Consistent assimilation of multiple data streams in a carbon cycle data assimilation system

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Abstract

Data assimilation methods provide a rigorous statistical framework for constraining the parametric uncertainty of land surface models (LSMs), with the aim of improving our predictive capability as well as identifying areas in which the models need improvement. The increase in the number of available datasets in recent years allows us to address different aspects of the model at a variety of spatial and temporal scales. However, combining data streams in a DA system is not a trivial task. In this study we highlight some of the challenges surrounding multiple data stream assimilation, with a particular focus on the carbon cycle component of LSMs. We examine the impact of biases and inconsistencies between the observations and the model (resulting in non Gaussian error distributions) and the impact of non-linearity in model dynamics. In addition we explore the differences between performing a simultaneous assimilation (in which all data streams are included in one optimisation) and a step-wise approach (in which each data steam is assimilated sequentially), given the assumptions inherent to the inversion algorithm chosen for this study. We demonstrate some of these issues by assimilating synthetic observations into two simple models: the first a simplified version of the carbon cycle processes represented in many LSMs, and the second a non-linear toy model. We further discuss these experimental results in the context of recent
studies in the carbon cycle data assimilation literature, and finally we provide some perspectives and advice to other land surface modellers wishing to use multiple data streams to constrain their models.

Keywords: data assimilation, carbon cycle, biogeochemical cycles, land surface model

1 Introduction

The carbon cycle is an important component of the Earth system, especially when considering the climatic impact of rising greenhouse gases concentrations from fossil fuel emissions and land use change. It is estimated that the oceans and land surface absorb approximately half of the CO$_2$ emissions due to anthropogenic activity, but uncertainties remain in the strength and location of sources and sinks, as well as in predictions of future trends (Ciais et al., 2013). Observations allow us to understand the system up until the present day, but they cannot tell us about the future, and can be limited in their spatial coverage. They also cannot distinguish between the complex interactions that may occur between different processes. Incorporating our current knowledge of physical mechanisms of biogeochemical cycles, including carbon, C, dynamics, into Land Surface Models (LSMs) represents a promising approach to analyse these interacting effects, to upscale observations to larger regions, and to make future predictions. However, the models can be limited by the lack of process representation, either due to gaps in our knowledge or in our technical and computing capability. As a result, model evaluations reveal that not all variables are well-captured by the model under current conditions (Anav et al., 2013), and the spread between model projections is still very large (Sitch et al., 2015).

Aside from model structural and forcing errors, one source of uncertainty is related to the parameter (i.e. fixed) values of a model. Model-data fusion, or data assimilation (DA), allows the calibration, or optimisation, of these values by reducing the model-data misfit while accounting for the uncertainties inherent in both the model and data in a statistically rigorous framework. The C cycle component of most LSMS is complex and contains a large number of parameters. Luckily however, there are an increasing number of in-situ and remote sensing-based data streams that can be used for parameter optimisation. These data bring information on different spatial and temporal scales, such as:
• Atmospheric CO₂ concentration data measured at surface stations at continental to global scales, which provide information from synoptic timescales to inter-annual variability (IAV) and long-term trends.

• Eddy covariance net CO₂ (net ecosystem exchange – NEE) and latent (LE) and sensible heat fluxes measured at half-hourly intervals at many sites across different ecosystems/regions, providing information at seasonal to inter-annual timescales.

• Satellite-derived measures of vegetation dynamics, including “greenness” indices (i.e. the Normalised Difference Vegetation Index – NDVI), fraction of absorbed photosynthetically active radiation (FAPAR) and leaf area index (LAI) at global scales and at daily time step spanning more than a decade, thus capturing IAV and long-term trends (though usually with a trade-off between spatial and temporal resolution).

• Satellite-derived measurements of soil moisture and land surface temperature at the same temporal and spatial scales as the satellite-derived observations of vegetation productivity.

• Aboveground biomass measurements are currently taken at only one or a few points in time at plot scale up to regional scale from aircraft and satellite data, or are estimated from allometric relationships at each site.

• Soil C stock estimates usually are only taken at one point in time at plot scale.

• Ancillary data on vegetation characteristics such as tree height or budburst – one measured at certain well-instrumented sites.

Increasingly, researchers are attempting to bring these sources of information together to constrain different parts of a model at different spatio-temporal scales within a multiple data stream assimilation framework (e.g. Richardson et al., 2010; Keenan et al., 2012; Kaminski et al., 2012; Forkel et al., 2014; Bacour et al., 2015). However, whilst the potential benefit of adding in extra data streams to constrain the C cycle of LSMs is clear, multiple data stream assimilation is not as simple as it may seem. When using more than one data stream there is the option to include all data streams together in the same optimisation (simultaneous approach), or to take a sequential (step-wise) approach. Mathematically, the optimal approach is the simultaneous, but computational constraints related to the inversion of large matrixes or
the requirement of numerous simulations (especially for global datasets), and/or the weight of different data streams in the optimisation, may complicate a simultaneous optimisation. On the other hand, in a step-wise assimilation the parameter error covariance matrix has to be propagated at each step, which implies that it can be computed. If the parameter error covariance matrix can be properly estimated and is propagated between each step, the step-wise approach can be mathematically equal to simultaneous. However, many inversion algorithms (e.g. derivative based methods that use the gradient of the cost function to find its minimum) require assumptions of model (quasi-) linearity and Gaussian parameter and observation error distributions. If these assumptions are violated, or the error distributions are poorly defined, it is likely that the step-wise will not be equal to the simultaneous, and that information will be lost at each step. An incorrect description of the observation (~ model) error distribution could result from the wrong assumption about the distribution of the residuals between the observation and the model, a poor characterisation of the error correlations, an incompatibility between the model and the data (possibly due to a model structural issue or differences in how a variable is characterised), or a bias in the observations that is not unaccounted for (i.e. is treated as a random error). Whilst a simultaneous optimisation is mathematically more rigorous in the sense that the error correlations are treated within the same inversion, if the prior distributions are not properly characterised any bias may be aliased to the wrong parameters (Wutzler and Carvalhais, 2014), more so than in a step-wise approach.

This tutorial-style paper demonstrates some of the challenges of multiple data stream assimilation discussed above with two simple models: one a simplified version of the carbon dynamics included in many LSMs, and the other a “toy” model designed to demonstrate the issues that arise with complex, non-linear models. Section 2 provides a description of these models, the inversion algorithm used to optimise the model parameters and the experiments performed, followed by the results for each test case. Section 3 further discusses the challenges outlined in Section 2 with reference to recent carbon cycle multiple data stream assimilation studies in the literature. Finally Section 4 provides some advice to land surface modellers wishing to carry out multiple data stream assimilation.
2 Demonstration with two simple models and synthetic data

2.1 Methods

2.1.1 Simple carbon model

To demonstrate the challenges of multiple data stream assimilation in a carbon cycle context, we have chosen a test model that represents a simplified version of the carbon cycle dynamics typically implemented in most LSMs. The model has been well-documented in Raupach (2007) and has been used previously in the OptIC DA inter-comparison project (Trudinger et al., 2007). It is based on two equations that describe the temporal evolution of two carbon pools, \( s_1 \) and \( s_2 \):

\[
\frac{ds_1}{dt} = F(t) \left( \frac{s_1}{p_1 + s_1} \right) \left( \frac{s_2}{p_2 + s_2} \right) - k_1 s_1 + s_0 \tag{1}
\]

\[
\frac{ds_2}{dt} = k_1 s_1 - k_2 s_2 \tag{2}
\]

In this model formulation, \( s_1 \) and \( s_2 \) are approximately equivalent to above- and belowground biomass stocks. The unknown parameters \( p_1, p_2, k_1 \) and \( k_2 \) will be optimised in the inversions. The first term on the right-hand side of Eq. (1) corresponds to the Net Primary Production (NPP) i.e. the carbon assimilated into the system as a function of time, \( F(t) \), weighted by factors that account for the size of both pools in order to introduce a limitation on NPP (the two fractions in parentheses). The litterfall is an output of \( s_1 \) and an input to \( s_2 \) and is a constant fraction of the aboveground carbon reserve as represented by \( k_1 s_1 \). Heterotrophic respiration (Rh) is an output of \( s_2 \) and is represented \( k_2 s_2 \). The constant \( s_0 \) is a “seed production” term set to 0.01 (i.e. not optimised) to ensure the model does not verge towards zero. A more detailed description of the properties of the model is given in Trudinger et al. (2007) and an in-depth analysis of the model behaviour is provided in Raupach (2007). Synthetic observations of both \( s_1 \) and \( s_2 \) variables were used to optimise all the unknown parameters in the model (see Section 2.1.5).

