10 April 2017

Re: GMD Paper gmd-2016-303
Authors: Tony E. Wong et al.

Dear Dr. Olivier Marti,

We have carefully addressed all the referees’ comments, suggestions, and questions. The reviews pointed out important ways to improve the exposition of the methods and assumptions, as well as expand and better define our model framework. While the additional analyses suggested by the referees changed slightly some of the ensemble statistics presented here (<5 cm over a century), none of the main conclusions of our study have changed. We are thankful for this opportunity to improve the quality of the manuscript.

Based on the instructions provided, we uploaded responses to each of the Referee Comments in the Interactive Discussion portal. We have also uploaded the flattened changes file of the revised manuscript and the electronic source files to the online submission site, and include a point-by-point response to the Referee Comments here, as well as a tracked changes version of the manuscript. We have typeset the original referee comments in black and our responses in blue, to more easily distinguish the two. We have structured the point-by-point response in the order: **Comment, Reply, Action** (where appropriate).

We would like to take this opportunity to express our sincere thanks to the three reviewers who identified areas of our manuscript that needed corrections or modification. We would like to also thank you and the peer-review team for allowing us to resubmit a revised copy of the manuscript and for the service to our community.

We hope that the revised manuscript is found to be suitable for publication in Geoscientific Model Development.

Sincerely yours,

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Referee #1

Comment #1
The submitted manuscript, “BRICK, a simple, accessible, and transparent model framework for climate and regional sea-level projections,” presents a good example for other geophysical modelers to follow. The authors describe a modular and transparent framework for projecting changes in regional sea level under different uncertain future scenarios, and they also give an example of how the framework can be used to plug in other modules enabling decision support based on the climate model outputs. While the flood risk management example is simplistic, it is illustrative of the potential for the BRICK model to be leveraged in a variety of useful applications. Further, the authors make a nice case for the value and importance of open-source, transparent, and simple modeling.

I believe the paper is of high quality and nearly ready for publication, but I have included a number of suggested revisions or comments on certain elements as outlined below:

Reply
Thank you very much. We address the suggestions below.

Action
None required

Comment #2
P2, line 25 – difficulty can be due to a variety of things: closed platforms, reliance on databases or other inputs that are less portable than source code, etc.

Reply
We agree that it is important to stress that good coding practice is not the sole requisite for reproducibility.

Action
In response, we added the statement of:
"Studies based on simple, mechanistically-motivated models have the potential to be transparent and reproducible when presented in open platforms and when the underlying data are readily available. Yet, although there ...“

Comment #3
P3, line 26 – Is that intended to be conservative, rather than "underconfident"? Worth explaining underconfident about what, exactly.

Reply
Conservative estimates and underconfidence are certainly related. In our view conservative estimates are deliberately risk-adverse (e.g. by using wide uncertainty ranges) whereas underconfidence refers to the tendency to have more outcomes within the estimated probabilistic uncertainty range than expected.

Action
We add the following short explanation: "..., e.g. by applying conservative estimates in the sense of being risk-averse”

Comment #4/5
I am unqualified to comment on the fundamental dynamics described in section 3. However, this section provides what appears to be an appropriate level of detail, and the components are based in reputable sources and representations from other well vetted models.
P14, 1-5 – Ability to easily recalibrate model in future with new data and/or methods is a very nice feature that should provide more longevity to the model

Reply
That is what we aimed for. Yet, we hope that the accessibility and flexibility will help others and ourselves to test alternative model choices and assumptions, as well as data.

Action
None required.

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Comment #6
P18, 21-22 – “With respect to dike heightening, the expected investments are a linearly increasing function”: this is not strictly accurate, as written, and should be explained more clearly. Jonkman (2009) makes a reasonable assumption that construction costs are proportional to the length of levee being constructed or upgraded, but the resulting calculations appear to show that investment costs are linear with respect to the log of the return period of level of protection provided. This is also a bit different than what readers might reasonably interpret the highlighted phrase to mean. Raw material costs when upgrading levees scale with the square of the levee height, because when raising the height, the base must also be widened.

Reply
The focus of this manuscript is especially the transparency, the accessibility and flexibility of the BRICK framework. The simple approximation of Jonkman et al. (together with its extremely clear description) is designed to fit this purpose. The reviewer is, of course, absolutely correct that our description should be as clear as possible too. Besides, the description contained a small error.

Action
We rephrased it as follows:
“In this simplified model, the investment costs only depend on dike heightening and are approximated by linear interpolation between data points provided by Jonkman et al. (and linear extrapolation for dike heightenings outside this range).”

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Comment #7/8
P18, 27 – what does the “exponential flood frequency constant” represent? Is this related to the amount the probability of flooding is reduced per meter of increased dike height?

P18, 27-28 – What factors are rolled up into the net discount rate? Jonkman (2009) assumes a real interest rate, net of inflation, and then makes further reductions for economic growth and changes in the yearly probability of flooding due to sea level rise. This is perhaps a small point because the discount rate is treated as uncertain, but if the intent is to follow Jonkman, it should be noted that the flood probability due to SLR is now endogenized in the BRICK analysis, rather than being an exogenous factor for Jonkman.

Reply
This is a great point, and in the revised manuscript we elaborate on details of the parameters of the flood risk module. Specifically, we now have added two separate paragraphs that provide a more detailed explanation about the uncertain parameters, including their assumed sampling distributions.

Action
We point to specific textual examples in our response to Comment #9, as those changes address both points.

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Comment #9
P18, 30-31 – How were the plausible ranges for each of these parameters chosen? Some of the choices seem a bit odd, such as assuming that the top end of the range for the investment cost uncertainty is 1. Why was the particular mean probability of flooding chosen? I acknowledge that this is not particularly important, given the illustrative nature of this simplified example, but some
additional explanation of the experimental setup would be helpful to put it on par with the level of thoroughness given to previous sections.

Reply
We completely agree that a more thorough discussion and description of this simple model formulation would benefit the manuscript. As the reviewer notes, we acknowledge that some of the ranges are somewhat ad hoc. They are meant, of course, to serve as a demonstration of model capability and not to inform on-the-ground decisions.

Action
We have clarified the choices for these parameter ranges in the revised text. A summary of these motivations has been added to the revised manuscript’s text, and is included below. This new text also includes a more thorough description of the flood risk parameters (addressing the reviewer’s comments #7/8 above).

"The uncertain parameters considered in this cost-benefit analysis include the initial flood frequency with no heightening (y^-1); the exponential flood frequency constant (m^-1); the value of goods protected by the dike ring (billion US dollars); the net discount rate (%); the uncertainty in investment costs (a unitless multiplicative factor); and the land subsidence rate (m y^-1). The central estimates for the exponential flood frequency constant (alpha) and the initial flood frequency with 0 heightening (p0) are taken from Van Dantzig (1956). The exponential flood frequency constant relates the increase in flood probability that results from an increase in sea level relative to the dike height. We make the assumption that this factor should scale (to first order) relatively well from Dutch case considered by Van Dantzig (1956) to the test case of New Orleans considered presently. The initial flood frequency with 0 heightening (p0) may not translate directly between these two cases, but highlights our intent for this experiment to serve as an example of future applications of the BRICK model to inform decision analyses. The admittedly ad hoc distributions assumed for alpha and p0 were selected to sample tightly around the central estimates from Jonkman et al. (2009). A more detailed treatment of this risk management problem would include using methods from extreme value theory to address the risks posed by storm surges (Coles et al. 2001).

The investment uncertainty considered in the sensitivity tests of Jonkman et al. (2009) included a base case, 50% lower, and 100% higher than the base case. We use this range for the investment uncertainty, applied as a multiplicative factor ranging from 0.5 to 2. The range for the value of goods protected by the dike ring is taken from Jonkman et al (2009), where the lower bound is the lowest estimate of value of goods protected by the three dike rings considered in that work (US$5 billion), and the upper bound is the estimated combined value protected by all three dike rings (US$30 billion). The net discount rate range is centered at 4%, the estimate from Jonkman et al (2009) accounting for inflation and interest rate. Those authors’ net discount rate is decreased to 2% due to economic growth (1%) and increased flooding probability due to sea-level rise (1%). Our demonstrative example endogenizes the effects of sea-level rise and accounts for parametric uncertainty in the value of goods protected by the dike ring. Hence, we center our range for the net discount rate at 4% but allow for +/-2% uncertain range. The rate of land subsidence is based on the estimates of Dixon et al. (2006), with mean 5.6 mm/y and standard deviation 2.5 mm/y. We transform this to a log-normal distribution to disallow negative rates of land subsidence.

We sample the uncertainty in these parameters via Latin hypercube, where the population size is given by the number of sea-level rise ensemble members that are present (10,671 for the control BRICK ensemble). ..." (proceeds as in original manuscript)

Additional notes:
We also have revised the notation in Table 1 to more clearly convey how the Lw factor translates to uncertainty in the investment costs for dike heightening. We changed the notation to Lw in [0.5, 2], which is more precisely conveys 50% lower to 100% higher.

Comment #10
4.4.2. – Given the local example, it would be nice to say something about the sea level rise encountered by Louisiana here. Otherwise, this section seems a bit out of place. In the previous section, it is stated that results (for the decision-analysis module) are presented for RCP 8.5, but then this section dives into sea level rise elsewhere in the world and also in RCPs 2.6 and 4.5. The
authors may wish to consider i) removing this section, ii) making more clear that the sea level rise serves as an input into the flood risk module and integrating it better into the rest of the section 4.4 discussion, or iii) moving this section back to 4.3 or elsewhere, then mentioning the local sea level rise in Louisiana as part of 4.4.1, in relation to being an input to the flood risk module.

Reply
This is a great point, and we appreciate the opportunity to streamline the manuscript.

Action
We have revised the first sentence of Section 4.4.2 in order to make clear how the maps of regional sea level changes are related to the flood risk experiment of Section 4.4:

"In order to link projections of sea-level rise to problems of local coastal adaptation, regional sea level is projected to 2100 under the climate change scenarios of RCP2.6, 4.5, and 8.5 (Fig. 4)."

We have also revised the transition from 4.4.2 to 4.4.3 by modifying first sentence of Section 4.4.3:

"We now focus on the regional sea-level projections for the gridcell containing New Orleans, Louisiana (29° 57' N, 90° 4' W) under RCP8.5 (Fig. 4c), to demonstrate the use of these sea-level projections in a common local flood risk management example."

