_{n-1/2}^{z}}{\Delta z} = 0, \]

where \( n, i, j, k \) and \( \Delta t, \Delta x, \Delta y, \Delta z \) are the grid node numbers and grid spacing in \( t, x, y, z \), respectively. This algorithm gives possibilities to select physically appropriate solutions of the equations (Lax, 1957; Richtmayer and Morton, 1967). It keeps the numerical scheme stability and allows us consideration of non-smooth solutions of nonlinear AGW equations. In addition we exploit a staggered grid, where temperature, pressure and density are specified at the same nodes, but for the velocity components \( u, v, w \) the mesh points are half grid spacing shifted along axes \( x, y, z \), respectively. To compute \( s_{n+1/2}^{x} \) at the first time half step we apply the implicit equation

\[ \frac{s_{n+1}^{x} - s_{n}^{x}}{\Delta t} + \frac{X_{n+1/2}^{x} - X_{n-1/2}^{x}}{\Delta x} + \frac{Y_{n+1/2}^{y} - Y_{n-1/2}^{y}}{\Delta y} + \frac{Z_{n+1/2}^{z} - Z_{n-1/2}^{z}}{\Delta z} = 0, \]

This substantially complicates simulations, but Kshevetskii (2001a, b, c) found that such structures of finite-difference schemes do not accumulate errors caused by acoustic waves.

In this study, we employ vertical profiles of background \( T_0, \rho_0, \) and \( p_0 \) given by the model of standard atmosphere MSIS-90 (Hedin, 1991) for average geomagnetic activity in January. The average spacing of height grid is about 170 m, but it is varying from 12 m near the ground (because of high gradients in the boundary layer) to about 1.2 km at altitudes of about 500 km depending on inhomogeneities of vertical temperature profiles. The horizontal grids spacing is 1/60 of horizontal wavelengths taken in the wave source Eq. (2). Time spacing is automatically determined to guarantee stability of the numerical algorithm and is equal to 0.14 s and 0.24 s for analyzed in this study AGWs having period \( \tau = 2 \times 10^3 \) s and horizontal phase speeds 30 m/s and 100 m/s, respectively.

The numerical model involves kinematic molecular heat conductivity and viscosity increasing versus altitude inversely proportional to the background density. We also include background turbulent heat conductivity and viscosity taking their vertical profiles with the maxima of 10 m²/s near the ground and at altitude of 100 km and the minimum of ~ 0.1 m²/s in the stratosphere. The model does not include some effects, for example, wave dissipation caused by ion drag and radiative heat exchange, which are less important for modeling high-frequency AGWs.
3. AGW polarization relations.

The comparisons considered in this paper used relations obtained from a theoretical model of monochromatic AGWs in the plain rotating atmosphere. 

Conventional theories suppose that wave components $v', p', \rho'$, and $T'$ are small deviations from stationary background values $v_0, p_0, \rho_0$, and $T_0$. In agreement with Hines (1960), Beer (1974), and Matsuno and Shimazaki (1981), we can look for solutions to atmospheric wave equations for AGW spectral components in the following form

$$
\frac{u'}{U} = \frac{v'}{V} = \frac{w'}{W} = \frac{p'}{p_0P} = \frac{\rho'}{\rho_0R} = \frac{T'}{T_0T} \sqrt{\frac{p_0}{p_0}} e^{i(\sigma t - \phi)}, \quad \phi = -kx - mz,
$$

where $p_0$ is the surface pressure; axis $x$ is directed along horizontal wave phase velocity; $\sigma, k, m$ are frequency, horizontal and vertical wave numbers; $U, V, W, P, R, T$ are complex amplitudes of respective values. Assuming homogeneity of $v_0$ and $T_0$, one can obtain (see Hines, 1960; Beer, 1974) a dispersion equation relating frequency and wave numbers, which can be written in the form of:

$$
m^2 = \frac{N^2 - \omega^2}{\omega^2 - f^2} - \frac{\omega_a^2 - \omega^2}{c^2},
$$