2.1.2 Non-linear toy model

Although the simple carbon model contains a non-linear term it is essentially still a quasi-linear model. In order to illustrate the challenges associated with multiple data stream
data assimilation for more complex non-linear models, we defined a simple non-linear toy model based on two equations with two unknown parameters:

\[ s_1 = a \exp(x) + at^2 \]  
\[ s_2 = \sin(10a + 10b) + 10t^2 \]

where \( s_1 \) and \( s_2 \) also correspond to two model state variables (as for the simple C model), \( a \) and \( b \) are the unknown parameters included in the optimisation, and \( t \) is the independent variable, which could represent time in a real-world scenario. Note that this model is not based on any particular physical process associated with land surface biogeochemical cycles, but it does contain typical mathematical functions that are observed in reality and implemented in LSMs. For example, the sinusoidal function (Eq. 4) could represent diurnal variations of various processes such as photosynthesis and respiration. Exponential response functions (such as in Eq. 3) are also observed for certain processes, including the temperature sensitivity of soil microbial decomposition. As for the simple carbon model, synthetic observations corresponding to the \( s_1 \) and \( s_2 \) variables were used to optimise both parameters (see Section 2.1.5).

2.1.3 Bayesian inversion algorithm

Most data assimilation approaches follow a Bayesian formalism which, simply put, allows prior knowledge of a system (in this case the model parameters) to be updated, or optimised, based on new information (from the observations). In order to achieve this we define a “cost function” that describes the misfit between the data and the model, taking into account their respective uncertainties, as well as the uncertainty on the prior information. If we follow a Bayesian formalism and least-squares minimisation approach, and assume Gaussian probability distributions for the model parameter and observation error variance/covariance, we derive the following cost-function (Tarantola, 1987):

\[ J(x) = \frac{1}{2}[(H(x) - y)^T \cdot R^{-1} \cdot (H(x) - y) + (x - x^a)^T \cdot B^{-1} \cdot (x - x^a)] \]  

where \( y \) is the observation vector, \( H(x) \) the model outputs given parameter vector \( x \), \( R \) the observation error covariance matrix (including measurement and model errors), \( x^a \) the a priori
parameter values, and B the prior parameter error covariance matrix. This framework leads to a Gaussian posterior parameter probability distribution function.

The aim of the inversion algorithm is to find the minimum of this cost function, thereby achieving the best possible fit between the model simulations and the measurements, conditioned on their respective uncertainties and prior information. For cases where there is a strong linear dependence of the model to the parameters (at least for variations in $x$ of the size of those expected in the data assimilation system), and where the dimensions of the problem are not too large, the solution can be derived analytically. If not, as is usually the case with LSMs, there are different numerical methods to find the most optimal parameter values. These include global search methods that randomly search the parameter space and test the likelihood of a particular parameter set at each iteration, and derivative methods, which calculate the gradient of the cost function at each iteration to find its minimum. In this study we use the latter class of methods. More specifically we use a quasi-Newton algorithm that uses both the gradient of the cost function and its derivative (Hessian) to evaluate if the minimum has been reached (i.e. where the gradient is zero). Thus we obtain the following algorithm for iteratively finding the minimum (Tarantola, 1987, p195):

$$x_{i+1} = x_i + \epsilon_i \left[ H^T R^{-1} H + B^{-1} \right]^{-1} H^T R^{-1} \left( y - H(x) - B^{-1}(x_i - x^*) \right)$$  \hspace{1cm} (6)$$

where $i$ is the iteration number and $H$ is the Jacobian, or first-order derivatives, of $H$, which in this study is determined using a finite difference method. Note that as we are potentially dealing with non-linear models, the quasi-Newton method has been slightly adapted to include the constant scaling factor $\epsilon_i$ (with a value <1.0) to ensure that the algorithm will converge.

Of course no inversion algorithm is perfect, and therefore it is possible that the true "global" minimum of the cost function has not been found. Derivative methods in particular can get stuck in so-called "local minima", preventing the algorithm from finding the true minimum. To address this issue we carry out a number of assimilations with different random first guess points in the parameter space. If they all result in the same reduction in cost function value, we can have more confidence that the true minimum has been found.

Once the minimum of the cost function has been found, the posterior parameter error covariance can be approximated (using the linearity assumption) from the inverse Hessian of
the cost function around its minimum, which is calculated using the Jacobian of the model at
the minimum of \( J(x) \) (for the set of optimized parameters), \( H_{yx} \), following Tarantola (1987):

\[
A = \left[ H^T R^{-1} H_x + B^{-1} \right]^{-1}
\]

(7)

Note that the posterior error covariance matrix can be propagated into the model space to
determine the posterior uncertainty on the simulated state variables as a result of the
parametric uncertainty (as shown in the coloured error bands in the time series plots – Figures
1 and 5) using the following matrix product and the hypothesis of local linearity (Tarantola,
1987):

\[
R_{post} = H_x A H^T_x
\]

(8)

However, we do not detail the propagated posterior uncertainty on the state variables further
in this study; rather, we describe the impact of the optimisation on the model–data fit in terms
of the RMSE value and also in terms of the relative uncertainty reduction on the parameters.

2.1.4 Step-wise versus simultaneous assimilation

**Step-wise approach**

In the step-wise approach each data stream (in our cases \( s_1 \) and \( s_2 \), see above) is
assimilated sequentially, and the posterior error covariance matrix of Eq. (7) is propagated to
the next step as the prior in Eq. (6). Note that the error covariance matrix can only be
propagated if it is calculated within the inversion algorithm, which is the case here but may
not be possible in other studies. The following details an example for two data streams.

Step 1: Assimilation of the first data stream, \( s_1 \). The prior parameters, including their values
and error covariance (\( x^0 \) and \( B \)), are optimised to produce a first set of posterior
optimised parameters \( x_1 \) with error covariance \( A_1 \).

Step 2: Assimilation the second data stream, \( s_2 \). The parameters, \( x_1 \), and their error
covariance, \( A_1 \), are used as a prior to the optimisation system and further optimised
to produce the second (and final) set of posterior optimised parameters, \( x_{post} \), and the
associated error covariance \( A \).
Simultaneous approach

Both data streams $s_1$ and $s_2$ are included in the optimisation and all parameters are optimised at the same time. The prior parameters, including their values and error covariance ($x^b$ and $B$) are optimised to produce the posterior parameter vector ($x_{\text{post}}$) and associated uncertainties $A$.

2.1.5 Optimisation set-up: parameter values and uncertainty, and generation of synthetic observations

In this study we used synthetic observations that were generated by running the model with known (or ‘true’) parameter values and adding random Gaussian noise corresponding to the defined observation error for both $s_1$ and $s_2$ (see Table 1). The true values of all parameters for both models are given in Table 1, together with their upper and lower bounds. The parameter uncertainty (1 sigma) was set to 40% of the parameter range following recent studies (e.g. Bacour et al., 2015). Prior values were chosen from a uniform random distribution bounded by the parameter bounds. We assumed independence (i.e. uncorrelated errors) for both the parameters and observation covariance matrices, thus the $R$ and $B$ matrices were diagonal.