Referee #2

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General comment
The manuscript, "BRICK v0.1, a simple, accessible, and transparent model framework for climate and regional sea-level projections, describes and open-source, modular modeling framework to investigate change in global and regional sea level. The authors details the modeling framework well all with conveying the value of this type of modeling. I suggest publication with a few minor comments to be addressed below.

Reply
Thank you very much.

Action
We address the suggestions below

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Comment #1
My main concern is that the model is contained in a zip file. This made it difficult to look at the structure and code without downloading the whole package. For maximum visibility and reproducibility it would be great to publish this model on github or bitbucket. This would allow for easy code review, version control, and issue tracking etc.

Reply
Thank you for pointing this out – it is exactly our intent *not* to distribute the model widely using a zip file or tarball. Indeed, this would go against our stated interest in reproducing, longevity, and transparency. Our codes are maintained on Github, and we only put a preliminary version out to accompany the GMD Discussions manuscript as a tarball. In hindsight, this seems to have been a poor choice, which we have rectified in our revised manuscript.

Action
In our updated manuscript, Code and Data Availability section, we point to a Github site where the codes will be maintained for the long term:

"All BRICK v0.2 code is available at https://github.com/scrim-network/BRICK under the GNU general public open source license. Large parameter files as well as model codes forked from the repository to reproduce this work (including the sea level projections) may be found at https://download.scrim.psu.edu/Wong_etal_BRICK/.”

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Comment #2
It would be useful to the reader to make it clear upfront that you are coupling multiple, already published, models together.

Reply
The reviewer is, of course, correct.

Action
In the abstract we now have:
"Here, we introduce a simple model framework (largely building on existing models) for projections of ...”

And in the introduction: "In this model framework, we present a set of existing, well-tested, and easy-to-couple simple models for...”

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Comment #3
A description of the inputs and outputs along with the spatial and temporal scales and rough run times would be useful. For example, does the model take in an emission pathway? Concentrations? Only CO2?
**Reply**

All of the component-models are zero-dimensional, with the following exceptions. The ocean-model is a 3-layer 1D model. The Antarctic ice sheet model (DAIS) considers a two-dimensional axisymmetric geometry. These exceptions are noted in the original manuscript, Page 10 Line 7 (DAIS) and Page 7 Line 4 (ocean). In Section 2.2.4 in the original manuscript, we give rough estimates of the run times (order of thousandths of a second per 1850-present hindcast simulation).

At page 7 line 25 in the original manuscript, we point out that the sea-level rise model uses a one-year time step: "The differential equations for the GIC, GIS, AIS, and TE contributions to global mean sea level (below) are integrated in BRICK using first-order numerical integration schemes with a one-year time step." The annual time scale can be easily adjusted.

In the revised manuscript, we have added text to make clear that the climate component uses a one-year time step as well, and state the required forcing is a radiative forcing time series: "We use a one-year time step for the DOECLIM model, and the required input to drive the model is the radiative forcing time series (W m\(^{-2}\))."

For projections, this uses Representative Concentration Pathways (as seen in the presentation of the results) and for the hindcasts, we use the same data as Urban and Keller (2010) and Urban et al. (2014).

The other component models are driven by global temperature and the Antarctic ice sheet contribution and the local sea-levels also require sea-level contributions from all sea-level components.

**Action**

We have revised the text within Sections 3.1 and 3.2 to make these details more clear:

"DOECLIM is a zero-dimensional energy balance model coupled to a three-layer, one-dimensional diffusive ocean model."

"We adopt a simple zero-dimensional sub-model for the contribution to global sea-level rise from Glaciers and Ice Caps (GIC) from Wigley and Raper (2005)."

"BRICK uses the mechanistically-motivated, zero-dimensional SIMPLE (Simple Ice-sheet Model for Projecting Large Ensembles) model as the parameterization for the Greenland ice sheet (GIS) contribution to global mean sea level change (Bakker et al., 2016a)."

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**Comment #4**

Does the user have to calibrate the model? Or does the model come already calibrated?

**Reply**

We will provide the larger calibrated parameter sets at a download server if users wish to use the sea-level projections showcased in the revised manuscript; we have run a larger ensemble since the initial submission, but the resulting conclusions have not changed. We hope that these projections, along with the "BRICK\_LSL.R" script to fingerprint to local sea level at a user-defined latitude and longitude, will be useful for readers to incorporate numerous uncertainties in sea level projections into their own work. Thus, the model may be used already calibrated, but the model’s accessibility enables easy experiments with alternative calibration schemes.

**Action**

Additionally, we have added a “Code Example” for using our sea-level projections and fingerprinting to yield local sea-level projections in a new README.md file, available from the Github repository.

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**Small points**
Comment
pg1 ln17  ‘easier to reproduce’ Easier than what? -> change in “easy”

Reply
We thank the reviewer for this nice textual suggestion.

Action
Corrected in the revised Abstract.

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Comment
Section 2.2.2 - are there instructions on how to incorporate new datasets?

Reply
We aim for a transparent easy to access framework. To test this, we need feedback from users and we will try (maybe with the help of other users) to incorporate this feedback.

Action
We have added a note about this to the “README_calibration” file in the “calibration” directory:

"Additional observational datasets for calibration may be introduced by making the following modifications.
In the calibration directory:
(1) [submodel]_readData.R – read the dataset, match the model and observational data indices (using “compute_indices” function)
(2) BRICK_calib_driver.R – add to the obs.all, obs.err.all, midx.all, oidx.all, ind.norm.data lists (these tell the BRICK_assimLikelihood.R routines how to compare the model and data)
(3) BRICK_assimLikelihood.R – calculate a likelihood function value for these data and add to “log.lik” routine. Note that simply adding the log-likelihood from the new dataset assumes independence between residuals from one dataset to the next.
In the data directory:
(1) Add the dataset, and make sure to point to this in [submodel]_readData.R”

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Comment
pg7 ln14  – what data is used for this comparison?

pg7 ln14-21  – I suggest expanding this paragraph a bit more.

Reply
In the original manuscript, we mention these data sets at Page 7 Line 17: “We add the heteroscedastic observational error estimates from Morice et al. (2012) and Gouretski and Koltermann (2007)”

Action
We have expanded this to specify that the data are temperature and ocean heat uptake in the revised text, as suggested:
“We add the heteroscedastic observational error estimates for global mean surface temperature from Morice et al. (2012) and for ocean heat uptake from Gouretski and Koltermann (2007)”

This type of calibration approach has been used previously, and we point to those studies for further details at the end of the paragraph in question:
“This type of calibration approach for DOECLIM has been implemented previously in the literature (Urban and Keller, 2010; Urban et al., 2014), and we direct the interested reader to these studies for further details.”

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Comment
pg9 ln31  - are the projections in the manuscript all relative to this mean?

Reply
Yes, that is correct.

**Action**
None required.

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**Comment**
*Figure 2* – the coloring on my end was hard to see.

**Reply**
We appreciate this opportunity to improve the clarity of our figures.

**Action**
We have revised the color schemes for both Figure 2 and Figure 3.
Referee #3

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General comment
This manuscript outlines design choices for the simplified contribution-based sea level model BRICK, lists the underlying equations and shows some of its features. Further, it discusses a simple application deriving regional flood risk. I expected the manuscript to be a model description paper, (which is also the chosen GMD category), but it is not. The reader is directed to another manuscript (Bakker16b, https://arxiv.org/pdf/1609.07119.pdf), which is currently under review elsewhere. Figures on calibration of sea level components as well as global sea level projections for the RCP scenarios, which I expected in this manuscript, are instead found in Bakker16b. It is therefore difficult for me to judge what is new and original in this paper (except for the applications) and would provide sufficiently substantial advance for publication in its current form. Both the manuscripts seem to point to the same source code and I think that attribution of the code needs to be clarified. Therefore, the reader does not have means to build trust in the calibration even when reading the Bakker16b paper. As I guess there is no possibility to merge the two manuscript (which I would find ideal), I therefore find it necessary that the title and abstract are adjusted so that it becomes clear that this is an application paper of the model (with the extra of presenting the equations, which are missing in Bakker16b). Alternatively, to make this an original contribution as model description paper, it clearly has to be highlighted what is new and different in this paper as compared to Bakker16b. I would then like to see the figures for the calibration and projections of the sea level components repeated (I expect they are not completely the same). On a positive note, I highly appreciate the effort of the authors to be as transparent as possible, providing input data, calibration data and source code. I quickly managed to reproduce the core figures. I acknowledge the open-source approach, which is missing for still too many of the climate modeling papers published. See also the specific comments.

Reply to general comment
Thank you for pointing out this avenue to clarify the difference in scope between this manuscript and Bakker16b. Whereas Bakker16b focuses on the model (i.e. set of equations) and its calibration, this study focuses on the code behind the model which has been specifically designed to support transparency, accessibility and flexibility. We note that the GMD description of a model description paper contains the statement:

“In addition to complete models, this type of paper may also describe model components and modules, as well as frameworks and utility tools used to build practical modelling systems, such as coupling frameworks or other software toolboxes with a geoscientific application.” In particular, we interpret this to mean that describing and demonstrating a useful coupling framework for pre-existing models does qualify ours as a GMD model description paper.

One may argue that this should be common practice in scientific modelling and we couldn’t agree more. Yet, in our assessment, this is not common practice. Transparency, accessibility and flexibility are (interrelated) modeling values that are of utmost importance for the scientific process (which obviously continues after successful peer-review and publication of a manuscript).

For example, the specific comments below contain some well justified and well considered concerns about the model choices of Bakker16b. The model values behind BRICK can facilitate discussing, exploring and testing such concerns. We hope that good coding practice, with care for the mentioned model values, will be to the advantage of the scientific modelling. As a result, this paper, in our assessment, fits nicely the category “model description paper”.

Action
Although not perfect, all reviewers express their appreciation for our attempts to be transparent, accessible and flexible. In the revised manuscript, we try to better clarify the scope, model values and coding practice. Further, we feel that the model description and the small modifications with respect should be better explained in order to improve transparency (see replies to specific comments).

To address the reviewer’s concern regarding clarity regarding the originality of the models, in the revised abstract, we now write:
"The BRICK model framework is written in R and Fortran and expands upon a recently published model setup. BRICK gives special attention to the model values of transparency, accessibility and flexibility in order to mitigate the above-mentioned issues, while ...."