where $f$ is the Coriolis parameter, $N$ is the isothermal Brunt-Vaisala frequency, $c$ is the sound velocity, $\omega_a$ is highest frequency of acoustic waves, $\omega = \sigma - kiu$. Beer (1974) found that Eq. (36) could be appropriate approximation for slowly varying background temperature and wind if one use the following expressions:

$$
N^2 = \frac{g}{T_0} \left( \frac{\partial T_0}{\partial z} + \gamma_a \right), \quad \omega_a^2 = \frac{c^2}{4H^2} \left( 1 + 2 \frac{\partial H}{\partial z} \right),
$$

where $g = g/c_p$, $g$ is the acceleration by gravity, $H$ is the atmospheric scale height, $c_p$ is the heat capacity at constant pressure. Applying technique by Beer (1974) we can get the following polarization relations.
\[ U \propto \omega (\omega^2 - f^2 - k^2 c^2), \quad W \propto \omega (\omega^2 - f^2 - k^2 c^2), \]
\[ V \propto i\omega (\omega^2 - f^2), \quad P \propto \gamma (\omega^2 - f^2)(m - i\Gamma), \]
\[ U \propto \omega (\omega^2 - f^2 - k^2 c^2), \quad W \propto \omega (\omega^2 - f^2 - k^2 c^2), \]
\[ V \propto i\omega (\omega^2 - f^2), \quad P \propto \gamma (\omega^2 - f^2)(m - i\Gamma), \]
\[ R \propto (\omega^2 - f^2)(m - i\alpha) + ik^2 c^2 N^2 / g, \]
\[ \Theta \propto (\omega^2 - f^2)(m + i\alpha) - ik^2 c^2 N^2 / g, \]
\[ \Theta \propto (\omega^2 - f^2)(m + i\alpha) - ik^2 c^2 N^2 / g, \]

where \( \alpha = 1/(2\Gamma); \quad \Gamma = (2-\gamma)/(2\gamma) \). Eq. (5) does not allow calculating wave amplitudes, but give opportunity to find their ratios. At \( f = 0 \) Eq. (58) are equivalent to the polarization relations obtained by Hines (1960). In nondissipative atmosphere, according Eq. (25), AGW amplitudes should grow with altitude, so that

\[ W = W_0 \sqrt{\rho_0 / \rho_0} \]

An important AGW characteristic is the wave momentum flux, vertical component of which, \( F_{\text{mz}} \), is as follows

\[ F_{\text{mz}} = \rho_0 (\text{Re}(U W^*)) = \rho_0 \text{Re}(U W^*) / 2, \]

\[ (9) \]

4. Comparisons of the numerical model and polarization relations.

In this study, using the high-resolution nonlinear numerical model described in sect. 2, we simulated hydrodynamic fields produced by spectral AGW components and compared ratios of their amplitudes with those predicted by the analytical polarization relations Eqs. (4)–(7), (8). To make calculations close to assumptions of Eq. (2) of conventional simulations.
matching the linear AGW theory (see Eq. (5)), we modelled nonlinear AGWs having forms of plane waves and suppose horizontally periodical distributions of vertical velocity at the Earth’s surface moving along axis $x$ of the form of

$$ (w)_{z=0} = W_0 \cos[k(x - c_x t)], $$

(8)

where $k = 2\pi/\lambda_x$ and $c_x$ are horizontal wavenumber and phase speed along the horizontal axis $x$ in the direction of the wave propagation; $W_0$ is the amplitude. Eq. (8) represents plane wave of vertical velocity at the lower boundary, which may correspond to spectral components of convective and turbulent AGW sources (Townsend, 1965, 1966). Medvedev and Gavrilov (1995) studied AGW generation caused by nonlinear interactions in meteorological and turbulent atmospheric processes. They found variety of wavelengths, amplitudes and other parameters of created AGWs. In this paper, we describe simulations for wave modes having $c_x = 30$ m/s and $c_x = 100$ m/s with unchanged period $\tau = 2 \times 10^3$ s and amplitudes $W_0 = 0.3$ cm/s. The modeling was performed beginning from the MSIS initial state (zero wave fields) and the windless background flow at $t = 0$, when the wave source Eq. (8) was triggered at the lower boundary.