2.1.6 Experiments

Table 2 details the experiments that were carried out based on all possible combinations for assimilating the two data streams. Three approaches were compared: i) separate – where only one data stream was included in the optimisation; ii) step-wise – where each data stream was assimilated sequentially; and ii) simultaneous – where both data streams were included in the optimisation. All parameters for both models were optimised in all experiments, therefore in the step-wise cases the parameters were optimised twice. Tests for the step-wise were also carried out with and without the propagation of the full posterior parameter error covariance matrix, $A_1$, in between steps 1 and 2 (test cases 2b and d – see Table 2) – i.e. for these tests only the posterior variance was propagated. An additional test was included for the simultaneous assimilation in order to test the impact of having a substantial difference in the number of observations for the data stream included in the optimisation; therefore in test case 3b, only one observation was included for data stream $s_2$. 
The differences in the parameter values and the theoretical reduction in their uncertainty 
\((1 - (\sigma_{\text{post}} / \sigma_{\text{prior}}))\) were examined for all eight test cases, as well as the fit (RMSE) to both 
data streams after the optimisation. For the step-wise approach we investigated if the fit to the 
first data stream is degraded in the second step by comparing the RMSE after each step. Note 
that the reduction in uncertainty is a theoretical, or approximate, estimate of the real 
uncertainty reduction because of the assumptions made in the inversion scheme.

In a second stage the impact of an unknown, un-accounted for bias in the model was 
examined. This bias could be a systematic bias in the observations due to the algorithm used 
for their derivation, the result of missing or incomplete processes in the model, or an 
incompatibility between the observations and the model, for example due to differences in 
spatial resolution or an inconsistent characterisation of a variable between the model and the 
observations. To test the impact of such an occurrence, we introduced a uniform scalar bias 
into the modelled \(s_2\) variable with a value of 10 (i.e. twice the magnitude of the defined 
observation uncertainty). All eight experiments were repeated, but a bias was introduced into 
the model calculation of \(s_2\) that was not accounted for in the cost function (i.e. the error 
distributions retained a mean of zero). This was treated as an unknown bias, and therefore not 
corrected or accounted for in the inversion scheme and the defined observation uncertainty 
(Table 1) was not changed for this set of experiments.

In all experiments for both models twenty assimilations were performed starting from 
different random “first guess” points in the parameter space. As discussed in Section 2.1.3 
this was done to test the ability of the algorithm to converge to the true global minimum of the 
cost function. Note that the global minimum and possible reduction in \(J(x)\) will be different 
for each experiment, as each is based on a different cost function.

### 2.2 Results

The twenty random first guess assimilations were examined for each set of experiments 
for both models (before the results for each test were examined in more detail), in order to 
check that the algorithm converged to a global minimum. As shown in the supplemental 
information (Fig. S1), a high proportion of the twenty first guess assimilations across all test 
cases for both models resulted in a similar reduction in \(J(x)\), even though the overall 
magnitude of the reduction was sometimes different between tests. This indicates that the
The algorithm does not easily get stuck in any local minima (if they exist). The examples shown in the results below were taken from one first guess parameter set for each model that belonged to the cluster that had the highest cost function reduction. Any differences seen in the parameter values, their posterior uncertainty or the resultant RMSE reduction described below therefore are due to the specific details of each test and not the inability of the algorithm to find the minimum.

2.2.1 Typical performance with a quasi-linear model and no bias

Figures 1a and b show the simple carbon model simulations for test case 3a (in which both data streams are assimilated simultaneously) for the $s_1$ and $s_2$ variables. A large reduction in RMSE is achieved after optimisation (blue curve) with respect to the observations (black curve). Overall, there is a good reduction in RMSE for all test cases (including the individual assimilations 1a and 1b) with a reduction of ~80% for $s_1$ and $s_2$. In addition, the optimisation of the $s_1$ and $s_2$ variables resulted in a good or moderate reduction in RMSE for variables not included in any assimilation: ~60% for the litterfall (Eqn. 1) and ~16% for the heterotrophic respiration ($R_h$ – Eqn. 2) across all test cases (not shown), although there was already a good prior fit to the data. As would be expected from these results, the parameter values and the theoretical reduction in parameter uncertainty do not vary between the tests (Figures 2a and b blue symbols), except for a slight difference in the value of the $k_2$ parameter in test cases 1a and 3b, for which there is also a lower reduction in uncertainty (~82% compared to >95%). Note that Fig. 2a shows the normalised parameter values to account for differences in the magnitude of the different parameters and their range (the zero line represents the “true” parameter value – see caption). In this situation therefore, where we have a relatively simple linear model and two data streams to which the model parameters are highly sensitive, we see that the differences between the step-wise and simultaneous approaches are minimal. This is even the case when the error covariance is not propagated between the two steps (test cases 2b and d), suggesting that under this assimilation set-up both $s_1$ and $s_2$ individually contain enough information to retrieve the true values of all parameters, as we can see from the separate test cases 1a and b.
In Section 2.2.1 we saw that there is little difference between a step-wise and simultaneous optimisation if there is no bias in the model or observations, and if the model is quasi-linear and therefore the critical assumptions behind the inversion approach were not violated. However, it is not uncommon to have a bias between your observations and model that is not obvious and therefore not accounted for in the optimisation, as the cost function used in most inversion algorithms (and in this study) assume Gaussian error distributions with zero mean. Note that this is also the case when defining a likelihood function for accepting or rejecting parameter values in a global search method. To test the impact of a bias, we added a uniform value to the simulated $s_2$ variable in a second test (see Section 2.1.6) that was treated as an unknown bias, and therefore not corrected or accounted for in the inversion scheme. The impact of this bias on $s_1$ and $s_2$ is shown in Figures 1c-d, and the reduction in RMSE between the model and observations is seen in Fig. 3 for all variables (including $R_h$ and litterfall). The red symbols in Fig. 2 show the resultant parameter values and theoretical reduction in uncertainty as a result of the bias. The inversion cannot accurately find the correct values for all parameters in any test case and there are now considerable differences between the simultaneous and step-wise approach. Furthermore the order in which the data streams are assimilated in the step-wise cases also results in different posterior parameter values (test cases 2a and b versus 2c and d in Fig. 2a and Fig. 3). Nevertheless the optimisation results in a similar reduction in uncertainty on the parameters, except in test case 1b where only $s_2$ data are assimilated (Fig. 2b).

The main impact of the bias in the modelled $s_2$ variable is on the value of $k_2$ parameter (Fig. 2a), which is consistently offset from the true value (dashed line in Fig. 2a) in all test cases. This was expected given that it is the parameter most directly related to the calculation of $s_2$. However, in test cases 2a and 3a, the values of $p_1$ and $p_2$ are also incorrect (and $p_1$ for test case 2b). Note that these parameters only indirectly influence the $s_2$ pool in the model, and therefore we might have expected that they would be less affected by the bias. This nicely demonstrates one issue that could arise in all DA studies, where the bias in a particular variable (in the observations or the model) is aliased onto another process in the model (Wutzler and Carvalhais, 2014). Such an aliasing of bias onto indirectly related parameters is even more evident when only $s_2$ is included in the assimilation and $s_1$ does not provide any
constraint (test case 1b) – in this case all parameters are incorrect but the $p_2$ parameter in particular shows a strong deviation from the true value (Fig. 2a). As a result we see a deterioration in the RMSE for the $s_1$, litterfall and Rh variables in test case 1b and in the step-wise cases where $s_2$ is assimilated in the second step (Figures 3a, c and d – test case 1b, 2a and 2b). However, the RMSE reduction remains high for the $s_2$ variable for these test cases (Fig. 3b), as the inversion has found a solution that accounts for the bias even though all inferred parameter values are incorrect. The assimilation of $s_1$ in the second step lowers the reduction in RMSE for $s_2$ gained in the first step to ~70%, but it is not a considerable degradation.

Even though the posterior parameter values are incorrect, and despite the fact that the first step results in a degradation, the final reductions in RMSE are largely the same than the situation with no bias for all variables when $s_1$ is included in a simultaneous assimilation or optimised in the second step (test cases 2c, d and 3a in Fig. 3). This shows that the inclusion of $s_1$ observations can find a solution to counter the bias in $s_2$ and prevents a degradation in the fit to the data. If $s_2$ is assimilated in the second step there is a negative impact on all other variables as discussed above, demonstrating again that the order of data stream assimilation can matter when there are biases or inconsistencies between the data and the model.

