S1 Supplementary Material

S1.1 Planetary Waves

The azonal component describes quasi-stationary planetary waves. The calculation depends on the level of height. At the equivalent barotropic level, azonal components of horizontal velocity are computed employing the definition of the stream function

\[
\langle u^*_{EBL}(z) \rangle = -\nabla \phi \langle \psi^*_{EBL} \rangle \tag{S1}
\]

\[
\langle v^*_{EBL}(z) \rangle = \nabla \lambda \langle \psi^*_{EBL} \rangle \tag{S2}
\]

whereby the azonal component are computed assuming isothermal expansion of air parcels in planetary waves

\[
\langle p^*_e \rangle = \langle p^*_{EBL} \rangle \exp \left( \left( z - z_{EBL} \right)/H_0 \right) + \frac{\rho \cdot g}{\Gamma R} \exp \left( -z/H_0 \right) \left\{ \ln \left( \frac{T(z)}{T(z_{EBL})} \right) - \ln \left( \frac{T(z_{EBL})}{T(z_{EBL})} \right) \right\} \tag{S3}
\]

and

\[
\langle p^*_{EBL} \rangle = \rho \left\{ \langle u^*_{EBL} \rangle \right\} \nabla \phi \langle \psi^*_{EBL} \rangle + 2 \left( \frac{\langle u^*_{EBL} \rangle}{a \cos(\phi)} + \Omega \right) \sin(\phi) \langle \psi^*_{EBL} \rangle \tag{S4}
\]

S1.1.1 Planetary Waves – orographically stream function:

For the waves excited by the orography, the stream function is calculated by

\[
\beta \nabla \lambda \langle \psi^*_{or,0,EBL} \rangle = -\frac{f}{H_0} \langle w_{or} \rangle + \frac{f^2 \partial (u'v')^*}{g} \frac{\partial}{\partial z} \tag{S5}
\]

where \( f \) is the Coriolis parameter and \( \beta = \nabla \phi f \) and
\[ w_{or} = (u)\nabla \varphi h_{or} + (v)\nabla A h_{or} + a_{std} (\langle u \rangle^2 + \langle v \rangle^2 + \langle u'^2 \rangle + \langle v'^2 \rangle)^{1/2} h_{std}. \tag{S6} \]

The variable \( h_{or} \) describes the grid cell averaged orography height \( h_{std} \) the subgrid scale standard deviation of the height of mountains, and \( a_{std} \) is an additional tuning parameter.

The azonal component describes quasi-stationary planetary waves and is subdivided into a geostrophic and ageostrophic term:

\[ u^* = u^*_{geo} + u^*_{ageo} \]

\[ v^* = v^*_{geo} + v^*_{ageo} \]

S1.2 Derivation of the zonal mean meridional wind velocity

The zonal mean meridional wind velocity \( \langle v(z, \phi) \rangle \) which also accounts for convective contribution is calculated by

\[
\langle v(z, \phi) \rangle = d_1 \ast (-2 \tan(\phi) \left( \langle u^* v^* \rangle + \langle u' v' \rangle \right)) + d_2 \ast \left( \frac{\partial}{\partial \phi} \left( \langle u^* v^* \rangle + \langle u' v' \rangle \right) \right) + d_3 \ast \left( \left( - \frac{dK_z}{z} + \frac{K_z}{H_0} \frac{\partial \langle u \rangle}{\partial z} \right) \langle u \rangle \right) + d_4 \ast (A)
\]

\[ = n_1 \ast (\tan(\phi) \langle u \rangle) + n_2 \ast \left( \frac{\partial \langle u \rangle}{\partial \phi} \right) + n_3 \ast \left( 2\Omega a \sin(\phi) \right). \tag{S7} \]

With \( K_z = 0.005 \) and \( \ln(4) \)

\[ A = \left( \frac{P_{conv} L}{(\Gamma_a - \Gamma)} - \frac{1}{H_0} \right) \langle u_{z,profile} \rangle \]

Whereby \( \Gamma_a \) is the lapse rate in the troposphere calculated by using the formula from Petoukhov (Petoukhov et al., 2000), \( P_{conv} \) by the cloud module implemented by Eliseev et al. (Eliseev, n.d.) and

\[ \langle u_{z,profile} \rangle = \begin{cases} 2, & |\phi| > 40 \\ -2 \cos \left( \phi \frac{\pi}{40°} \right), & \text{otherwise} \end{cases} \]

The additional calculating of \( \langle u_{z,profile} \rangle \) instead of using the calculated surface zonal velocity is done to avoid instabilities.