In the revised Introduction, we additionally emphasize that BRICK is built from "existing, well tested" simple models.

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**Specific comment #1**

Though the authors refer to Bakker16b for details on calibration, p13,L9 mentions that the calibration has been modified. This is also evident from the posterior ranges in Tables A1-A5 as compared to Bakker16b Table S3. Therefore, even with Bakker16b at hand, it is not easily possible for the reader to assess the quality of the here presented numbers. This needs thorough further discussion, see general comment above.

**Reply**

We have modified the calibration relative to Bakker et al. 2016b by including a contribution from land water storage (per the reviewer's later suggestion) and using rejection sampling to join the Antarctic ice sheet model parameters (calibrated using a paleo simulation of 240,000+ years) with the rest of the model parameters (calibrated using a modern simulation from 1850-2009).

**Action**

We have added this point to the overview of the sea-level rise module in the revised manuscript: "BRICK accounts for land water storage contributions to global mean sea level using mass balance trends from the International Panel on Climate Change (IPCC) Fifth Assessment Report (AR5, Church et al., 2013) and from the work of Dieng et al. (2015)."

We have also added the land water storage term to the sea-level mass balance in Eq. (1):

$$\frac{dS}{dt}(t) = \frac{dS_{GISC}}{dt}(t) + \frac{dS_{GIS}}{dt}(t) + \frac{dS_{AIS}}{dt}(t) + \frac{dS_{TE}}{dt}(t) + \frac{dS_{LWS}}{dt}(t),$$

where $S_{LWS}$ is the sea level contribution from changes in land water storage.

We have moved the sentence the reviewer mentioned (original manuscript at p13, L9) to the following paragraph, and clarify that the use of rejection sampling and subtraction of land water storage contributions (estimated from the IPCC AR5 (Church et al., 2013, Table 13.1)) are the key differences between this work and that of Bakker et al. 2016b:

"We combine these two disjoint sets of parameters to form concomitant full BRICK model parameters sets, and calibrate these to global mean sea level data (Church and White, 2011) using rejection sampling (Votaw Jr. and Rafferty, 1951). Prior to rejection sampling, contributions from land water storage are estimated using trends from the IPCC (Church et al., 2013) and subtracted from global mean sea level. When projecting global mean sea-level rise, we estimate land water storage contributions by extrapolating using the 2003-2013 trend of 0.30+/−0.18 mm/y found by Dieng et al. (2013). This approximation may not hold in reality (Wada et al., 2012), but serves as a starting point for future model developments. The use of rejection sampling and the estimation of land water storage contributions to sea level are the two aspects in which our calibration approach differs from that of Bakker et al. (2016b). In this rejection sampling step, each full BRICK parameter set is constructed by..."

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**Specific comment #2**

You shortly discuss overparametrization but I find your argumentation not yet convincing. In p16 L8: Wouldn’t a lower BIC for the full BRICK model be a stronger indicator for the full model being superior? The higher BIC than BRICK-GMSL actually hints to overparametrization, right? Also, on p16L11 you discuss that missing annual variability is of little concern. Shouldn’t your more complex full BRICK model with 39 parameters better capture the dynamics and thus also better capture the shorter timescales of variability than the 13 parameter BRICK-GMSL model? Please discuss this and include some hints on where “variability got lost” and on potential improvements.
Reply
Thanks for pointing out this opportunity to clarify the exposition. You are, of course, correct, that the AIC for the full BRICK model is lower than for the Rahmstorf 2007 emulator, which indicates that the full BRICK model fits “better”, but the BIC for the Rahmstorf emulator is lower, which suggests the contrary. The BIC more heavily penalizes based on the number of parameters, which we note at Page 15 Line 30 in the original manuscript. At Page 16 Line 8 of the original text, we address this mixed result, but aim to make this point clearer in the revised text by writing:

“These mixed results for the model comparison metrics indicate that using the full BRICK sea-level rise module is not unreasonably over-parameterized; if the full BRICK model were obviously over-parameterized, we would expect the AIC for the GMSL emulator experiment to be lower than for the full BRICK model.”

As to the point about capturing the variability: The full BRICK model in these experiments is not directly calibrated using GMSL data. Rather, the GMSL data are invoked in the rejection sampling step that joins the paleoclimate (Antarctic ice sheet parameters) with the modern (rest of the model components’ parameters) calibrations. Thus, the full BRICK model ensemble captures the individual components of sea-level rise (and temperature and ocean heat uptake), then is culled via rejection sampling to only those ensemble members which also match GMSL data. This can readily be seen in Figure 3, that the interannual variability in glaciers and ice caps contribution to sea level is better captured. By averaging over the ensemble and the four major contributions to global mean sea level, this variability is – as expected – smoothed.

Action
We completely agree with the reviewer that this is an important point that the original manuscript was in need of improvement. We have revised our discussion of this experiment in Section 4.2.3 of the revised manuscript:

“The full BRICK simulation does not capture the annual variation in global mean sea level that the BRICK-GMSL simulation successfully captures. This is attributed to the smoothing effect of averaging over the model ensemble the four major contributions to global mean sea level, as opposed to calibrating the BRICK-GMSL simulations directly to global mean sea level data.”

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Specific comment #3
You use the model for thermal expansion that uses global mean temperature as input (equ. 15) though the DOECLIM model explicitly provides ocean heat uptake, which could be used to calculated thermal expansion. Why did you not go the DOCLIM way? Can you compare the two approaches and discuss the difference?

Reply
This would be a nice experiment indeed. The reason why we did not do this is the difficulty to obtain ocean heat uptake data that match the spatial and temporal resolution of the model. That means that we cannot separately test the proposed model to estimate expansion from ocean heat. In this paper, we focus on observational data sets for calibration as opposed to modeled reconstructions (which are more widely available). Also note our response to Specific Comment #7 below.

Action
No further action required. We note that our model framework is designed specifically to enable the interested user to perform such experiments as this one with relative ease, as demonstrated by our GIC-SIMPLE/GIC-MAGICC experiment.

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Specific comment #4
Your model equations 3-7 cannot be easily related to equations 8-9, which are the relevant ones for model calibration and slr contribution from Greenland. They are not in Bakker16a. Your sentence on p9L20 “SIMPLE algebra ...” is not enough to understand the simplification from equs 3-7 to 8-9. Please outline this derivation clearly. Equ 3-7 may be moved to an Appendix together with such outline as they are not fully necessary to understand your equ 8-9 model.

Reply
You are absolutely correct. Equations 3-7 cannot be easily related to equations 8-9. Our original text neglected several further approximations, hence and the two sets of equations are not fully interchangeable.

**Action**

We removed the equations 3-7 (originally intended to clarify) from the manuscript, as suggested, and slightly reordered the section.

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**Specific comment #5**

Land water storage changes through dams and groundwater pumping plays a role for past and future sea level rise, see the papers of Yoshihide Wada for example. Ignoring such influences your ensemble selection as you use past global mean sea level rise as a criterion. It will also add to future sea level rise and thus flood risk. If not included in the model this should at least be discussed appropriately. It would be good to shift in ensemble members if LWS is subtracted from global mean sea level rise.

**Reply**

We thank the reviewer for this nice insight.

**Action**

We have revised the rejection sampling step of our calibration to global mean sea level (GMSL) data (Church and White, 2011) such that contributions from land water storage estimated from IPCC AR5 (Church et al., 2013; Table 13.1, Ch. 13, p.1151) are subtracted from the GMSL data set prior to rejection sampling.

We have added a rudimentary estimation of the land water storage contributions to global mean sea level to our projections as well. We use the 2003-2013 trend of 0.30 +/- 0.18 mm/y from Dieng et al. (2013), and assume this trend continues to 2100. We sample annual contributions to sea level from land water storage normally with mean 0.30 mm and standard deviation 0.18 mm. This addition shifts the ensemble 5-95% range for projected GMSL by 2100 from 0.91-1.73 m to 0.95-1.74 m in RCP8.5, for example. We appreciate the suggestion and opportunity to include land water storage contributions in at least a rudimentary way in our model framework.

We note the limitation of these assumptions in the revised text. Namely, that extrapolation of the trend of Dieng et al. (2013) may not hold in reality (Wada et al., 2012). The following text is added to Section 4.1:

"We combine these two disjoint sets of parameters to form concomitant full BRICK model parameters sets, and calibrate these to global mean sea level data (Church and White, 2011) using rejection sampling (Votaw Jr. and Rafferty, 1951). Prior to rejection sampling, contributions from land water storage are estimated using trends from the IPCC (Church et al., 2013) and subtracted from global mean sea level. When projecting global mean sea-level rise, we estimate land water storage contributions by extrapolating using the 2003-2013 trend of 0.30+/-.0.18 mm/y found by Dieng et al. (2013). This approximation may not hold in reality (Wada et al., 2012), but serves as a starting point for future model developments. The use of rejection sampling and the estimation of land water storage contributions to sea level are the two aspects in which our calibration approach differs from that of Bakker et al. (2016b). In this rejection sampling step, each full BRICK parameter set is constructed by…”

We also point to this in the overview of the sea-level rise module in the revised manuscript in Section 3.2:

"BRICK accounts for land water storage contributions to global mean sea level using mass balance trends from the International Panel on Climate Change (IPCC) Fifth Assessment Report (AR5, Church et al., 2013) and from the work of Dieng et al. (2015).”

===

**Specific comment #6**

Similarly, not all sea level change can be attributed to climate change since the start of industrialization as the ocean, glaciers and ice sheets all have longer memory. If I see this correctly, you assume global mean temperature change being the sole driver, thus attributing all sea level change to temperature change since preindustrial. This has been a main critique to so-
called semi-empirical models and you should comment on this here or, best, do some sensitivity tests.

Reply
Only the DOECLIM model to estimate global temperature assumes that the initial (pre-industrial) temperature was close to the equilibrium temperature belonging to the then atmospheric composition. The other models are not necessarily in equilibrium at the start of the calculations.

Action
No further changes necessary.

===

Specific comment #7
You do not mention how thermal expansion (or more generally: ocean dynamics) enters your regional sea level projections though it is an important contribution. I see in your code that you assume constant thermal expansion around the globe. This should a) be mentioned and b) be justified.