The analysis of the impact of the bias presented here is specific to this model and the type and magnitude of the bias that was added, but the broader findings can be generalised to any situation in which there is a bias or inconsistency between a model and data that is not accounted for in the assigned error distributions. Exactly what might constitute a bias or inconsistency is discussed more in Section 3.2. Also note that it is important to examine the impact on the other variables. For the separate test case 1b in which only $s_2$ data are used to optimise the model, the negative impact on the other variables (Fig. 3) would have been concealed if we had only examined the posterior reduction in RMSE for the $s_2$ variable. Again this is a concern that is inherent to all DA experiments, whether single- or multi-data stream, but we can see from these results (i.e. by comparing the separate test cases 1b with 2a and b) that adding another data stream in a multi-constraint approach does not always reduce the problem.
2.2.3 Difference between the step-wise and simultaneous approaches in the presence of a non-linear model

As discussed in Section 2.2.1, there is little difference between the step-wise and the simultaneous assimilation approaches for simple, relatively linear models, unless the observation error (including measurement and model errors) distribution deviates strongly from the Gaussian assumption. However in reality, large-scale, complex LSMs may contain highly non-linear responses to certain model parameters. To demonstrate the impact of non-linearity in a multiple data stream assimilation context, we used a non-physically based toy model chosen for its non-linear characteristics (see Section 2.1.2).

Fig. 4a shows the posterior parameter values for both the \(a\) and \(b\) parameters of the non-linear toy model for all test cases. The values were not normalised as both parameters have the same range. The horizontal dashed line shows the “true” known values of the parameters (both equal to 1.0) that were used to generate the synthetic observations. Note that no bias has been introduced into the model in the results described here. The prior and posterior model \(s_1\) and \(s_2\) simulations for the non-linear toy model are compared to the synthetic observations in Fig. 5 for both step-wise cases in which the posterior error covariance matrix from step 1 (\(A_1\) – see section 2.1.4) was propagated to step 2 (experiments 2a and c – Fig. 5a-d) and both simultaneous cases 3a and b (Fig. 5 e-h). Finally Fig. 6 summarises the reduction in RMSE between the simulated and observed \(s_1\) and \(s_2\) variables for the non-linear toy model for all test cases and, in the step-wise cases, the reduction in RMSE after both the first and second steps (light versus dark green bars).

Assimilating each data stream individually (test cases 1a and b) does not result in an accurate retrieval of the posterior parameters (Fig. 4a), nor in a strong constraint on either parameter, as shown by the lack of theoretical reduction in the parameter uncertainty after the optimisation (Fig. 4b). Despite this, there is a 91-92% reduction in RMSE for the data stream that was included in the optimisation (i.e. for \(s_1\) in test case 1a – Fig. 6a, and \(s_2\) in test case 1b – Fig. 6b). However, the improvement on the other data stream is much less (28% reduction in RMSE for \(s_1\) when \(s_2\) is assimilated) or even results in a degradation compared to the prior fit (e.g. in the case of \(s_2\) when \(s_1\) is assimilated – Fig. 6b). Lack of improvement, or even degradation, in the RMSE of other variables in the model is a common issue for data assimilation in general – one that is not often evaluated in model-data fusion studies.
Only the simultaneous case, in which all $s_1$ observations have been included in the cost function (test case 3a), manages to retrieve the correct parameter values after the optimisation. All other posterior parameter values are incorrect, and are considerably different between each case, unlike for the simple carbon model (without a model bias). Most step-wise test cases (particularly 2b-d) do not result in the same parameter values as the simultaneous test case 3a in which all the observations are included (Fig. 4a), highlighting that strong non-linearity in the model sensitivity to parameters together with the use of an algorithm that is only adapted to weakly non-linear problems, as well as the assumption of linearity in calculating the posterior error covariance matrix at the minimum of the cost function, can result in differences between a step-wise and simultaneous approach in multiple – data stream assimilation (see Section 1).

In the simultaneous optimisation in which all observations are included (test case 3a) the posterior fit to the data dramatically improves for both the $s_1$ and $s_2$ data streams after the assimilation (blue dashed line in Fig. 5e and f). This was expected given that the correct values of the parameters were found. For the step-wise cases (test case 2a in Figures 5a and b, and test case 2c in Fig. 5c and d), the black dashed line shows the prior, and the posterior after step 1 is shown by green dashed line. In the step-wise assimilation we see two different scenarios depending on which data stream was assimilated first. In the first step the results are the same as the case where each individual data stream is assimilated separately. In both cases the first step results in a good fit to the data that was included in the optimisation in that step. When the $s_1$ data was assimilated in the first step (Fig. 5 first row), the fit to $s_2$ deteriorated after the optimisation (Fig. 5b green dashed line and Fig. 6b – test case 2a_s1), but when the $s_2$ data were assimilated first (Fig. 5 second row) the optimisation step did manage to achieve an improvement in the $s_1$ data stream (Fig. 5c green dashed line and Fig. 6a – test case 2c_s1).

In the second step the optimisation of $s_2$ in test cases 2a and b does not degrade the fit to $s_1$ when the full parameter error covariance matrix ($A_1$) is propagated between step 1 and 2 (Figures 5a blue curve and 6a 2a_s2). Furthermore optimising $s_2$ in the second step reverses the deterioration in $s_2$ caused by assimilating $s_1$ in the first step (Figures 5b blue curve and 6b 2a and b dark green bars). However, when $s_1$ data were assimilated in the second step (test cases 2c and d), we found that the good fit achieved with $s_2$ observations in the first step was effectively reversed (Fig. 5d blue curve). Therefore assimilating $s_1$ in the second step degraded the fit to the $s_2$ observations, even compared to the prior case (Fig. 6b, dark green
bars for test cases 2c and d). This nicely highlights one of the main possible issues with a step-wise assimilation framework.

The fact that the final reduction in RMSE values after both steps was ~90% for most cases, even though the values were not correct for all but case 3a (Fig. 4), indicates that the error correlation between the two parameters (~ -1.0 – calculated from the posterior error covariance matrix but not shown) led to alternative sets of values that resulted in a similar improvement to the data – a phenomenon known as model equifinality.

### 2.2.4 Order of assimilation of data streams and propagation of parameter error covariance matrices in a step-wise approach

Comparing the step-wise cases 2a and b with 2c and d for the non-linear toy model reveals that neither order in the assimilation, \( s_1 \) then \( s_2 \), or \( s_2 \) then \( s_1 \), results in the correct posterior parameter values that match the simultaneous test case (Fig. 4a). This is not a result that can be generalised to all step-wise assimilations as it will depend on the data stream involved and whether they contain enough information to accurately constrain all the parameters included in the optimisation, as well as any biases in the model or observations (as discussed in Section 2.2.2) or model non-linearity (section 2.2.3). In the case of the non-linear toy model, neither \( s_1 \) nor \( s_2 \) find the right parameter values when assimilated individually, therefore it is not surprising that neither order manages to achieve the right posterior parameter values. Nevertheless, the theoretical uncertainty of both parameters is reduced by >95% for the step-wise cases in which \( A_1 \) from step 1 is propagated between step 1 and 2 (test cases 2a and c – Fig. 4b), even though the posterior values for the step-wise cases are incorrect. This demonstrates that a good theoretical reduction in uncertainty is not always indicative that the right parameters have been found by the optimisation. The lower theoretical reduction in parametric uncertainty for cases 2b and d (Fig. 4b) demonstrates that information is lost between the steps if the posterior error covariance terms of \( A_1 \) after step 1 are not propagated to step 2, and therefore cannot be used to further constrain the optimisation.

From a mathematical standpoint the most rigorous approach is to propagate the full parameter error covariance matrices between each step. Without that constraint not only is information lost in the second step, but the information contained in the second data stream
may have a stronger influence compared to a simultaneous or step-wise case with a propagated error covariance matrix. The inversion may therefore be more vulnerable to any strong biases or incompatibilities between the model and the observations of the second data stream, or indeed the particular sensitivity of its corresponding model state variable to the parameters. This is one possible explanation for the degradation seen in $s_1$ in the non-linear toy model when $s_2$ is optimised in the second step and $A_1$ is not propagated between the steps (Fig. 6a test case 2b $s_2$). The same was also true for the simple carbon model for test case 2b when a bias was introduced into the $s_2$ simulation (see Section 2.2.2 and Fig. 3a).