Gavrilov and Kshevetskii (2014a, b) demonstrated that after triggering the wave source at $t = 0$, fast acoustic and very long gravity wave modes would quickly reach very high heights. Simulations demonstrate that in the horizontally periodic approximation of Eq. (1), these initial pulses can reach altitudes of 100 km and higher in a few minutes and form quasi-vertical wave fronts analogous to those in Fig. 1a,b,c of the paper by Gavrilov and Kshevetskii (2013, 2014a). These initial waves dissipate because of molecular viscosity and heat conduction. When time increases, more and more of the waves with longer vertical wavelengths are taken away by dissipation, therefore vertical wavelengths should decrease in time at a given height in the middle atmosphere (Heale et al., 2014). After some transition time, initial AGW wave modes disappear and wave vertical structure matches to the main spectral component of the wave source (8) having horizontal wave number $k$ and phase speed $c_x$.

To estimate AGW amplitudes in the numerical model solution we calculated standard deviations of corresponding wave fields over all nodes of the horizontal grid at considered altitude. For sinusoidal wave component, this standard deviation is equal to a half
AGW amplitude. Therefore, ratios of amplitudes of horizontally homogeneous stationary sinusoidal AGWs should be equal to the ratios of corresponding standard deviations. Simulated standard deviations of wave fields in horizontal planes located at different heights grow in time throughout transition intervals after activating the wave forcing and then tend to constant values different at each height (see Gavrilov and Kshevetskii, 2014b). In the horizontally periodical approximation of Eq. (1), these standard deviations are approximately equal to a half wave amplitudes at large $t$, when the AGW process tends to become quasi-stationary. For a plane spectral AGW component with vertical wavelength $\lambda_z$, the vertical group velocity is $c_g \approx \lambda_z/\tau$, and the time of its energy arriving to altitude $z$ is $t_e = z/c_g$. For considered main spectral components of the wave source (8) with $\tau = 2 \times 10^3$ s and average $\lambda_z \sim 10$ km for $c_s = 30$ m/s, and $\lambda_z \sim 35$ km for $c_s = 100$ m/s. Therefore, one can get $t_e/\tau = z/\lambda_x \sim 1, 6, 10$ and $t_e/\tau \sim 0.3, 1.7, 2.9$ at heights 10, 60, and 100 km, respectively, for both $c_s$. Thus, lengths of the transition intervals are longer for smaller $c_s$. These intervals grow with altitude and may be longer than ten wave periods at height of 100 km.

Table 1 represents standard deviations at different altitudes calculated with the numerical model and with analytical polarization relations and their ratios for AGW with $c_s = 30$ m/s. The numerically modeled standard deviations Table 1 contains simulated SDs at each altitude calculated periodically and averaged over model outputs during the initial transient interval $t < t_e$ (bottom part of Table 1) and for quasi-stationary waves $t > t_e$ (upper part of Table 1). Respective data numbers $n$ for each altitude are presented in Table 1. Respective values obtained from analytical linear AGW theory (see section 3) are calculated using average background values and are placed to the columns labeled as “Lin” at each altitude in Table 1. Consideration of Fig. 5 of the paper by Gavrilov and Kshevetskii (2014b) shows that standard deviations of wave fields simulated with the numerical model vary in time due to definite variations and irregular perturbations. Standard deviations of each average numerically simulated parameter are given in Table 1.

For comparisons of numerically simulated values with analytical ones in Table 1, we use standard t-test giving probability of the null hypothesis about equity of averages of two irregular quantities (Rice, 2006). Approximately, the probability of equity of two respective average values in Table 1 is larger 95%, if difference between them is less than 1.96 multiplied by the standard deviation of the average value (Rice, 2006). In this study, we considered only cases, when the standard deviations in Table 1 are smaller than 0.15 of...
respective average values. Pairs of AGW parameters, which we can consider equal with
confidence larger than 95%, are marked with bold font in Table 1. The numbers of those pairs
are largest in the upper part of Table 1 at altitudes 30 and 60 km, which correspond to quasi-
stationary AGWs in the free atmosphere considered in conventional AGW theory described in
sect. 3. Reasonable agreements between simulated and analytical wave parameters in
atmospheric regions, which correspond to the scope the limitations of the nondissipative
linear AGW theory, may be considered as evidences of adequate descriptions of wave
processes by the used nonlinear numerical model.