Reply
Thank you for this excellent point. We are currently not aware of a method to estimate (the effect on changing) ocean dynamics (on local sea-level rise) by means of simple semi-empirical models. It may take a while before a satisfying simple model has been developed. In the meantime, emulators of GCM’s may prove useful. Our stated aim with the BRICK model, however, is to avoid emulating other models but rather employ preferentially observational data. In order to resolve these ocean dynamics, a depth- and latitudinally-resolved ocean model would be required.

Action
We have added text to the revised manuscript at Section 3.2.4 (Thermal Expansion) to make clear these assumptions and modeling choices:

"BRICK uses a simple parameterization for the contribution of thermal expansion (TE) of the Earth's oceans to sea-level rise. We make the simplifying assumption that thermal expansion of the oceans occurs uniformly around the globe. While this is, of course, not strictly true, the next obvious step up in model complexity would be to use a vertically- and latitudinally-resolved model for thermal expansion, incorporating the DOECLIM model output for ocean heat uptake. This two-dimensional ocean model is beyond the scope of the simple model framework described presently, but an excellent subject for future work. Here, we employ a simple zero-dimensional thermal expansion emulator based on the parameterizations of the sea-level rise sub-models of (Mengel et al., 2016) and was originally used by (Grinsted et al., 2010) to model the total global mean sea level changes."

===

Specific comment #8
Equation 13, p30 is unclear to me. Sea level rise and Antarctic ice volume loss should be related by a constant factor. Instead, your right side of the equation is a sum. I think this is wrong. Please correct or explain.

Reply
We thank the reviewer for catching this typo. Indeed, they should be related by the constant factor (57 m SLE)/(V_{0,AIS} m^3). That is, the "1" in our original equation 13 should not have been there.

Action
We have corrected this error in the revised manuscript. Our apologies.

===

Specific comment #9
One important last point: you provide the source code and the data as a zip file (though section 2.3 highlights the importance of version tracking.) Transparency and accessibility (as highlighted in sec 2.2 would profit if you’d follow your words: using one of the gitlab/github/bitbucket sites would make your code easier accessible and changes to it transparent. I think this is a precondition for
publication of the manuscript if you want to keep section 2. Such repository should hold a README.md similar to your current readme, which names the additional R packages needed, i.e. Deoptim, ncdf4, gplots, fields. http://joss.theoj.org/about#reviewer_guidelines provides guidelines for such readme. A short and illustrative example with global sea level projections would be great. Why not creating a notebook for such? See https://github.com/tanyaschlusser/Jupyter-with-R/blob/master/example-Jupyter-R.ipynb as example. I guess such gitlab/github/bitbucket repository is on your plan after publication, as also indicated in Bakker16b.

**Reply**

This is, again, an excellent point – it is exactly our intent *not* to distribute the model widely using a zip file or tarball. Indeed, this would go against our stated interest in reproducibility, longevity, and transparency. Our codes are maintained on Github, and we only put a preliminary version out to accompany the GMD Discussions manuscript as a zip file.

**Action**

In our updated manuscript Code and Data Availability section, we point to a Github site where the codes will be maintained for the long term:

"All BRICK v0.2 code is available at https://github.com/scrim-network/BRICK under the GNU general public open source license. Large parameter files as well as model codes forked from the repository to reproduce this work (including the sea level projections) may be found at https://download.scrim.psu.edu/Wong_etal_BRICK/.”

We have also added a README.md file – this was a great suggestion. This file can be found in the top-layer directory at the Github link above.

```
## Minor comments

**Minor comment #1**

Section 2: Framework design As said before, I highly appreciate your efforts to be open source and transparent, but I think this section can be shortened considerably here as it does not contribute to the understanding of the model. You could address the points mentioned in a more direct way as outlined in the last paragraph of the specific comments.

**Reply**

We appreciate the reviewer’s understanding of our stated epistemic modeling values. It is specifically these sections of text, elaborating upon the needs for accessibility, transparency, efficiency and flexibility, that we feel are an important part of our message and contribution to the greater modeling community.

**Action**

Perhaps our release of a zip file of model codes instead of providing the Github link immediately sent the wrong message, and we have corrected this in the revised manuscript (see above reply to Specific Comment #9). We have also expanded to include a README.md file, as suggested above. We strongly feel, however, that these sections are relevant to put the model coupling framework (the key development here that makes this a model description paper) into context.

```

**Minor comment #2**

There is a zoo of reference periods, including 1850-1870, 1850-1970, 1961-1990,

**Reply**

This is true, and a result of the different observational datasets and assumed reference periods for the sub-models. Often, these sub-models include parameters whose values rely on preserving these reference periods. Our codes aim to keep track of these in a user-friendly way by passing explicitly a list object in R that keeps track of reference periods for each sub-model and dataset, and avoiding global variables (when possible) which may hide these types of bugs.

**Action**

The 1850-1970 reference period was another nice typo catch, which has been corrected in the revised manuscript in Section 3.2.3 (Antarctic Ice Sheet). We apologize.
Minor comment #4
1986-2005 and 1960. I wonder if this could be reduced for clarity.
Timeseries figures: Think about your color scheme: pink and violet may not be the best combination.

Reply
We use 1961-1990 for the hindcast reference period because all observational time series cover this period (the glaciers and ice caps data extend only to 2003 (Dyurgerov and Meier, 2005)). For the projections, we use 1986-2005 as the reference period, following the examples of Mengel et al. (2016), Church et al. (2013), and others.

Action
We have revised the color scheme used for Figures 2 and 3.

Minor comment #5
Citations: The introduction includes a lot of references to co-authors. It is therewith a bit difficult to assess the paper’s position within the field. Could you be broader?

Reply
Thank you for the pointer. In our view, there are two important aspects to cover in the Introduction: (1) [semi-empirical] modeling and (2) communication/connecting to decision-making. With respect to these aspects, we

(1) include references to: Hartin et al. (2015), Meinshausen et al. (2011a), Jevrejeva et al. (2016), Rahmstorf (2007), Mengel et al. (2016), and Nauels et al. (2016), as well as the co-author references to Applegate et al. (2012), Urban et al. (2014), Urban and Keller (2010), and Bakker et al. (2016a and 2016b).

(2) include references to: Herman et al. (2015), Weaver et al. (2013), and Lempert et al. (2004), as well as the co-author references to Hall et al. (2011), Garner et al. (2016) and Goes et al. (2011).

The most relevant citations in our introduction to place our model within the realm of other semi-empirical sea-level rise models are to the groundbreaking works of Mengel et al. (2016) and Nauels et al. (2016), which are not co-author citations. We note as well the need in all scientific literature to communicate relevant references (in this case, some works our co-authors have contributed to).

Action
We have added a references to Grinsted et al. (2010) and Kopp et al. (2016), regarding semi-empirical modeling and uncertainty quantification.

We have added references to Gauderis et al. (2013), Fischbach et al. (2012), and Johnson et al. (2013), regarding uncertainty and coastal risk management.

Minor comment #6
Introduction: If you expand this to be a model description paper, I would like to see a recap of the state of the art of sea level projections. What about the past, what data is available, what can large climate models do ... ?

Reply
This comment is addressed largely by our "Reply to general comment" above. To recap, our manuscript is well within the boundaries of a model description paper, as outlined by the GMD website:

"In addition to complete models, this type of paper may also describe model components and modules, as well as frameworks and utility tools"
used to build practical modelling systems, such as coupling frameworks or other software toolboxes with a geoscientific application.”

The model philosophy behind BRICK is such that it is relatively easy to address this kind of questions. However, this is not the scope of this paper (see reply to general comment). The focus of the present manuscript is to present the model framework and demonstrate its flexibility and –as the reviewer’s “Source Code Comment” points out- transparency and relative ease-of-use. Hence, we leave discussion of sea-level hindcasts and projections to Bakker et al. (2016b, “Sea-level projections accounting for deeply uncertain ice-sheet contributions”), which is the more appropriate manuscript to elaborate on the sea-level projections.

In our view, a comparison of a semi-empirical modeling framework such as BRICK against a large climate model (e.g., the NCAR Community Earth System Model) would be to compare apples and oranges; their purposes are quite different. We note in the Introduction the trade-off between physical model complexity and statistical model complexity (Page 2 Line 30 to Page 3 Line 3), and specify our aim to support decision-making with a nimble model capable of thoroughly exploring the low-probability, high-risk tails of distributions.

“...what data is available...” – Each sub-section of Section 3 (Model Components) includes a reference for the dataset used for calibration of BRICK. It is our intention that the assimilation of additional datasets is made simple by our transparent modeling framework.

**Action**
No further action required.

---

**Textual comments**

**Comment**

**p1**: L18: useful for uncertainty quantification: repeats the “pivotal role in the quantification of uncertainties: : :” of L16. Rephrase or delete this sentence L23: “aims to help mitigate”: maybe two verbs would be enough. L23: “these issues”: I can guess what you mean, but it is not clear. Be more precise.

**Reply**
These are good points.

**Action**
L16/18: We have rephrased L18 to read: “These qualities also make simple models useful for the characterization of risk.”

L23: We have rephrased this to read ”...BRICK gives special attention to the model values of transparency, accessibility, and flexibility in order to mitigate the above-mentioned issues, while...”

---

**Comment**

**p2**: L9: “allotment” is this “allocation”? L32: “there is a wide range ...”: I would move such outlook to the end of the paper.

**Reply**
These are also nice textual suggestions.

**Action**
L9: We have revised the word “allotment” to read “allocation”, as suggested.

L32: The aim of this paragraph is to link our epistemic modeling values to making our model useful to inform decision-making, as well as a wide range of other useful applications. In our view, this is a key aspect of the BRICK model framework (flexibility), and we would very much like to keep these key points in the Introduction.

---
Comment
p3: L10: "They simulate climate ...": they simulate global mean temperature change would be more appropriate at this level of complexity I think. L15: "drive high-risk events" suggests some physical driver. this is not true I think. rather "represent" or similar L17: "its flexibility": not clear -> "the flexibility of ..."

Reply
We thank the reviewer for the nice ways to improve specificity and clarity of the manuscript.

Action
L10: We have revised this to read "They simulate global mean surface temperature and contributions to global mean sea-level rise."
We have also revised in the Conclusion (Sect. 5) to read "The main physics (global mean temperature and sea-level rise) codes are also..."
L15: We have replaced "drive" with "represent", as suggested. We have replaced the previous use of "represent" in this sentence with "resolve."
L17: Thank you for pointing out this inclarity. We have revised this to read "Yet, the flexibility of the BRICK model framework also enables the ..."