For the derivation we start with the differential equation of the zonal wind component
\[
\frac{du}{dt} = \frac{\tan \phi}{a} uv + f v - \frac{1}{\rho} \Delta_3 p + F_u
\]

Whereby \(a\) is the Earth radius, \(f\) is the Coriolis factor and \(F_u\) is the frictional force in \(u\)-direction. Multiplying the equation with \(\rho\) and using that \(\rho \frac{du}{dt} = \frac{d(\rho u)}{dt} - u \frac{d\rho}{dt} = \frac{\partial (\rho u)}{\partial t} + V \cdot \Delta (\rho u)\) and \(V \cdot \Delta (\rho u) = \Delta (\rho u V) - (\rho u) \Delta \cdot V\), we get

\[
\frac{\partial (\rho u)}{\partial t} + \Delta (\rho u V) - u \left( \frac{d\rho}{dt} + (\rho u) \Delta \cdot V \right) = \frac{\tan \phi}{a} \rho uv + f \rho v - \Delta_3 p + \rho F_u
\]

With the continuity equation and using spherical coordinates, the equation simplifies to

\[
\frac{\partial (\rho u)}{\partial t} + \frac{1}{a \cos \phi} \frac{\partial (\rho u^2)}{\partial \lambda} + \frac{1}{a \cos \phi} \frac{\partial (\rho \cos \phi \rho v)}{\partial \phi} + \frac{\partial (\rho w u)}{\partial z} = \frac{\tan \phi}{a} \rho uv + f \rho v - \frac{1}{a \cos \phi} \frac{\partial p}{\partial \lambda} + \rho F_u
\]

We calculate the zonal average \(\bar{\cdots}\), take into account that \(\frac{\partial \rho}{\partial \lambda} = 0\) and assume a vertical dependence of the density \(\rho = \rho_0 (z)\):

\[
\frac{\bar{\partial} (\rho_0 u)}{\partial t} + \frac{1}{a} \frac{\bar{\partial} (\rho_0 uv)}{\partial \phi} + \frac{\bar{\partial} (\rho_0 w u)}{\partial z} = 2 \frac{\tan \phi}{a} \rho_0 uv + f \rho v + \rho_0 F_u
\]

We split the wind variables into a synoptic scale waves, planetary waves and zonal mean wind \(u = \bar{u} + u^* + u'\). Under the assumption that \(\bar{u}\) and \(v^*\) are independent, the result of the zonal mean over the azonal component is zero:

\[
\bar{u} \bar{v} = \bar{u} \bar{v} + \bar{u} v^* + \bar{u} v' + u^* \bar{v} + u^* v + u' \bar{v} + u' v + u' v^* + u' v' = \bar{u} \bar{v} + u^* v^* + u^* v' + u' v^* + u' v'
\]

We average eq. (10) over time and phase speed \((\bar{\cdots})\). By assuming independency of the variables, we can simplify the terms \(\langle \bar{uu'} \rangle = \langle u' \bar{v} \rangle = 0\). In addition, it is \(\bar{u} \bar{v} = \bar{u} \bar{v}\) due to quasi stationarity of both terms. It is also \(\frac{\partial \bar{x}}{\partial t} = 0\)
and \( \langle \vec{u}\vec{v} \rangle = \langle \vec{u} \rangle \langle \vec{v} \rangle \) since the oscillations of \( \vec{u} \) and \( \vec{v} \) are very small and independent of each other. By using the continuity equation