However, the reverse is also true – if the first data stream contains strong biases then the associated error correlations will be also propagated with $A_1$. If autocorrelation in the observation errors, or indeed correlation between the errors of the data streams, is not accounted for it is likely that the posterior simulations are over-tuned, i.e. we will overestimate the reduction in parameter uncertainty. If this is the case and the first step results in incorrect parameter values, the propagation of $A_1$ could restrict the parameter values to the wrong location in the parameter space and thus inhibit the ability of the inversion to find the correct global minimum. These issues are likely to be more considerable for non-linear models, as seen by the lack of difference between test cases 2a-d in the simple carbon model example (Fig. 2).

2.2.5 Lessons to be learned when dealing with non-linearity

Most optimisation studies with a large-scale LSM use derivative methods based on a least-squares approach, and therefore rely on assumptions of Gaussian probability and linear model sensitivity. However, if the model is weakly non-linear within the probability distribution around the point in parameter space that is being analysed (see Tarantola, 1987, p72), it is possible to use an iterative algorithm, such as the one described in Eq. (6), to find the minimum of the cost function (i.e. the maximum likelihood of the posterior parameter estimation (minimum of $J(x)$)) of the parameters can be used to calculate the posterior error covariance (see Eq. (6)). If the model is too strongly non-linear and therefore these assumptions are not met, it may not be possible to find the true global minimum of the cost function and the characterisation of the posterior probability distribution will be incorrect.
This is a particular problem if the posterior parameter error covariance matrix is then propagated in a step-wise approach, although these issues are relevant to both step-wise and simultaneous assimilation. Note that performing a number of tests starting from different random “first guess” points in parameter space can help to diagnose if the global minimum has been reached, as outlined in Section 2.1.6 and discussed at the beginning of the results (Section 2.2).

It is possible to avoid dealing with issues related to non-linearity in the model sensitivity and non-Gaussian error distributions by using a global search method (e.g. Markov Chain Monte Carlo or a genetic algorithm) that randomly, but effectively, searches the entire parameter space. However in large dimensional problems, as is likely the case when optimising a LSM at large scales with multiple data streams, using a global search method is likely not feasible due to computational time constraints. In these cases, a derivative method is likely the only option.

An important finding of the results presented for the non-linear toy model in Section 2.2.3 is that degradation in another data stream is not necessarily the result of a bias or incompatibility between the observations and the model. Rather if the model sensitivity to the parameters is very non-linear, multiple combinations of parameter values may exist that result in a similar reduction of the cost function (multiple minima), but provide a different fit to each data stream. Even though all data streams may be sensitive to all the parameters, the information content of each will not be the same. Finding the true global minimum in this instance may require a bit more careful thought in planning the assimilation set-up, and may depend on having a reasonable idea of the level of information each data stream can bring to constrain each parameter. It may be the case that one data stream has a higher non-linear sensitivity to the parameters and therefore may act as an “troublemaker” and pull the parameters in a direction that results in a degradation to the other data streams, as seen in Section 2.2.3. If a simultaneous optimisation is not possible, it may be useful under such circumstances to identify any “troublemaker” data streams, and assimilate them in the first step of the optimisation. In the second step “peacemaker” data streams, with a lower non-linear sensitivity to the parameters, will then find a compromise set of parameter values that can fit both data streams well, provided the full posterior parameter error covariance matrix is propagated between the steps in order to retain all the information brought by the first data stream. As discussed this could be an explanation for the results seen for the non-linear toy.
model test case 2a where $s_1$ was assimilated prior to $s_2$ (Figures 6a and b) as discussed in Section 2.2.3.

3  Examples from existing carbon cycle data assimilation studies

3.1  Extra constraint from multiple data streams

Most site-based carbon cycle data assimilation studies have used eddy covariance measurements of NEE and LE fluxes to constrain the relevant parameters of ecosystem models. However, a few studies have also made use of chamber flux soil respiration data and field measurements of vegetation characteristics (e.g. tree height, budburst, LAI) or estimates of litterfall and carbon stocks as ancillary information (e.g. Keenan et al., 2012; Thum et al., in review; Van Oijen et al., 2005; Richardson et al., 2010; Williams et al., 2005). Two recent studies combined high-resolution satellite-derived FAPAR data and in-situ eddy covariance measurements to optimise parameters related to carbon, water and energy cycles of the ORCHIDEE and BETHY LSMs (Bacour et al., 2015; Kato et al., 2013, respectively).

At global scales the number of studies that use multiple data streams from satellites or large-scale networks to optimise LSMs has been increasing in recent years, although this remains a relatively new area of research. CCDAS-BETHY was the first global carbon cycle data assimilation system (CCDAS) making use of the high-precision measurements of the atmospheric CO$_2$ concentration flask sampling network (Rayner et al., 2005; Scholze, 2003) to constrain process parameters of the prognostic terrestrial carbon cycle model BETHY (Knorr, 2000). Since its first application assimilating atmospheric CO$_2$ concentration data only, CCDAS-BETHY has been further developed to consistently assimilate multiple data streams both at local and global scales. In particular, Kaminski et al. (2012) optimised 70 process parameters plus one initial condition by simultaneously assimilating a satellite-derived FAPAR product derived from the Medium Resolution Imaging Spectrometer (MERIS; Gobron et al., 2008) and flask samples of atmospheric CO$_2$ at two sites from the GLOBALVIEW product (GLOBALVIEW-CO2, 2008) on a coarse resolution. More recently, Scholze et al. (2015) demonstrated the added value of assimilating remotely sensed soil moisture data in addition to observations of atmospheric CO$_2$ concentration from the flask-sampling network. They used the same coarse resolution set-up of CCDAS as Kaminski et al. (2012) and CO$_2$ observations from 10 sites of the GLOBALVIEW product (GLOBALVIEW-

Two other global CCDAS based on different LSMs have been developed in recent years (Peylin et al., 2016; Schürmann et al., 2016). Schürmann et al. (2016) optimized model parameters and initial conditions of the land component JSBACH (Raddatz et al. 2007) of the MPI Earth System Model (ESM) (Giorgetta et al. 2013) using atmospheric CO2 concentration data and the TIP-FAPAR product (Pinty et al., 2007) as joint constraints over a 5 year period in addition to evaluating the mutual benefit of each data stream in a fully factorial design. Peylin et al. (2016) used three different data streams as global constraints for the ORCHIDEE LSM (Krinner et al., 2005), which forms the land surface component of the IPSL ESM (Dufresne et al., 2013), in a multi-site step-wise assimilation approach. First, satellite-derived vegetation index data (NDVI) from the MODIS instrument was used to constrain the phenology parameters at 60 sites for the temperate and boreal deciduous PFTs, followed by NEE and LE observations at 78 FLUXNET sites for 7 PFTs to optimise all the carbon-related parameters, and finally atmospheric CO2 concentration measurements from 53 sites in the GLOBALVIEW network (GLOBALVIEW-CO2, 2013), which predominantly provided a constraint on the initial magnitude of the soil carbon reserves in the model. Atmospheric CO2 concentration observations are one of the most accurate, long-term data sets in environmental science and they provide important information about the global CO2 sink capacity by land and ocean. These three global multiple data stream CCDAS have allowed an improvement in both the mean seasonal cycle as well as the trend of net land surface CO2 exchange, especially with the inclusion of the atmospheric CO2 data (Kaminski et al., 2012; Peylin et al., 2016; Schürmann et al., 2016).