Many numerically simulated AGW parameters do not match to the respective analytical
values in Table 1. No matches are in the bottom part of Table 1, which corresponds to the
initial transition time interval. Gavrilov and Kshevetskii (2014b) showed that vertical
structures of transient waves are different from those predicted by the linear AGW theory
during the transition interval after activating the surface wave source Eq. (81). Bottom part
of Table 1 shows that numerically simulated wave amplitude $W$ is smaller than that predicted
by AGW theory at high altitudes, because these values refer to small $t < t_e$, when energy of
the main wave component does not yet reach considered altitude. Numerical and analytical
amplitude ratios are also substantially different in the bottom part of Table 1 for $t < t_e$.

In the upper part of Table 1 for quasi-stationary AGWs at $t > t_e$, the numerically simulated
AGW amplitudes $W$ are slightly smaller than the analytical values at altitudes up to 60 km.
This can be caused by small AGW dissipation at low altitudes and by partial reflections of the
wave energy from inhomogeneities of background atmospheric fields in the numerical model.
Wave dissipation becomes larger at altitude 100 km due to grows in cinematic viscosity and
heat conductivity, therefore simulated amplitude $W$ in the upper part of Table 1 become much
smaller than that predicted by nondissipative AGW theory. In addition, one can see
substantial differences in numerically simulated and analytical ratios of some AGW
amplitudes, which can be due to influences of dissipative effects. At low altitudes, differences
in simulated and analytical ratios of AGW amplitudes can reflect the influence of lower
boundary conditions. In particular, the condition $u = 0$ at the Earth’s surface makes AGW
amplitudes of horizontal velocity at low altitudes smaller than that predicted by the AGW
theory for free atmosphere. The upper part of Table 1 for $t > t_e$, shows that the best
agreements exists between numerical and analytical values of the ratio $R/\Theta \approx 1$ at all altitudes.
Table 1 reveals numerically simulated AGW momentum fluxes $F_{mz}$, Eq. (710) calculated as
\[ \rho_0 u' \rho_0 \langle \omega' w'' \rangle \]
averaged over horizontal planes at fixed altitudes and over respective time intervals. For comparisons, in Table 1 given contains also momentum fluxes $F_{mz}$ given by
Eq. (10) and calculated using the right formula of Eq. (7) from numerically calculated simulated amplitudes $W$ and $U$. The upper part of Table 1 shows that at $t > t_e$ wave momentum flux $F_{mz}$ is almost constant at altitudes 10 – 60 km due to relatively small dissipation and reflection of wave energy. At altitude of 100 km wave dissipation increases and $F_{mz}$ decreases producing strong wave accelerations of the mean flow, which are proportional to the vertical gradient of $F_{mz}$. In the bottom part of Table 1 for $t < t_e$, values of $F_{mz}$ are much smaller than respective $F_{mz}$ values for $t > t_e$, because during initial transition interval, energy of the main AGW modes of the wave source (811) does not yet reach high altitudes.

Table 2 is the same as Table 1, but for AGW components with $c_x = 100 \text{ ms}^{-1}$, which has longer vertical wavelength. In the upper part of Table 2 for $t > t_e$, we have smaller number of pairs equal with confidence 95% (marked with bold font), than that in the upper part of Table 1. This may be connected with stronger influence of vertical inhomogeneities of background temperature profile on faster AGW with longer vertical wavenumber and with larger partial reflection of faster AGW energy. Stronger reflections lead to smaller amplitudes $W$ at altitudes below 100 km in the upper part of Table 2 compared to that in Table 1. On the other hand, $W$ at altitude 100 km in the upper part of Table 2 is larger than that in Table 1 due to smaller dissipation of longer AGWs. Therefore, waves with longer vertical wavelengths can better penetrate faster propagate from the surface to the upper layers and less dissipate in the middle atmosphere, where they can produce have larger dynamical and thermal effects amplitudes than those with shorter vertical wavelengths (see Gavrilov and Kshevetskii, 2014b). Similar to Table 1, we have larger amounts of equal (with 95% confidence) numerically simulated and analytical AGW parameters at altitudes 30 and 60 km. At low and high altitudes and at $t < t_e$ (in the bottom part of Table 2) numbers of equal pairs are smaller due to influence of the lower boundary conditions, larger dissipation and AGW transience, respectively.