Comment
p4: L3: "to simulate climate change", as before: you rather try to model the response of global mean temperature to perturbations in the radiative forcing. "simulating climate change" is bigger than this. L4 rather "simulated temperature and sea level rise"

Reply
The reviewer is indeed correct.

Action
L3: We have revised this to read "The essence of the BRICK physical model is to simulate changes in global mean surface temperature and sea level, in response to perturbations in radiative forcing."
L4: Revised to "temperature and sea-level changes", as suggested.

Comment
p.5: L9: "through a clear outlet for coupling to socioeconomic models": I think you talk about a stable and well documented API (application programming interface).

Reply
In broad terms, yes, this is our intention. Within the context of the manuscript, however, we only aim to demonstrate how linking the BRICK projections for global mean sea level may be connected via the regional sea level fingerprinting to local coastal risk management problems (for example). This is a very nice suggestion and nudge into a direction to employ more sophisticated software engineering than is currently implemented in the BRICK model. Our intention is to use a high level programming language (R) as the user interface, in order to make the model accessible and comfortable to use for a broad audience.

Action
None required, although by providing the access to the Github repository along with the revised manuscript, we encourage users to become involved in future model developments, including a stable, well-documented, and more sophisticated API.

Comment
p7: L26: "below" can go I think L27-29: "Initial conditions : : : earliest year of the simulation"
These two sentences do not make sense to me. Why do you start at "certain years" and why would you integrate backwards? If this is not the standard forward in time modeling, you should explain this in more detail.

Reply
The reviewer makes some good points.

L27-29: This numerical modeling choice was motivated by the ability of this scheme to implement an initial condition for each sub-model at the reference point for the initial condition assumed by that particular sub-model. For example, as detailed in Wigley and Raper (2005), the glacier and ice cap sub-model assumes the parameter V0 is given in the year 1990. It would be possible to initialize the model in 1850, say, but this begs the question: what value should be used in this year? The most straightforward way to integrate the sub-model of Wigley and Raper (2005) is to integrate forward in time (their equation 4/5). However, the glacier data (Dyurgerov and Meier, 2005) spans 1961-2003. Solving the backwards integration problem is a trivial rearrangement of our first-order differential equations.

Action
L26: We have removed the parenthetical comment "(below)", as suggested.

L27-29: We understand and apologize for the ambiguity in our phrasing. To clarify this, we have revised the text here to read:

"Initial conditions are specified at a year dictated by the sub-model’s assumed reference point. This differs, in general, among the sub-models and some model parameters depend on preserving this reference year. Starting from this initial condition, a first-order explicit numerical integration method integrates forward in time to the end of the simulation and a first-order implicit (backward differentiation) method integrates backward in time to the earliest year of the simulation."

===

Comment
p8: L14: Why do you not calibrate the uncertain glacier equilibrium temperature -0.15C?

Reply
This is a good point. This was a modeling choice motivated by the need to balance computational feasibility and thoroughness. Several other temperature-related parameters exist in the Antarctic ice sheet model, and adding three more parameters (especially two to the already quite heavily parameterized AIS model) seemed to be too much.

Action
We have clarified at several points in the revised manuscript the notions of overparameterization, as the reviewer suggested.

===

Comment

Reply
See our response to "Specific comment #4", above.

Action
No further action required.

===

Comment
p8-p9, equations 3-7 How do these equations relate to the model you use? The relation is also not evident from Bakker16a. See specific comment above.

Reply
See our response to "Specific comment #4", above.
**Action**
No further action required.

---

**Comment**

**p10**: L10: Why include the time rate of sea level change? L27: 14 parameters: I think over-parametrization should be discussed also here.

**Reply**

L10: This is described in greater detail by Shaffer (2014) (his equations 13 and 14), but the time rate of change in sea level arises from accounting for the isotstatic adjustment of the Antarctic ice sheet, and in particular the effect of that adjustment (ice displacement) on sea level.

L27: This is a good point and overparameterization may seem to be a concern. However, our aim is to account for a wide a range of model uncertainties as possible, and constrain our simulations using observational data. Parametric uncertainty plays a large role in this accounting of uncertainty, and the Antarctic ice sheet model parameters (Shaffer, 2014; his Table 1) are no exception. If we were to assume that these parameters were known with certainty when in fact, they are not, then we would be potentially cutting off decision-relevant upper tails of the distributions of (for example) sea-level rise.

**Action**

We have added a sentence to address this:

"The heavily parameterized Antarctic ice sheet module reflects our focus on including a broad range of model and observational uncertainties, and consideration of the critical role of the Antarctic ice sheet in driving substantial uncertainty in future sea levels (Church et al., 2013)."

---

**Comment**

**p11** L3: "Each mass ...” This is about fingerprints and valid for all contributions. I would suggest to mere it into the more general section 3.3. L13: “is the main equation ...”

**Reply**

The reviewer is correct, these are nice areas for refining and streamlining the text.

**Action**

L3: We have revised this text to read:

"Antarctic shore-average local mean sea level functions as the input to DAIS when run as a sub-model of the coupled BRICK model. This is estimated as described in Sect. 3.3."

And we have added the following text to Section 3.3, as suggested:

"We couple changes in global sea level to the Antarctic ice sheet model using an Antarctic shore-average fingerprint ratio of -1.0 for the AIS contribution to global sea level, and Antarctic shore-average fingerprint factors of 1.0 for the other contributions to sea-level rise from all BRICK submodels (Slangen et al., 2014). Preliminary experiments indicated that our results are not sensitive to the precise choices of these fingerprints."

L13: Corrected to “is the main equation”, as suggested.

---

**Comment**

**p12** L18: You assume the fingerprints to be constants, they would not be so in reality. As you explain later, this assumption is ok here.

**Reply**

Quite true.

**Action**

No change necessary.
Comment

p13 L9: There seems to be a modification to the approach of Bakker16b, it is however unclear how this changes your results. See general comment.

Reply
Indeed this is true – see “Reply to general comment”.

Action
No further action required.

Comment

p14 "Examining BRICKs and full sea-level rise module intercomparison” This heading is rather confusing to me. In the first part you talk about plugging in a global sea level model. In the second part you discuss several goodness-of-fit measures. You can be more precise in the heading. And have a subheading for the goodness of fit paragraph.

Reply
This is a good observation – we agree this is unclear in the original text.

Action
In the revised text, this section heading has been updated to “Testing alternative model components: a sea-level rise module intercomparison”. We have also added a subsection (4.2.2) for the goodness of fit paragraph, as suggested.

Comment

p15 L5: “this specific emulator ...” refer to Rahmstorf once again here, otherwise unclear.

Reply
The reviewer is correct – this is a nice place to improve clarity.

Action
We have revised this in the manuscript revision to read: “Note that the Rahmstorf (2007) emulator is arguably not the state-of-the-art anymore...”

Comment

p16 L8: "These mixed results ...": I think this sentence has no strong basis. You should explain better why you think your model is not overparametrized if you get “mixed results.” Wouldn’t a lower BIC for the full BRICK model be a stronger indicator for the full model being superior? The higher BIC actually hints towards overparametrization, right? See also specific comments.

Reply
See “Reply to specific comment #2” above.

Action
No further action required.

Comment

L11: Paragraph about variability “: : : missing annual variability is of little concern.” You are running over this, but you should not. Your 39 parameter model captures much less short term variability than the GMSL model. You add complexity just to note that you can resolve less the dynamics of SLR? You should find a good reasoning here to justify this.
Reply
See “Reply to specific comment #2” above.

Action
No further action required.

===

Comment
p19, paragraph 4.4.3: You should name somewhere Fig. 5 as I think that is what you are talking about here.

Reply
We thank the reviewer for pointing out this oversight.

Action
We have revised the second and third sentences of this section to read “We find the economically-efficient (i.e., cost-minimizing) dike heightening to be 1.45 m (ensemble mean; 90% range is 0.75 to 2.95 m; Fig. 5). This heightening corresponds to a return period of about 1270 years (ensemble mean; 90% range is roughly 200-3000 years; Fig. 5).”

===

Comment
Fig. 2: I think you here name “BRICK-R07” what you normally call “BRICK-GMSL”.

Reply
Quite right.

Action
It has been corrected in Figure 2 of the revised manuscript.

===

Source Code Comment:
Just to let you know how a person new to the code may address this: I had a look into the code and found the READMEs and comments within the code files, great! I did not get the model running straight away, but almost. Here is my way: First, look into ./README: Ok, I need to compile fortran files. This was easy after reading fortran/README and deleting the *so and obj/* files. I think it is better to not deliver them with the code, as they are platform dependent (at least). As I did not want to do the full calibration, I wanted to test the projections. I searched for projections and you write in ./README to have a look into /calibration/README_projections, which I did. However, the script described therein, run_BRICK.R, is not given in the repository, so I could not run the projections. I went back to the ./README, followed the text and read further about ./calibration/processingPipeline_BRICKexperiments.R, which I got running after an install.packages("ncdf4"). I adjusted the plotdir, needed to install.packages("fields") and install.packages("gplots") and could then source("analysis_and_plots_BRICKexperiments.R"). Nice!

Reply
We thank the reviewer very much for the nice code review! This is precisely the level of scrutiny we hoped ours and future codes may be evaluated with. We greatly appreciate the reviewer’s comments and suggestions here, and encourage future referees to follow this reviewer’s nice example.

Action
In the Github repository accompanying the revised manuscript, we have removed the *so and obj/* files (added to .gitignore), included all required routines (we apologize – this was an oversight in the codes accompanying the original manuscript), and in the top-level README file, we provide a list of the R packages needed, which may be copy-pasted into an R terminal from the README.
Simple models can play pivotal roles in the quantification and framing of uncertainties surrounding climate change and sea-level rise. They are computationally efficient, transparent, and easy to reproduce. These qualities also make simple models useful for the characterization of risk. Simple model codes are increasingly distributed as open source, as well as actively shared and guided. Alas, computer codes used in the geosciences can often be hard to access, run, modify (e.g., with regards to assumptions and model components), and review. Here, we introduce a simple model framework (largely building on existing models) for projections of global mean temperature as well as regional sea levels and coastal flood risk (BRICK: Building blocks for Relevant Ice and Climate Knowledge). The BRICK model framework is written in R and Fortran and aims to help mitigate these issues, while maintaining a high degree of computational efficiency. We demonstrate the flexibility of this framework through simple model intercomparison experiments. Furthermore, we demonstrate that BRICK is suitable for risk assessment applications by using a didactic example in local flood risk management.
**BRICK** v0.2, a simple, accessible, and transparent model framework for climate and regional sea-level projections

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## 1 Introduction

Simple, mechanistically-motivated Earth system models often play a pivotal role in climate and flood risk management (Hartin et al., 2015). For example, they are used for uncertainty quantification (Bakker et al., 2016b; Grinsted et al., 2010; Urban et al., 2014; Urban and Keller, 2010), complex model emulation (Applegate et al., 2012; Bakker et al., 2016a; Hartin et al., 2015; Meinshausen et al., 2011a), and incorporated in integrated assessment models (Hartin et al., 2015; Meinshausen et al., 2011a).