Many of the aforementioned studies reported that adding extra data streams helped to constrain unresolved sub-spaces of the total parameter space. Scholze et al. (2015) found that adding SMOS soil moisture data to the assimilation simultaneously with CO2 observations reduced the ambiguity in the solution space when assimilating CO2 only, and the multiple data constraint was able to resolve a much larger sub-space in parameter space (about 30 parameters out of the 101 compared to 15 without SMOS data). Bacour et al. (2015) and Schürmann et al. (2016) both reported that the addition of FAPAR data bought extra information on the phenology-related processes in the model, and therefore retrieved different posterior C flux-related parameter values than when assimilating NEE or atmospheric CO2.
data alone. An interesting aspect of the Kaminski et al. (2012) study was that the inclusion of FAPAR in addition to atmospheric CO$_2$ concentration samples resulted in a particular improvement for the hydrological fluxes in the model, thus demonstrating the importance of assessing the potential benefit for model variables that may not have been the main target of optimisation. Richardson et al. (2010) concluded that using ancillary information (e.g. woody biomass increment, field-based LAI and chamber measurements of soil respiration) as orthogonal constraints to NEE data provided a valuable added constraint on many model parameters, which improved both the bias in model predictions and reduced the associated uncertainties. Thum et al. (in review) also found that the addition of aboveground biomass stocks brought a longer-term constraint on allocation parameters and mortality/turnover processes. However, they noted an incompatibility when assimilating both annual increment and total biomass data, as the total stocks take into account losses related to disturbance and management (e.g. canopy thinning) – processes that were not included in that version of the model. Keenan et al. (2012) also argued that the use of such ancillary constraints is essential for a better partitioning of net carbon fluxes into their gross components. However, Williams et al. (2005) observed that one-off, or rarely taken, measurements of carbon stocks were unable to constrain components of the carbon cycle to which they were not directly related.

This calls into question the issue of the influence of different data streams in a joint assimilation, especially if the number of observations for each is vastly different which is the case when assimilating both half-hourly C flux data in addition to soil C stock observations that are typically available at an annual time scale. The spatial coverage of each data stream is also important, especially for heterogeneous landscapes (Barrett et al., 2005). Test case 3b, in which only one observation was included for the $s_2$ data stream instead of the complete time-series, shows that a substantial difference in number of observations between the data streams can influence the resulting parameter values and posterior uncertainty (compare test cases 3a and b in Fig. 2 for the simple C model and Fig. 4 for the non-linear toy model) as each data stream will have a different overall “weight” in the cost function. However, the impact of having a different number of observations for each data stream in the cost function also depends strongly on the prescribed observation error and relative sensitivity of each corresponding model variable to the model parameters. If one variable has a greater sensitivity than the other, it will matter less if fewer observations of that variable are included in the cost function.
Xu et al. (2006), among others, have mentioned the possible need to weight the cost function for different data sets. Different arguments abound on this issue. Some contend that the cost function should not be weighted by the number of observations because the error covariance matrices (B and R) already define this weight in an objective way (e.g. Keenan et al., 2013). Certainly it should not be necessary to weight by the number of observations in the cost function if there is sufficient information to properly build the prior error covariance matrices (B and R). On the other hand, it is a difficult task to characterise the model structural uncertainty and the observation error correlations (see Kuppel et al., 2013 for practical solutions). Given this, our expert knowledge on the workings of the model processes and the sensitivity of the model to the parameters may permit us to specify a stronger weight to a data stream that could help to constrain a particular section of the model, but for which there are only a few data points. Clearly the definition of the prior error model, including for the covariance between errors of the data streams, is of the utmost importance (Trudinger et al., 2007) and merits close attention in future multiple data stream assimilation studies.

Although a number of multiple data stream assimilation studies exist at various scales, very few studies have specifically investigated the added benefit of different combinations of data streams, with a few notable exceptions (Barrett et al., 2005; Richardson et al., 2010; Kato et al., 2013; Keenan et al., 2013; Bacour et al., 2015; Schürmann et al., 2016). Kato et al. (2013) and Bacour et al. (2015) both evaluated the complementarity of eddy covariance and FAPAR data streams at site level, i.e. the impact of assimilating one individual data stream on the other model state variable, as well as when both data streams were included in the optimization (see discussion in Section 3.2). The study of Keenan et al. (2013) was particularly notable in its aim to quantify which data streams provide the most information and how many data streams are actually needed to constrain the problem. They reported that of the 17 field-based data streams available, projections of future carbon dynamics were well-constrained with only 5 of the data sources, and crucially, not with eddy covariance NEE measurements alone. These results may be specific to this site or type of ecosystem, but this study highlights the need for further research in this area, and in particular, for synthetic data experiments that allow us to understand which data will be the most useful for a given scientific question. This will also enable researchers to plan more efficient measurement campaigns with experimentalists, as also pointed out by Keenan et al. (2012).
3.2 Issue of bias and inconsistencies between the observations and the model

Despite the theoretical benefit of adding data streams into an assimilation system as orthogonal constraints, several of the aforementioned studies at both site and global scale have reported a bias or inconsistency either between the different observation data streams, or between the observations and the model. This is easily detected when the optimisation of one data stream results in a worse fit than the prior in one or more of the other data streams, as seen in Section 2.2.2. Kato et al. (2013) assimilated SeaWiFS FAPAR (Gobron et al., 2006) and eddy covariance LE measurements at the FLUXNET site in Maun, Botswana. They showed that the individual assimilation of each the two data streams resulted in a perfect (i.e. within the observational uncertainty) fit to the assimilated data set, but a considerable degradation of the fit to the non-assimilated data set compared to the prior. A comparison against eddy covariance measurements of gross carbon uptake (gross primary production – GPP) hinted to a bias problem with the FAPAR data because the fit to the independent GPP data was degraded after assimilating FAPAR data only, while the fit improved after assimilating the LE data only. Nevertheless, the simultaneous assimilation of both data streams achieved a compromise between the two suboptimal states reached after assimilating only one data stream. The calibration further limited the number of parameters with correlated errors, and yielded a higher theoretical reduction in parameter uncertainty and a decrease in the RMS difference by 16% for the GPP data compared to the prior.

Bacour et al. (2015) also noted that when assimilating both in-situ and satellite-derived FAPAR data (from the SPOT and MERIS instruments) and in-situ NEE and LE flux data from two French FLUXNET sites into the ORCHIDEE LSM both separately and together, the posterior parameter values changed significantly for the photosynthesis and phenology-related parameters, depending on the bias between the model and the observations and the correlation between the parameter errors. If NEE data were assimilated alone there was an even stronger positive bias (model–observations) in the start of leaf onset in the FAPAR data than in the prior simulations, and no improvement in the maximum value. This was likely due to the fact that there were enough degrees of freedom to fit the NEE without changing the phenology-related parameters. Similarly, the fit to the NEE was degraded when the model was only optimized with FAPAR data. The model was able to fit the maximum FAPAR but this resulted in an adverse effect on the carbon assimilation capacity of the vegetation. The
authors argued this was related to incompatibilities between the FAPAR and both the model and NEE measurements, possibly due to its larger spatial footprint of the satellite-derived FAPAR data and/or inaccuracies in the retrieval algorithm. However, given that assimilating in-situ FAPAR also degraded the fit to the NEE, another culprit may be an inconsistency between the model and the data. The authors suggested this could be due to the different assumptions or characterisation of a variable in a model compared to what is described in the data. For example, satellite-derived greenness measures (FAPAR/NDVI) also contain information on the non-green elements of vegetation, but the model only simulates green LAI. Furthermore parameters and processes in models have been developed at certain temporal and spatial scales. Vegetation is often simply represented as a “big leaf” model in LSMs, taking no account of vertical canopy structure or the spatial heterogeneity in a scene, which is an additional source of inconsistency with what is measured. The joint (simultaneous) assimilation of all three data streams in Bacour et al. (2015) reconciled the different sources of information, with an improvement in the model-data fit for NEE, LE and FAPAR. However, the compromise achieved in the joint assimilation was only possible when the FAPAR data were normalised to their maximum and minimum values, which thus partially accounted for any bias in the magnitude of the FAPAR or inconsistency with the model.