\[
\frac{1}{\rho_0} \frac{\partial}{\partial \phi} \left( \frac{\rho_0}{a} \frac{\partial \langle \vec{u} \rangle}{\partial \phi} \right) + \frac{\rho_0}{a} \frac{\partial}{\partial \phi} \left( \langle \vec{v}'\vec{u}' \rangle + \langle \vec{v}' \vec{u} \rangle \right) + \rho_0 \frac{\partial}{\partial z} \langle \vec{u} \rangle + \frac{\partial}{\partial z} (\rho_0 \langle \vec{w}'\vec{u}' \rangle + \langle \vec{w}' \vec{u} \rangle) = 0
\]

we can simplify eq. (S10) to

\[
\frac{1}{\rho_0} \frac{\partial}{\partial \phi} \left( \frac{\rho_0}{a} \frac{\partial \langle \vec{u} \rangle}{\partial \phi} \right) + \frac{\rho_0}{a} \frac{\partial}{\partial \phi} \left( \langle \vec{v}'\vec{u}' \rangle + \langle \vec{v}' \vec{u} \rangle \right) + \rho_0 \frac{\partial}{\partial z} \langle \vec{u} \rangle + \frac{\partial}{\partial z} (\rho_0 \langle \vec{w}'\vec{u}' \rangle + \langle \vec{w}' \vec{u} \rangle) = \frac{\tan \phi}{a} \rho_0 \langle \vec{u} \rangle + 2 \frac{\tan \phi}{a} \left( \langle \vec{v}'\vec{u}' \rangle + \langle \vec{v}' \vec{u} \rangle \right) + f \rho_0 \vec{v} + \rho_0 \vec{f}_u
\]

With the assumption that \( \rho_0 = e^{-z/H_0} \) and \( \rho_0 \vec{f}_u = \frac{\partial \tau}{\partial z} = \frac{\partial}{\partial z} \left( \kappa \rho_0 \frac{\partial \langle \vec{u} \rangle}{\partial x} \right) = \kappa \frac{\partial \rho_0}{\partial z} \frac{\partial \langle \vec{u} \rangle}{\partial z} + \rho_0 \frac{\partial \kappa}{\partial z} \frac{\partial \langle \vec{u} \rangle}{\partial z} + \rho_0 \kappa \frac{\partial^2 \langle \vec{u} \rangle}{\partial z^2} = -\kappa \frac{\rho_0}{H_0} \frac{\partial \langle \vec{u} \rangle}{\partial z} + \rho_0 \frac{\partial \kappa}{\partial z} \frac{\partial \langle \vec{u} \rangle}{\partial z} \tag{S11}
\]

we obtain

\[
\rho_0 \langle \vec{v} \rangle \left( \frac{1}{a} \frac{\partial \langle \vec{u} \rangle}{\partial \phi} - \frac{\tan \phi}{a} \langle \vec{u} \rangle - f \right) = 2 \frac{\tan \phi}{a} \left( \langle \vec{v}'\vec{u}' \rangle + \langle \vec{v}' \vec{u} \rangle \right) - \frac{\rho_0}{a} \frac{\partial}{\partial \phi} \left( \langle \vec{u}'\vec{v}' \rangle + \langle \vec{v}' \vec{u} \rangle \right) - \rho_0 \frac{\partial}{\partial z} \langle \vec{u} \rangle - \frac{\partial}{\partial z} (\rho_0 \langle \vec{w}'\vec{u}' \rangle + \langle \vec{w}' \vec{u} \rangle) - \kappa \frac{\rho_0}{H_0} \frac{\partial \langle \vec{u} \rangle}{\partial z} + \rho_0 \frac{\partial \kappa}{\partial z} \frac{\partial \langle \vec{u} \rangle}{\partial z} \tag{S12}
\]