Computational constraints often impose hard trade-offs between physical model complexity and statistical model complexity. For example, a sizable *allocation* of computational time could be spent running a small number of simulations using a high-complexity physical model. Highly detailed simulations are useful to better understand the complex system, but with just a small number of simulations, only weak ensemble statistics can be drawn. In contrast, numerous realizations of a less detailed physical model could be run. This would provide the opportunity for more advanced ensemble statistical techniques including the characterization and quantification of uncertainties. It is important in climate-related applications such as mitigation of greenhouse gas emissions or adaptation to sea-level rise that the relevant uncertainties are explored and communicated clearly to policy-makers (e.g., Garner et al., 2016; Gauders et al., 2013; Goes et al., 2011; Hall et al., 2012; Lempert et al., 2004).

Several studies have broken important new ground in tackling these challenges. For example, Nauels et al. (2016) present a platform of sea-level emulators (i.e. simple models of complex models) that efficiently produces future
projections and characterizes key model structural uncertainties using statistical calibration methods. Semi-empirical modeling (SEM) approaches trade detailed physics for a model that can efficiently project sea level using statistical, but mechanistically motivated, relationships between sea-level changes and climate conditions such as temperature and radiative forcing (Grinsted et al., 2010; Jevrejeva et al., 2010; Kopp et al., 2016; Rahmstorf, 2007).

Recent work has expanded upon the SEM approach to use simple models to resolve individual contributions to global sea level (Bakker et al., 2016b; Mengel et al., 2016; Nauels et al., 2016). Recent work has expanded upon the SEM approach to use simple models to resolve individual contributions to global sea level (Bakker et al., 2016b; Mengel et al., 2016; Nauels et al., 2016).

Studies based on simple, mechanistically-motivated models have the potential to be transparent and reproducible when presented in open platforms and when the underlying data are readily available. Yet, although there is an increasing tendency to share scientific code, it can be (perhaps surprisingly) hard to get the models running and to reproduce the results. A likely cause for this is that not enough attention is given to the scientific coding itself. Careful coding, documentation, and review require a dedicated commitment of time, but scientific incentives to do so can be weak.

Here we introduce BRICK v0.2 (“Building blocks for Relevant Ice and Climate Knowledge”), a new model framework that focuses on accessibility, transparency, and flexibility while maintaining, as much as possible, the computational efficiency that make simple models so appealing. There is a wide range of potential applications for such a model. A simple framework enables uncertainty quantification via statistical calibration approaches (Higdon et al., 2004; Kennedy and O’Hagan, 2001), which would be infeasible with more computationally expensive models. A transparent modeling framework enables communication between scientists as well as communication with stakeholders. This leads to potential application of the model framework in decision support and education (Fischbach et al., 2012; Johnson et al., 2013; Weaver et al., 2013). The present work expands on previous studies by (i) providing a platform of simple, but mechanically-motivated sea-level process models that resolve more processes, (ii) providing a model framework that can facilitate model comparisons (for example, between our models and those of Nauels et al. (2016)), (iii) exploring combined effects of key structural and parametric uncertainties, (iv) explicitly demonstrating the flexibility of our framework for interchanging model components, and (v) explicitly demonstrating the utility of our model framework for informing decision analyses.

In this model framework, we present a set of existing, well-tested, and easy-to-couple simple models for climate and flood risk management. They simulate global mean surface temperature and contributions to global mean sea-level rise. BRICK also includes a regional sea-level rise module, which translates the global mean sea level contributions to regional sea level at a user-defined location. We use these regional sea level projections to demonstrate how the physical model may be linked to decision-making and impacts. We implement a Bayesian...
calibration approach with an aim to adequately resolve the tails of the distribution of future sea level because these low-probability areas represent high-risk events. In robust decision-making approaches, it can be favorable to be underconfident as opposed to overconfident, e.g. by applying conservative estimates in the sense of being risk-averse (Herman et al., 2015). We hence include in our Bayesian approach wide, mechanistically-motivated prior parameter probability distributions (Bakker et al., 2016b). Yet, the flexibility of the BRICK model framework also enables the implementation of other calibration schemes. This paper is intended to showcase a useful model framework that is attractive for a sustainable approach to model development, for example by inspiring fellow researchers to contribute to the framework, to rethink their coding practice, and maybe even to adopt some of the demonstrated design objectives in their future research proposals.

The hindcast skill of the BRICK model has been previously demonstrated (Bakker et al., 2016b). Thus, the present work focuses on outlining a set of epistemic modeling values that we believe facilitates advances in the modeling community. The remainder of this work is organized as follows. In Sect. 2, we describe these values and the ways in which the BRICK model implementation strives to attain them. Section 3 contains an overview of the BRICK model components for climate and the contributions to sea-level rise. Section 4 describes and presents the results of a set of model experiments conducted to demonstrate how BRICK lives up to our epistemic modeling values. Section 5 summarizes the findings of this work and provides conclusions and guidance for future work.

2 Framework design

2.1 Model design

The essence of the BRICK physical model is to simulate changes in global mean surface temperature and sea level, in response to perturbations in radiative forcing. The socioeconomic impacts of the simulated temperature and sea-level changes may then be assessed. This is depicted in Fig. 1. The climate component, each individual contribution to global sea-level rise, and an impacts module are sub-models of BRICK, or “BRICKs.” We defer details of the specific sub-models to Sect. 3. The physical model (climate and sea-level rise) components of BRICK are intentionally simple. This choice is guided by the epistemic modeling values outlined below.

2.2 Epistemic modeling values

2.2.1 Accessibility

We selected R (R Core Team, 2016) as the base language for BRICK because it is (i) stable, (ii) freely available and open source, (iii) relatively easy to use, and (iv) easy to call subroutines written in faster languages. In the BRICK
source code accompanying this study, the physical sub-models within the climate and sea-level rise modules are all provided as both R and Fortran 90 routines. It is our aim that the full physical-statistical model of BRICK is accessible using a modern laptop. This means that sizable Monte Carlo simulations (on the order of a million samples) must be possible on a time scale of hours. This is made possible by calling Fortran 90 sub-models from the base code in R.

In addition to conceptual accessibility, it is our view that useful model codes are physically accessible too. Openness with scientific codes is likely to lead to higher quality codes (Easterbrook, 2014). In an effort to be truly open source and freely available, all codes – including the physical model, statistical model, and processing and plotting scripts used for the results shown here – are available through a download server as well as the Github repository provided in the Code Availability section of this article. Providing all code and data necessary to recreate this study is a critical component of reproducible research (Murray-Rust and Murray-Rust, 2014) and can help to build trust between the general public and scientific community (Easterbrook, 2014; Grubb and Easterbrook, 2011).

2.2.2 Transparency

We aim to achieve transparency in two areas: the physical modeling, including the related model code, and the communication of scientific findings.

In regards to transparent physical modeling, we use simple numerical integration schemes whenever possible. We use as few global variables as possible, in order to “write programs for people, not computers” (Wilson et al., 2014). The essence of these authors’ advice is that users should not be expected to remember more than a few pieces of information as they read and develop code. To this end, in BRICK we aim to give appropriately suggestive names to our variables within the code, such that a human intuitively understands what the quantity at hand represents. For example, when naming a logical or Boolean variable, we prefer for its name to read as a question that the variable itself answers, and begin the variable name with the letter “l” to imply it is a “logical” variable. One example of this in the BRICK source code is the variable “l.project”, which is true when the model is configured to make projections of future sea-level rise and climate, and false when the model is set up for hindcast simulations. While it may seem fussy to review these points, practices such as this will facilitate the sharing of scientific codes and enable the community to build stronger and more efficient collaborations.

Transparency also serves to link the findings of a physical model to decision-making and policy impacts. BRICK can be a useful tool to link climate changes (global temperature and sea-level rise) to decision-making frameworks through a clear outlet for coupling to socioeconomic models. Perhaps most importantly, the coupled physical-
statistical framework in BRICK incorporates many sources of uncertainty into the physical findings on which the decisions will be based. It is important that these uncertainties in climate projections are represented in the decision-making framework (Lempert et al., 2004).

### 2.2.3 Flexibility

A modular programming approach is taken with BRICK, which allows each component sub-model to be exchanged for alternative models. In this way, as the scientific forefront progresses, the BRICK sub-models may advance as well. The flexible BRICK framework also permits a quantitative evaluation of model structural differences, which is valuable in the event that it is unclear which of two candidate models should be chosen. In these cases, the BRICK framework is valuable for model comparison and quantification of structural uncertainty. As new data sets for the calibration of the sub-models become available, these can also be incorporated instead of or in addition to the current data sets. We demonstrate the flexibility of the BRICK framework through a series of modeling experiments (Sect. 4).

### 2.2.4 Efficiency

Code efficiency is enabled primarily through (i) the use of simple models and (ii) using model versions written in R for easy preliminary experimentation, and Fortran 90 versions for production simulations. This practice also follows the advice of Wilson et al. (2014) for code developers to “write code in the highest-level language possible, and shift to lower-level languages like C and Fortran only when they are sure the performance boost is needed.” This boost indeed enables the generation of production simulations on most modern laptops. The simulation of one million model iterations spanning from 1850 to present, performed on each of four CPUs (two cores and two threads per core) yields an ensemble of four million model realizations. This procedure requires less than an hour on a model year 2012 laptop with a 2.9 GHz dual-core processor with 16 GB of RAM. Paleoclimatic simulations require longer wall clock times, but can still be completed in less than a day. All simulations for this study were completed on this machine.