The story of biases and apparent inconsistencies in FAPAR data doesn’t end there. A bias correction was also necessary in the study by Kaminski et al. (2012) with CCDAS-BETHY using the MERIS FAPAR product in addition to atmospheric CO\textsubscript{2} data (see above). They found that optimisation procedure failed when using the original FAPAR product because the FAPAR values were biased towards higher values. Only after applying a bias correction on the FAPAR data before the assimilation procedure was the optimisation successful. Schürmann et al. (2016) also reported the need to reduce a prior model bias in FAPAR. Even though the assimilation corrected successfully for this FAPAR bias, an imprint of the prior bias was evident in the spatial patterns of the modelled heterotrophic respiration. Assimilating FAPAR data alone therefore resulted in a slight degradation in the net C flux and consequently led to incorrect simulations of the atmospheric CO\textsubscript{2} growth rate. The addition of CO\textsubscript{2} as a constraint prevented this degradation and resulted in a compromise in which FAPAR helped to disentangle these processes and find different parameter values compared to the CO\textsubscript{2}-only case, thus improving the fit to both data streams. Forkel et al. (2014) discovered an apparent inconsistency between satellite-derived FAPAR and GPP data in tundra regions when using these data (plus satellite-derived albedo) to optimise the LPJmL...
LSM. They too speculated that the data might be positively biased, in this case due to issues with satellite measurements taken at high sun zenith angles. However, they gave alternative suggestions, one being that an inadequate model structure may be at fault – for example, the LPJmL does not include vegetation classes corresponding to shrub, moss and lichen species that are dominant in these ecosystems. They also noted that the GPP product they used, which is based on a model tree ensemble up-scaling of FLUXNET data (Jung et al., 2011), might contain representation-related biases, given that there are very few FLUXNET stations in tundra regions. The issue of representation errors of sites has been touched upon before (e.g. Raupach et al., 2005). Alton (2013), who performed a global multi-site optimisation of the JULES LSM with a diverse range of data including satellite-derived LAI, FLUXNET, soil respiration and global river discharge, raised the point that FLUXNET sites are known to be large carbon sinks, which could potentially result in biased global NEE estimates. Resolving these apparent inconsistencies was beyond the scope of most of these studies, aside from applying a bias correction where one was evident. Nonetheless this issue clearly merits further attention if the increasing number of available datasets is to be fully utilised.

### 3.3 Step-wise versus simultaneous assimilation

The paper by Alton (2013) documents the only previous study to have used a step-wise assimilation approach with more than two data streams, stating that the final parameter values were independent of the order of data streams assimilated. No studies in the LSM community to date have explicitly examined a step-wise versus simultaneous assimilation framework with the same optimisation system and model. The step-wise assimilation with the ORCHIDEE-CCDAS detailed in Peylin et al. (2016) has been compared to a simultaneous optimisation using the same three data streams as part of an on-going study. At each step, the resulting simulations (using the posterior parameters) were compared to the data stream from the previous steps. The fit to the MODIS NDVI (used in a similar manner to FAPAR as a proxy for vegetation greenness) was unchanged after further optimization of the phenology-related parameters in the second and third steps using in-situ flux and atmospheric CO₂ concentration data. In the simultaneous optimisation, the addition of NEE or atmospheric CO₂ concentration measurements resulted in a lower improvement to the fit to MODIS NDVI. As the NDVI data were normalised this was not a result of a simple bias in the magnitude of the data. Rather, it was likely due to inconsistencies between the model and data as discussed by...
Bacour et al. (2015) and in Section 3.2. It is important to reiterate that there should be no difference between the step-wise and the simultaneous given an adequate description of the error covariance matrices and compliance with the assumptions associated with the inversion algorithm used. However, in practice it is very difficult to define a PDF that properly characterises the model structural uncertainty and observation errors accounting for biases and non-Gaussian distributions. This leads to issues within a simultaneous assimilation, particularly if the information content of one data stream is much higher, and a greater risk of differences between a step-wise and simultaneous assimilation. As discussed in Section 2.2.5 a step-wise assimilation may be useful on a provisional basis for dealing with possible inconsistencies. In the step-wise approach of Peylin et al. (2016) the error covariance of the phenology-related parameters was strongly constrained by the satellite data in the first step (and was propagated to the second step), the later assimilations with NEE and atmospheric CO₂ data in steps 2 and 3 found alternative solutions for the C flux-related parameters that provided a reasonable fit to all data streams. Wherever possible however, a simultaneous optimisation is favourable because the strong parameter linkages between different processes are maintained, and therefore biases and inconsistencies between the model and observations should be addressed prior to optimisation.

4 Advice for Land Surface Modellers

Although it is clear that in many cases, increasing the number of observations in a model optimisation provides additional, orthogonal constraints, challenges remain that should not be ignored. Based on the simple toy model results presented in this study, in addition to lessons learned from existing studies, we recommend the following points when carrying out multiple data stream carbon cycle data assimilation experiments:

- Devote time to characterising the error structure for the observations and parameter error distributions, including their correlations (Raupach et al., 2005). For the observations this should include the model structural errors (Kuppel et al. 2013), the temporal or spatial autocorrelation and correlation between different data streams.
- In the case of non-Gaussian error distributions consider performing a transformation to make the distributions more Gaussian, or avoid a least squares
formulation and instead use a method that avoids outliers (e.g. absolute deviations – Trudinger et al., 2007).

- Analyse and correct for biases in the observations, or approximately account for it in the observation error covariance matrix, $R$, using the off-diagonal terms or inflated errors (Chevallier, 2007), or by using the prior model-data RMSE to define the observation uncertainty.

- Investigate potential incompatibilities between your model and data. Take time to understand which physical quantities your data correspond to and whether that is consistent with the description of the equivalent variable in the model. As for the previous point, one way of attempting to account for unknown inconsistencies between the model and data is to set the observation uncertainty, $R$, the prior RMSE between the model and the data.

- Evaluate the impact on other model variables with independent observations, and if the optimisation degrades the fit compared to the prior, investigate the reasons behind the inconsistency and address them as above.

- Assess the non-linearity of your model (multiple first guess tests can help in this regard), and if strongly so, avoid a least squares formulation of the cost function or use global search algorithms for the optimisation – although at the resolution of typical LSM simulations ($\geq 0.5 \times 0.5^\circ$) this will likely only be computationally feasible at site or multi-site scale.

- Prior information is key in a Bayesian framework. Effort should be put into better constraining the prior parameter bounds of all parameters based on literature wherever possible.

- Conduct preliminary sensitivity analyses to determine which parameters should be constrained by each data stream.

- Set up experiments with synthetic data, as in this study, to understand the constraints posed by the different data streams you will include in the experiment.

- If technical constraints require a step-wise approach is used it is preferable (from a mathematical standpoint) to propagate the full parameter error covariance matrix.
between each step, if it can be calculated, and carefully consider the order of the assimilation of data streams (a synthetic experiment will aid in this regard).

- Be aware that a good theoretical reduction in model or parameter uncertainty can be misleading, as it is not necessarily indicative that the right parameter values have been found. If this is the case, it could impact predictions made outside the spatio-temporal window included in the optimisation.

Many of these issues are relevant to any data assimilation study, including those only using one data stream. However, most are more pertinent when considering more than one source of data. The impact of bias in the magnitude of satellite-derived FAPAR data has featured highly in past multiple data stream assimilation studies. Aside from simple corrections, Quaife et al. (2008) and Zobitz et al. (2014) suggested that LSMs should be coupled to radiative transfer models to provide a more realistic and mechanistic observation operator between the quantities simulated by the model and the raw radiance measured by satellite instruments. This proposition followed the experience gained in the case of atmospheric models for several decades (Morcrette, 1991).

Other promising directions could also be considered to help constrain the problem of lack of information in resolving the parameter space, including the use of other ecological and dynamical “rules” that limit the optimisation (see for example Bloom and Williams, 2015), or the addition of different timescales of information extracted from the data such as annual sums (e.g. Keenan et al., 2012). Of course, optimising the parameters of the model will not account for all the uncertainty in a model. Inaccurate or incomplete process representation is likely a key factor that may also bias the posterior values retrieved in any optimisation. Keenan et al. (2012) reflected that despite using multiple different constraints and different time increments in the cost function, the inter-annual variability and long-term trend of carbon uptake at Harvard forest FLUXNET site in the USA could not be reproduced without a temporal variation of the parameters, suggesting a missing process in the model. However, as this paper shows, the complexities of model-data fusion require that we continue to develop DA techniques alongside development of LSMs, with the hope of converging upon more reliable and accurate predictions of the global C budget in the near future. Finally we should also seek to develop collaborations with researchers in other fields who may have advanced further in a particular direction. Members of the atmospheric and hydrological modelling
communities, for example, have implemented techniques for inferring the properties of the
prior error covariance matrices, including the mean and variance, but also potential biases,
autocorrelation and heteroscedasticity, by including these terms as “hyper-parameters” within
the inversion (e.g. Michalak et al. 2005; Evin et al., 2014; Renard et al., 2010; Wu et al.
2013). Of course this extends the parameter space – making the problem harder to solve
unless sufficient prior information is available (Renard et al., 2010), but such avenues are
worth exploring.

