Reply to Referee#1

We appreciate your careful reading of the manuscript and thoughtful comments for
improving the manuscript. Please find below a point-by-point response to each of the
comments. The original comments are in italics.

Specific comments:

1) With regard to the general comment above, on line 11 of page 3727, the statement
about verifying the model’s ability to “treat surface elevations associated with the vertical
terrain-following coordinate” should probably be reserved for the new section describing the
“Schar mountain” experiment.

=> Following your suggestion, we will revise the manuscript.

2) In line 27 of page 3729, what is meant by “degrees of freedom (DOF)”? A short
explanation would be good here. DOF is also mentioned in Fig. 6. If DOF means number of
GLL grid points, it doesn’t make sense that the DOF is the same in both 5th and 8th order
experiments, as mentioned in the Fig. 6 caption. For a given resolution (i.e., delta-x), it seems
the 8th order experiments have more DOF. Please explain.

=> “DOF” in the manuscript means “total number of GLL grid points in the physical
domain”. The experiment is designed to use the same number of GLL grid points in the given
physical domain for both 5th and 8th order experiments, remaining a given mean delta-x. It is
achieved by using lower number of elements in 8th order experiment than that of 5th because
the number of grid points at a given level becomes ne*np, which ne refers to the number of
elements and \( np \) denotes the polynomial order of the elements. To make it clearly, we will revise the manuscript.

3) In lines 15-17 of page 3730, the authors state that the 8th order polynomials at the coarsest resolution are “relieved from the deviation from the converged solution” in Fig. 6c. However, when I compare Figs. 6b and 6c, I don’t see much difference between the amount of deviation between the 5th and 8th order solutions at 400m or 200m. Your statement could use some clarification.

=> We focus on the 400m resolution profiles of the 5th and 8th order experiments. In comparison with each other, we observe more deviations from the converged solution (the 50m resolution results) in the 5th order experiment than in the 8th order experiment. We would like to take examples such as the profile over the range from 4km to 8km, and the first peak of the profile near 10km location. To make it clearly, we will revise the manuscript.

4) In line 26 page 3732 to line 4 of page 3733, there is mention of numerical diffusion applied to the momentum and potential temperature, however, there is no mention of how it is implemented. It seems that a diffusion term applied with the mixed SEM/FDM methods would not be very trivial. It would be good if the authors described their method and/or reference an already published technique if it was used. Also, it would be good to describe what order of diffusion is used, i.e., del-squared or del4 (hyperdiffusion?).

=> We use an explicit Laplacian (del-squared) on coordinate surface for spatial (horizontal/vertical) diffusion. The horizontal Laplacian operator in SEM is described in Denis et al. (2011). The vertical Laplacian operator in FDM is basically described in
Skamarock et al. (2008). The only difference between our study and Skamarock et al. (2008)
comes from the hybrid-sigma vertical coordinate. We will add the references and the
description of what order of diffusion is used in the revised manuscript.

For your information, we briefly describe how to implement the Laplacian in this study.

Let me explain horizontal SEM first, then vertical FDM later.

In order to implement \( f = K_h \nabla^2 (\mu \partial_a) \) for a model flux variable \( \mu \partial_a \) (In 2D
framework of our study we consider \( \frac{\partial^2}{\partial x^2} \) as \( \nabla^2 \)), we multiply by the basis function as a

\[
\psi, \text{ and integrate using the divergence theorem to yield the weak form equation}
\]
as the following:

\[
\int_{\Omega^e} \psi f \, d\Omega^e = K_h \left( \int_{\Gamma^e} \psi n \cdot \nabla (\mu \partial_a) \, d\Gamma^e - \int_{\Omega^e} \nabla \psi \cdot \nabla (\mu \partial_a) \, d\Omega^e \right),
\]

where \( K_h \) denotes the horizontal eddy viscosity coefficient and the term with \( \Gamma^e \) is a
boundary integral which accounts for internal faces (neighboring elements share faces). Since
we use the periodic boundary condition in this study, the boundary integral term of the right
hand sides can be ignored in all elements, which allows to rewrite the equations as

\[
\int_{\Omega^e} \psi f \, d\Omega^e = -K_h \int_{\Omega^e} \nabla \psi \cdot \nabla (\mu \partial_a) \, d\Omega^e.
\]

After introducing the polynomial expansions such as \( \partial (x, t) = \sum_{k=1}^{n+1} \psi_k(x) \partial_n (x_k, t) \), the
integrals of the above equation are approximated with the SEM (section 3.1.1 in the
manuscript).

For the vertical Laplacian operator, we add the diffusion term for a model flux variable

\( \mu \partial_a \), which is given as
\[
\frac{\partial}{\partial t} \left( \mu g h \right) = \ldots + K_v g^2 \left( \mu g h \right)^{-1} \frac{\partial}{\partial \eta} \left( \left( \mu g h \right)^{-1} \frac{\partial \left( \mu g h \right)}{\partial \eta} \right),
\]

where \( K_v \) denotes the vertical eddy viscosity coefficient and \( \alpha \) is the inverse density. It is noted that the above term is nothing more than \( K_v \frac{\partial^2 \left( \mu g h \right)}{\partial z^2} \). The vertical derivative term \( \frac{\partial}{\partial \eta} \) is discretized by the centered finite difference.

**Technical corrections:**

1) On line 12 of page 3721, the definition of eta-dot is written as a partial derivative w.r.t. time. Instead it should be written as the material (substantial) derivative w.r.t. time (i.e., \( D-\eta/DT \)).

\( \Rightarrow \) Thank you for your comment. We will correct it.

2) In line 8 of page 3727, should the reference to Eq. (24) instead be to Eq. (21)?

\( \Rightarrow \) Thank you for your comment. We will change the equation number.

3) On line 11 of page 3729, what are \( x_r \) and \( x_c \) set to?

\( \Rightarrow \) The center of the bubble \( (x_c, z_c) \) is set to \( (0,3000) \) m. The size of the bubble is defined by the parameters \( (x_r, z_r) = (4000,2000) \) m. We will add this description.

4) In Equation (25) on page 3731, should \( ac \) be \( xc \)?
We missed the mention about $a_c$. No, it should not. $a_c$ is set to 5000 m. We will add this.

5) In Equation (26) on page 3732, what is $r_c$ set to?

=> Thank you for pointing out. $r_c$ is set to 250 m. We will add this too.

References