Providing computationally efficient code simplifies its use. For example, there may be limitations on the computing resources allocated for a particular project, or an instructor might be interested in enhancing coursework by incorporating computer modeling exercises. In these cases, transparency is critical (as mentioned above), but also the model must be sufficiently efficient that it neither (i) expires the computational allotment for the experiment, nor (ii) takes too long to be of any educational use. Our epistemic modeling values of accessibility, transparency, flexibility, and efficiency motivate the choice of a relatively simple physical modeling framework. Accordingly, a
detailed statistical calibration framework is implemented. Within this framework, physical model and statistical model parameters are calibrated using observational data sets and mechanistically-motivated prior ranges. The statistical model is reviewed at greater length by Bakker et al. (2016b), so we provide only an overview in Sect. 4.1.

2.3 Code review and sharing

We invite the readers to download and test our code, as well as provide feedback on how best to further develop BRICK to fulfill the four epistemic values outlined above. Frequent and thorough code review by other team members as well as outside agents is another critical step towards good scientific coding practices (Wilson et al., 2014), and “peer review needs to be supplemented with a number of other mechanisms that help to establish the correctness and credibility of scientific research” (Grubb and Easterbrook, 2011). Wilson et al. (2014) also note that a number of high profile research articles have been retracted or revised because of errors in the code. The likelihood of these errors may be greatly reduced by thoroughly testing other group members’ codes. In our own experience conducting the experiments described in this study, we have anecdotal evidence for the value of testing one another’s code. Some errors were corrected through this process, and many more pieces of code were modified for clarity. We continue to invite all comments and suggestions for improvements and modifications (to the corresponding author).

The use of a version control system greatly expands the accessibility of a code base, and also facilitates continuous improvement of the modelling framework itself. This is true and useful before, during, and after the peer-review process. Mistakes are inevitable and we assume that BRICK still contains some minor errors, ambiguities, and pieces of code that do not fully comply to our own standards. Openly sharing the code and documentation will help to address these issues. It is our hope that BRICK may be further developed as a community modeling tool, and that other users may contribute to the framework through added or revised models and data, or improved functionality. The use of a version control system facilitates this type of community effort (Wilson et al., 2014).

3 Model components

3.1 Global mean climate

We adopt DOECLIM (Diffusion Ocean Energy balance CLIMate model, (Kriegler, 2005)) as a starting point for a simple climate model (Fig. 1). DOECLIM is a zero-dimensional energy balance model coupled to a three-layer, one-dimensional diffusive ocean model. The DOECLIM physical model outputs are global mean surface temperature anomaly (°C) and ocean heat uptake ($10^{22}$ J). Calibration data for DOECLIM include both global...
surface temperature (Morice et al., 2012) and ocean heat uptake (Gouretski and Koltermann, 2007). We use a one-year time step for the DOECLIM model, and the required input to drive the model is the radiative forcing time series (W m$^{-2}$). This forcing is partitioned into aerosol and non-aerosol components, to enable a representation of the uncertainty associated with these forcings. The BRICK model considers this as an uncertain model parameter denoted as the aerosol forcing scaling factor ($a_{DOECLIM}$). This aerosol scaling factor has been used elsewhere in the literature (Urban et al., 2014; Urban and Keller, 2010) and accounts for some uncertainty in the radiative forcing of aerosols (Meinshausen et al., 2011b). The interested reader is directed to Kriegler (2005) and Tanaka and Kriegler (2007) for more information about the DOECLIM model.

We fit a first-order autoregressive (AR1) error model to the model-data discrepancy between temperature and ocean heat uptake model output and calibration data. We estimate the first-order lag autocorrelation parameters ($\rho_1$ and $\rho_2$) and homoscedastic component of the AR1 innovation variance ($\sigma_T$ and $\sigma_H$) within the calibration framework as statistical model parameters. We add the heteroscedastic observational error estimates for global mean surface temperature from Morice et al. (2012) and for ocean heat uptake from Gouretski and Koltermann (2007) in quadrature to $\sigma_T$ and $\sigma_H$ (respectively) for the complete heteroscedastic temperature and ocean heat uptake error estimates. The model calibration approach implemented here assumes normally-distributed model-data residuals with mean zero (Higdon et al., 2004). The AR1 error model has the effect of “whitening” the residuals to satisfy this assumption. This type of calibration approach for DOECLIM has been implemented previously in the literature (Urban et al., 2014; Urban and Keller, 2010), and we direct the interested reader to these studies for further details.

3.2 Sea level components

The BRICK global mean sea level module calculates global sea level change as the sum of four individual components: glaciers and ice caps (GIC), Greenland ice sheet (GIS), Antarctic ice sheet (AIS), and thermal expansion (TE). These component sub-models are described in the following sections. BRICK accounts for land water storage contributions to global mean sea level using mass balance trends from the International Panel on Climate Change (IPCC) Fifth Assessment Report (AR5, Church et al., 2013) and from the work of Dieng et al. (2015). The differential equations for the GIC, GIS, AIS, and TE contributions to global mean sea level are integrated in BRICK using first-order numerical integration schemes with a one-year time step. Initial conditions are specified at a year dictated by the sub-model’s assumed reference point. This differs, in general, among the sub-models and some model parameters depend on preserving this reference year. Starting from this initial condition, a first-order explicit numerical integration method integrates forward in time to the end of the simulation and a first-order implicit (backward differentiation) method integrates backward in time to the earliest year of the simulation.
Preliminary experiments (not shown) demonstrated that the one-year time step is sufficiently short to maintain numerical stability. The total global mean sea-level rise from the coupled BRICK model is

\[
\frac{dS(t)}{dt} = \frac{dS_{\text{GIC}}}{dt}(t) + \frac{dS_{\text{GIS}}}{dt}(t) + \frac{dS_{\text{AIS}}}{dt}(t) + \frac{dS_{\text{TE}}}{dt}(t) + \frac{dS_{\text{LWS}}}{dt}(t),
\]

where \(S(t)\) is the global mean sea level (m) in year \(t\), \(S_{\text{GIC}}\) is the sea level contribution from GIC (m), \(S_{\text{GIS}}\) is the sea level contribution from the GIS (m), \(S_{\text{AIS}}\) is the sea level contribution from the AIS (m), \(S_{\text{TE}}\) is the sea level contribution from thermal expansion (m), and \(S_{\text{LWS}}\) is the sea level contribution from changes in land water storage.

We report projections of future sea level relative to the 1986-2005 mean.

### 3.2.1 Glaciers and ice caps

We adopt a simple zero-dimensional sub-model for the contribution to global sea-level rise from Glaciers and Ice Caps (GIC) from Wigley and Raper (2005). This same formulation is used in the MAGICC climate model (Meinshausen et al., 2011a). The parameterization for the GIC contribution to global sea-level rise is:

\[
\frac{dS_{\text{GIC}}}{dt}(t) = \beta_b(T_g(t) - T_{eq,\text{GIC}}) \left(1 - \frac{S_{GIC}(t)}{V_{0,\text{GIC}}}ight)^n.
\]

In Eq. (2), \(S_{\text{GIC}}\) is the cumulative sea level contribution from GIC (m), \(\beta_b\) is the initial mass balance sensitivity to global temperatures (m °C⁻¹ y⁻¹), \(T_{eq,\text{GIC}}\) is the theoretical equilibrium temperature at which the GIC mass balance is at steady state (°C), \(V_{0,\text{GIC}}\) is the initial total volume of GIC available in 1990 (m sea level equivalent (SLE)), and \(n\) is an exponent parameter for area-to-volume scaling. An initial condition, \(S_{0,\text{GIC}}\), is provided as an uncertain model parameter. \(T_{eq,\text{GIC}}\) is taken equal to -0.15°C (Wigley and Raper, 2005). Note that in this formulation for GIC contribution to sea-level rise, whether the GIC mass is increasing or decreasing depends only on \(T_g(t)\) relative to \(T_{eq,\text{GIC}}\); it is independent of the current state \(S_{\text{GIC}}(t)\). Within this model for the GIC sea-level contribution, \(T_g\) is relative to the 1850-1870 mean global surface temperature (Wigley and Raper, 2005).

The uncertain physical model parameters for GIC-MAGICC (which will be tested in Sect. 4.2) are \(\beta_b\), \(V_{0,\text{GIC}}, S_{0,\text{GIC}},\) and \(n\). We fit an AR1 model to the model-data discrepancy between GIC model output and calibration data (Dyurgerov and Meier, 2005) in the same manner as the temperature and ocean heat uptake calibration (Sect. 3.1). Uniform prior distributions are used for the GIC-MAGICC physical and statistical model parameters. These prior distributions, as well as calibrated posterior medians, 5%, and 95% quantiles, are given in Appendix A.

### 3.2.2 Greenland ice sheet

BRICK uses the mechanistically-motivated, zero-dimensional SIMPLE (Simple Ice-sheet Model for Projecting Large Ensembles) model as the parameterization for the Greenland ice sheet (GIS) contribution to global mean sea level.
level change (Bakker et al., 2016a). SIMPLE estimates the GIS response to changes in global mean surface temperature by first estimating an equilibrium ice sheet volume \(V_{\text{eq, GIS}}\) m SLE at which the sea level contribution from the GIS is zero, and estimating the e-folding time-scale of GIS volume changes due to changes in global temperature \(\tau_{\text{GIS}}\).

\[
V_{\text{eq, GIS}}(t) = a_{\text{GIS}} T_g(t) + b_{\text{GIS}}
\]

\[
\frac{1}{\tau_{\text{GIS}}}(t) = a_{\text{GIS}} T_g(t) + b_{\text{GIS}}
\]

In Eqs. (3) and (4), \(a_{\text{GIS}}, b_{\text{GIS}}, a_{\text{GIS}}, b_{\text{GIS}}\) are uncertain physical model parameters. \(a_{\text{GIS}}\) is the sensitivity of the equilibrium volume to changes in temperature (m SLE °C); \(b_{\text{GIS}}\) is the equilibrium volume \(V_{\text{eq, GIS}}\) for zero temperature. \(a_{\text{GIS}}\) is the sensitivity of the time-scale of GIS volume response to changes in temperature (°C\(^{-1}\) year\(^{-1}\)); and \(b_{\text{GIS}}\) is the equilibrium (\(T_g=0\) °C) time-scale of GIS volume response to changes in temperature (year\(^{-1}\)).