In atmospheric regions, where numerical and analytical AGW Tables 1 and 2 contains comparisons of the numerical results and linear polarization relations at altitudes below 100 km, where considered AGW modes are quasi-linear and almost nondissipative. At higher
1 altitudes growing wave amplitudes and molecular viscosity and heat conduction lead to fast
growing wave-induced mean flows, which violate assumptions of conventional AGW theories
and change ratios of wave amplitudes of different hydrodynamic fields. Therefore, we found
 poor agreement between numerical and analytical wave results above altitude 100 km and do
not include them into Tables 1 and 2. These disagreements become larger with increases in
amplitudes of the lower boundary wave sources due to higher nonlinear effects and faster
grows in the wave-induced jet streams above 100 km. To get better agreements, improved
analytical AGW theories taking into account transient processes, high wave dissipation and
fast changes in background fields are required.

2 In the areas of Tables 1 and 2, where numerical and analytical parameters are close, one can
use analytical formulae for descriptions and estimations of the wave fields. Opposite to that,
areas of substantial differences between numerical and analytical AGW parameters in Tables
1 and 2 reveal regions, where analytical theories give substantial errors and numerical
simulations of wave fields are required.

3 Relations of linear AGW theory are frequently used for simplified parameterizations of AGW
dynamical and thermal effects for their use in the numerical models of atmospheric general
circulations (e.g., Lindzen, 1981; Holton, 1983; Gavrilov, 1997; etc.). Similar
parameterizations are also developing for highly dissipative AGWs in the upper atmosphere
(e.g., Vadas and Fritts, 2005; Yigit et al., 2008). Sometimes, different parameterizations give
different results. Direct numerical simulation models of atmospheric AGWs may be useful
tools for testing and verifications of simplified parameterizations of wave effects.

4 5. Conclusions

5 In this study, we performed high-resolution numerical simulations of nonlinear AGW
propagation to the middle and upper atmosphere from a plane wave forcing at the Earth’s
surface and compared them with analytical polarization relations of linear AGW theory. Such
comparisons may be used for verifications of numerical models of atmospheric AGWs.

6 Numerical simulations show that after triggering the wave source Eq. (811) at \( t = 0 \), fast
acoustic and very long gravity wave modes would quickly reach very high heights. After
some transition time \( t_e \) (increasing with altitude), initial AGW wave modes disappear and
wave vertical structure matches to the main spectral component of the wave source Eq. (811)
having horizontal wave number \( k \) and phase speed \( c_\theta \). The numbers of numerically simulated
and analytical pairs of AGW parameters, which are equal with confidence 95%, are largest at
altitudes 30 and 60 km at \( t > t_e \). At low and high altitudes and at \( t < t_e \), numbers of equal pairs are smaller, because of influence of the lower boundary conditions, larger dissipation and AGW transience, which can produce substantial inclinations from conditions, assumed in conventional theories of linear nondissipative stationary AGWs in the free atmosphere.

Reasonable agreements between numerically simulated and analytical wave parameters in atmospheric regions, which correspond to the scope the limitations of the AGW theory, may be considered as evidences of adequate descriptions of wave processes by the used nonlinear numerical model. Areas of substantial differences between numerical and analytical AGW parameters reveal atmospheric regions, where analytical theories give substantial errors and numerical simulation of wave fields is required. Direct numerical simulation models of atmospheric AGWs may be useful tools for testing and verifications of simplified parameterizations of wave effects.

**Acknowledgements**

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**References.**


Table 1. Standard deviations and their ratios for AGW with $c_s = 30 \text{ m/s}$ calculated with the numerical model and with analytical polarization relations (labeled as Lin) at different altitudes averaged over the initial transient interval $t < t_e$ and for quasi-stationary waves $t > t_e$. Bold font shows the data pairs equal with probabilities larger than 95%.

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Table 2. Same as Table 1, but for AGW with $c_s = 100$ m/s.