5 Conclusions

In this study we have attempted to highlight and discuss some of the challenges
associated with using multiple data streams to constrain the parameters of LSMs, with a
particular focus on the carbon cycle. We demonstrated some of the issues using two simple
models constrained with synthetic observations for which the ‘true’ parameters are known.
We performed a variety of tests in Section 2 to demonstrate the differences between
assimilating each data stream separately, sequentially (in a step-wise approach) and together
in the same assimilation (simultaneous approach). In particular we focused on difficulties that
may arise in the presence of biases or inconsistencies between the data and the model, as well
as non-linearity in the model equations. In Section 3 we discussed the experimental results
with reference to similar difficulties that have been documented in recent C cycle assimilation
studies.

Many of the issues faced are inherent to all optimisation experiments, including those in
which only one data stream is used. It is of utmost importance to determine if the
observations contain biases, and/or if inconsistencies or incompatibilities exist between the
model and the observations, and to correct for this or properly account for this in the error
covariance matrices. Secondly it is crucial to understand the assumptions and limitations
related to the inversion algorithm used. Without these two points being met, there is a greater
risk of obtaining incorrect parameter values, which may not be obvious by examining the
posterior uncertainty and model-data RMSE reduction. Furthermore it is more likely that the
implementation of a step-wise versus simultaneous approach will lead to different results.

This study was not able to examine an exhaustive list of all possible challenges that may
be faced when assimilating multiple data streams, but we hope that this tutorial style paper
will serve as a guide for those wishing to optimise the parameters of LSMs using the variety
of C cycle related observations that are available today. Furthermore we hope that by increasing awareness about the possible difficulties of model-data integration, we can further bring the modelling and experimental communities together to work more closely on these issues.

**Code availability**

The model and inversion code will be made available via the ORCHIDAS website (upon registration): [https://orchidas.lsce.ipsl.fr/multi_data_stream.php](https://orchidas.lsce.ipsl.fr/multi_data_stream.php).

**Acknowledgements**

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Thum, T., N. MacBean, P. Peylin, C. Bacour, D. Santaren, B. Longdoz, D. Loustau and P. Ciais, The potential benefit of using forest biomass data in addition to carbon and water flux measurements to constrain ecosystem model parameters: case studies at two temperate forest sites. In revision for Agric. For. Meteorol.


Table 1: The optimisation set-up for both models, including the true parameter values, their range and the observation uncertainty (1 sigma). The parameter uncertainty (1 sigma) was set to 40% of the range for each parameter.

<table>
<thead>
<tr>
<th>Model</th>
<th>Parameter value (range)</th>
<th>Observation uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simple carbon</td>
<td>$p_1$ 1 (0.5,5) 1 (0.5,5) $k_1$ 0.2 (0.03,0.9) $k_2$ 0.1 (0.01,0.12)</td>
<td>$s_1$ 0.5 $s_2$ 5</td>
</tr>
<tr>
<td>Non-linear toy model</td>
<td>$a$ 1 (0,2) $b$ 1 (0,2)</td>
<td>$s_1$ 0.5 $s_2$ 0.5</td>
</tr>
</tbody>
</table>
Table 2: List of experiments performed for both models with synthetic data. All parameters are optimised in all cases (therefore in both steps for the step-wise approach).

<table>
<thead>
<tr>
<th>Test case</th>
<th>Step 1</th>
<th>Step 2</th>
<th>Parameter error covariance terms propagated in step 2?</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Separate</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1a</td>
<td>$s_1$</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>1b</td>
<td>$s_2$</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td><strong>Step-wise</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2a</td>
<td>$s_1$</td>
<td>$s_2$</td>
<td>yes</td>
</tr>
<tr>
<td>2b</td>
<td>$s_1$</td>
<td>$s_2$</td>
<td>no</td>
</tr>
<tr>
<td>2c</td>
<td>$s_2$</td>
<td>$s_1$</td>
<td>yes</td>
</tr>
<tr>
<td>2d</td>
<td>$s_2$</td>
<td>$s_1$</td>
<td>no</td>
</tr>
<tr>
<td><strong>Simultaneous</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3a</td>
<td>$s_1$ and $s_2$</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>3b</td>
<td>$s_1$ and only 1 obs for $s_2$</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>
Figure 1: Prior and posterior model simulations compared to the synthetic observations for the simple carbon model for test case 3a for a) $s_1$ and b) $s_2$ simulations without any model bias, and c and d) with bias in the simulated $s_2$ variable. The coloured error band on the prior and posterior represents the propagated parameter uncertainty (1 sigma) on the model state variables (in the equivalent colour as the mean curve). This is mostly visible for the prior model simulation (pink band) as there is a high reduction in model uncertainty reduction as a result of the assimilation.
Figure 2: a) Normalised posterior parameter values and b) posterior parameter error reduction for all parameters of the simple carbon model for each test case, and for both the simulations with no bias (blue) and simulations with a bias in the $s_2$ variable that was not accounted for in the inversion (red). In a) parameters values were normalised to account for differences in the magnitude of the different parameters and their range, thus it is a measure of the distance from the true value as a fraction of the range and is calculated as: (posterior value – true value / max parameter value – minimum parameter value). The closer the value to the zero dashed line represents a better match to the “true” parameter value. To give an indication of the optimisation performance, the following are the normalised first guess parameter values for this particular example test (compare with posterior values in Fig. 2a): $p_1$ 0.09, $p_2$ 0.29, $k_1$ 0.1, $k_2$ 0.15.
Figure 3: Reduction in RMSE for all test cases for simulations with a bias in the $s_2$ variable: a) $s_1$, b) $s_2$, c) litterfall and d) heterotrophic respiration (Rh). For the step-wise cases (2a, b, c and d) the reduction after both step 1 and step 2 are shown in light and dark green respectively, and are denoted in the x-axis labels with ‘_s1’ for step 1 and ‘_s2’ for step 2. The reduction (in %) is calculated as $1 - (\text{RMSE}_{\text{post}} / \text{RMSE}_{\text{prior}})$. 
Figure 4: Posterior parameter values of both the non-linear toy model $a$ and $b$ parameters for each test case for the simulations with no model bias. The y-axis range corresponds to the parameter bounds and the dashed horizontal line represents the “true” known value of both parameters. To give an indication of the optimisation performance, the following are the first guess parameter values for this particular example test (compare with posterior values in Fig. 4a): $a$ 0.87, $b$ 1.98. b) Posterior uncertainty reduction for both parameters for all test cases.
Figure 5: Prior and posterior model simulations compared to the synthetic observations for the non-linear toy model (with no bias) for both the $s_1$ (left column) and $s_2$ (right column) variables for a) and b) test case 2a (1st row) – step-wise approach with $s_1$ observations assimilated in the first step, followed by the $s_2$ observations in the second step; c) and d) test case 2c (2nd row) – step-wise approach with $s_2$ observations assimilated in the first step, followed by $s_1$ observations in the second step; and e) and f) test case 3a (3rd row) – the simultaneous case in which both data streams were included. For both step-wise examples $A_1$ was propagated between the 1st and 2nd steps. The coloured error band on the prior and posterior represents the propagated parameter uncertainty (1 sigma) on the model state variables (in the equivalent colour as the mean curve). This is mostly visible for the prior.
model simulation (pink band) as there is a high reduction in model uncertainty reduction as a result of the assimilation.
Figure 6: Reduction in RMSE for all test cases for both a) $s_1$ and b) $s_2$ variables for the non-linear toy model simulations with no model bias. For the step-wise cases (2a, b, c and d) the reduction after both step 1 and step 2 are shown in light and dark green respectively, and are denoted in the x-axis labels with '_s1' for step 1 and '_s2' for step 2. The reduction (in %) is calculated as $1 - (\text{RMSE}_{\text{prior}} / \text{RMSE}_{\text{post}})$. 

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