The contribution to the vertical exchange of the atmospheric momentum from stationary eddies described in our case by zonally averaged \( \langle \vec{w}'\vec{u}' \rangle \) is shown negligibly small (Hantel and Hacker, 1978). Also, the scale analysis attests that Also, the scale analysis attests that \( \langle \vec{w} \rangle \frac{\partial \langle \vec{u} \rangle}{\partial z} \) are small:

\[
-\rho_0 \frac{\partial}{\partial z} \langle \vec{w} \rangle \frac{\partial \langle \vec{u} \rangle}{\partial z} - \frac{\rho_0}{H_0} \frac{\partial \langle \vec{w}'\vec{u}' \rangle + \langle \vec{w}' \vec{u} \rangle}{\partial \phi} - \kappa \frac{\rho_0}{H_0} \frac{\partial \langle \vec{u} \rangle}{\partial z} + \rho_0 \frac{\partial \kappa}{\partial z} \frac{\partial \langle \vec{u} \rangle}{\partial z} \approx -\rho_0 \frac{\partial}{\partial z} \langle \vec{w}'\vec{u}' \rangle + \langle \vec{w}' \vec{u} \rangle - \kappa \frac{\rho_0}{H_0} \frac{\partial \langle \vec{u} \rangle}{\partial z} + \rho_0 \frac{\partial \kappa}{\partial z} \frac{\partial \langle \vec{u} \rangle}{\partial z} \tag{S13}
\]

Hence the eq. (S12) can be rewritten into

\[
\rho_0 \langle \vec{v} \rangle \left( \frac{1}{a} \frac{\partial \langle \vec{u} \rangle}{\partial \phi} - \frac{\tan \phi}{a} \langle \vec{u} \rangle - f \right) = 2 \frac{\tan \phi}{a} \left( \langle \vec{v}'\vec{u}' \rangle + \langle \vec{v}' \vec{u} \rangle \right) - \frac{\rho_0}{a} \frac{\partial}{\partial \phi} \left( \langle \vec{v}'\vec{u}' \rangle + \langle \vec{v}' \vec{u} \rangle \right) - \rho_0 \frac{\partial}{\partial z} \langle \vec{u} \rangle - \frac{\partial}{\partial z} (\rho_0 \langle \vec{w}'\vec{u}' \rangle + \langle \vec{w}' \vec{u} \rangle) - \kappa \frac{\rho_0}{H_0} \frac{\partial \langle \vec{u} \rangle}{\partial z} + \rho_0 \frac{\partial \kappa}{\partial z} \frac{\partial \langle \vec{u} \rangle}{\partial z} \tag{S13}
\]

With \( \langle \vec{u}'\vec{w}' \rangle = -\kappa' \frac{\partial \langle \vec{u} \rangle}{\partial z} \), whereby \( \kappa' \) is the coefficient of large-scale turbulent exchange for the momentum due to transient synoptic eddies (Williams and Davies, 1965), we get

\[
\rho_0 \langle \vec{v} \rangle \left( \frac{1}{a} \frac{\partial \langle \vec{u} \rangle}{\partial \phi} - \frac{\tan \phi}{a} \langle \vec{u} \rangle - f \right) = 2 \frac{\tan \phi}{a} \left( \langle \vec{v}'\vec{u}' \rangle + \langle \vec{v}' \vec{u} \rangle \right) - \frac{\rho_0}{a} \frac{\partial}{\partial \phi} \left( \langle \vec{v}'\vec{u}' \rangle + \langle \vec{v}' \vec{u} \rangle \right) - \kappa \frac{\rho_0}{H_0} \frac{\partial \langle \vec{u} \rangle}{\partial z} + \rho_0 \frac{\partial \kappa'}{\partial z} \frac{\partial \langle \vec{u} \rangle}{\partial z} + \rho_0 \frac{\partial \kappa}{\partial z} \frac{\partial \langle \vec{u} \rangle}{\partial z}
\]

With \( \kappa = \kappa' \) we can simplify the equation to
\[
\langle v(z, \phi) \rangle = \frac{-2 \tan(\phi) \left( \langle u^* v^* \rangle + \langle u' v' \rangle \right) + \frac{\partial}{\partial \phi} \left( \langle u^* v^* \rangle + \langle u' v' \rangle \right) + \left( -\frac{dK_z}{z} + \frac{K_z}{H_0} \frac{\partial \langle u \rangle}{\partial z} \right) a}{\tan(\phi) \langle u \rangle - \frac{\partial \langle u \rangle}{\partial \phi} + 2\Omega a \sin(\phi)}
\]
\( \ldots \) \( \text{(S13)} \)