The initial condition \(V_{\text{GIS}}(0)\) is provided as an uncertain model parameter (m SLE). Using this initial condition, designated in the year 1961, the sea-level rise due to the GIS is calculated as the change from \(V_{\text{GIS}}(0)\) to the current GIS volume, \(V_{\text{GIS}}(t)\). This formulation, of course, assumes that all GIS volume lost makes its way into the oceans. An AR1 model is fitted to the GIS model-data residuals. Due to poor convergence, the first-order lag autocorrelation parameter \(\sigma_{\text{res}}\) is held constant at a value determined by a preliminary model simulation that is optimized using a differential evolution optimization algorithm (Storn and Price, 1997). The GIS training data set does not provide heteroscedastic error estimates, so the AR1 innovation variance is taken to be the estimated statistical parameter \(\sigma_{\text{GIS}}\) added in quadrature to the provided error estimate (Sasgen et al., 2012). All GIS physical and statistical model parameters are assigned uniform prior distributions. The ranges for these priors and posterior distribution medians, 5, and 95% quantiles are given in Appendix A. Further details regarding SIMPLE are provided in Bakker et al. (2016a).

### 3.2.3 Antarctic ice sheet

We employ the Danish Center for Earth System Science Antarctic Ice Sheet (DAIS) model to simulate the Antarctic ice sheet contribution to global sea level (Shaffer, 2014). This is a two-dimensional model for the Antarctic ice sheet that assumes an axisymmetric geometry, shown graphically in Shaffer (2014), his Fig. 2. The DAIS model tracks changes in Antarctic ice sheet volume, considering contributions from (i) incident precipitation, (ii) runoff of meltwater, (iii) ice flow, and (iv) ice sheet disintegration from rising and warming sea levels. Input forcings for the
DAIS model include Antarctic surface temperature reduced to sea level ($T_{S}$, °C), high latitude ocean subsurface temperature ($T_{ASO}$, °C), global mean sea level (m), and the time rate of change of global mean sea level (m y$^{-1}$).

When calibrated as a stand-alone model, the DAIS forcings are provided based on temperature reconstructions (see Shaffer (2014)). When the DAIS model is run as a component in the coupled BRICK model, a separate sub-model is needed to convert the global mean surface temperature from the climate model (DOECLIM) to the Antarctic surface and ocean subsurface temperatures required by the DAIS model. The Antarctic surface temperature is estimated from a linear regression with global mean surface temperature (Morice et al., 2012; Shaffer, 2014). The Antarctic ocean temperatures ($T_{ANTO}$) are modeled through a simple relation with the global mean surface temperature, $T_{g}$ (relative to 1850-1900 mean). $T_{ANTO}$ is bounded below at the freezing point of salt water ($T_{f} = -1.4^\circ$C):

$$T_{ANTO}(t) = T_{f} + \frac{a_{ANTO}T_{g}(t) + b_{ANTO} + T_{f}}{1 + \exp\left[\frac{a_{ANTO}T_{g}(t) + b_{ANTO} - T_{f}}{b_{ANTO}}\right]}$$

Equation (6) is a modified linear regression between the global mean surface temperature $T_{g}$ and the Antarctic ocean temperature $T_{ANTO}$, such that the Antarctic ocean temperature is bounded below by the freezing temperature of sea water, $T_{f}$. In Eq. (6), $a_{ANTO}$ is the sensitivity of the Antarctic ocean temperature to global mean surface temperature (unitless), and $b_{ANTO}$ (°C) is the approximate Antarctic ocean temperature for $T_{g} = 0^\circ$C. $b_{ANTO}$ is the approximate ocean temperature because the relationship in Eq. (6) is not a simple linear regression. $a_{ANTO}$ and $b_{ANTO}$ are both estimated as uncertain model parameters. The DAIS model contains 11 physical and one statistical parameter, for a total of 14 Antarctic ice sheet parameters to be estimated. The heavily parameterized Antarctic ice sheet module reflects our focus on including a broad range of model and observational uncertainties, and consideration of the critical role of the Antarctic ice sheet in driving substantial uncertainty in future sea levels (Church et al., 2013).

Here, we use an updated and corrected version of the DAIS model (Ruckert et al., 2017; Shaffer, 2014). In the original formulation of the DAIS model, the input forcing from year $t$ is used to determine the AIS contribution to sea-level rise in year $t$. This implicit numerical scheme assumes sea level and temperatures for the current year are known during that model iteration. For this study, in which temperatures and sea level originate in other BRICK model components, the DAIS model is re-cast using an explicit numerical scheme. The sea level and temperatures from the year $t$ are used to calculate the AIS contribution in year $t$. Antarctic shore-average local mean sea level functions as the input to DAIS when run as a sub-model of the coupled BRICK model. This is estimated as described in Sect. 3.3.
The dynamical core of the DAIS model is more detailed than the GIC, GIS, and TE emulators given above. For this reason, we do not undertake a full review of the model equations here. The interested reader is directed to Shaffer (2014) and Ruckert et al. (2017) for further details regarding the DAIS model and its hindcast forcings. Equation (3) of Shaffer (2014) is the main equation of state for the Antarctic ice sheet volume ($V_{\text{AIS}}$, m$^3$):

$$\frac{dV_{\text{AIS}}}{dt}(t) = B_{100} (T_s, R) + F (S, R).$$

In Eq. (3), $B_{100}$ is the total rate of accumulation of mass on the Antarctic ice sheet (m$^3$ y$^{-1}$), $T_s$ is the Antarctic surface temperature reduced to sea level (°C), $S$ is the sea level (m), $R$ is the Antarctic ice sheet radius (m), and $F$ is the ice flux at grounding line (m$^3$ y$^{-1}$). Following Shaffer (2014), we take the present sea level equivalent Antarctic ice sheet volume to be 57 m SLE, and the initial ice sheet volume ($V_{\text{AIS}, 0}$, m$^3$) to be consistent with an initial ice sheet radius of 1.86x10$^6$ m. Thus, the Antarctic ice sheet contribution to global sea level may be calculated as

$$\frac{dS_{\text{AIS}}}{dt}(t) = -57 \mbox{ m} \cdot \frac{dV_{\text{AIS}}}{dt}(t) \frac{1}{V_{\text{AIS}, 0}}.$$

### 3.2.4 Thermal expansion

BRICK uses a simple parameterization for the contribution of thermal expansion (TE) of the Earth’s oceans to sea level rise. We make the simplifying assumption that thermal expansion of the oceans occurs uniformly around the globe. While this is, of course, not strictly true, the next obvious step forward in model complexity would be to use a vertically- and latitudinally-resolved model for thermal expansion, incorporating the DOECLIM model output for ocean heat uptake. This two-dimensional ocean model is beyond the scope of the simple model framework described presently, but an excellent subject for future work. Here, we employ a simple zero-dimensional thermal expansion emulator based on the parameterizations of the sea-level rise sub-models of Mengel et al. (2016) and was originally used by Grinsted et al. (2010) to model the total global mean sea level changes. First, an equilibrium sea level rise from thermal expansion, due to changing global surface temperature ($S_{\text{eq, TE}}$, m) is calculated as

$$S_{\text{eq, TE}}(t) = a_{\text{TE}} T_s(t) + b_{\text{TE}}.$$

In Eq. (4), $a_{\text{TE}}$ is the sensitivity of the equilibrium sea-level rise from thermal expansion, due to changing global surface temperatures (m °C$^{-1}$), and $b_{\text{TE}}$ is the equilibrium sea-level rise from thermal expansion with no temperature anomaly (m). The sea-level rise due to thermal expansion evolves with time as

$$\frac{dS_{\text{TE}}}{dt}(t) = \frac{1}{\tau_{\text{TE}}} \left( S_{\text{eq, TE}}(t) - S_{\text{TE}}(t) \right),$$

where the quantity $\tau_{\text{TE}}$ is the e-folding time-scale with which current sea-level adjusts to the equilibrium state, and $1/\tau_{\text{TE}}$ is taken as an uncertain model parameter. This parameter is assigned a gamma prior distribution with shape 1.81 and scale 0.00275, which places the 5th and 95th quantiles for $\tau_{\text{TE}}$ at 82 and 1,290 years (Mengel et al., 2016). This choice of prior distribution is motivated by the fact that $\tau_{\text{TE}}$ functions similarly to the uncertain time-scale
associated with an exponentially-distributed random variable. A gamma distribution is the conjugate prior for such a random variable. The initial condition \( S_{0c} \) is provided as an uncertain model parameter (\( m \)), designated in year 1850. To match this accounting for sea-level rise relative to pre-industrial, forcing temperature is taken relative to its 1850-1870 mean. We calibrate the thermal expansion component of sea-level rise using trends reported by the [IPCC (Church et al., 2013)](https://www.ipcc.ch/).

### 3.3 Regional sea-level patterns

In order to link the projections of global mean sea-level change from BRICK to a local coastal adaptation, information on regional sea level change is needed. Thus, the global mean sea level from BRICK is downscaled to regional sea level using previously published maps of scaling factors for the glacier and ice sheet components of sea-level change (Slangen et al., 2014). Any redistributions of mass between the cryosphere and the ocean (e.g. ice melt) leads not only to a change in the total mass of the ocean, but also to changes in regional sea level as a result of variations in the gravitational field of the Earth, which in turn affects the solid Earth and the rotation of the Earth (e.g., Mitrovica et al., 2001). This typically (and counterintuitively) leads to a sea-level fall close to the source of mass loss and larger-than-average sea-level rise at larger distances (> 6700 km) from the source. These so-called regional sea-level “fingerprints” are constant for the time scales used in this study, as long as the location of the ice mass change remains the same. The fingerprints can therefore be used to relate global glacier and ice sheet contributions to sea level (Sect. 3.2.1–3.2.3) to their regional sea level contribution. We couple changes in global sea level to the Antarctic ice sheet model using an Antarctic shore-average fingerprint ratio of 1.0 for the AIS contribution to global sea level, and Antarctic shore-average fingerprint factors of 1.0 for the other contributions to sea-level rise from all BRICK sub-models (Slangen et al., 2014). Preliminary experiments indicated that our results are not sensitive to the precise choices of these fingerprints.

The glacier fingerprint is based on projected changes in glacier mass in 2100 using a glacier model driven by temperature and precipitation information from the Fifth Climate Model Intercomparison Project database (Taylor et al., 2012) under the Representative Concentration Pathway 8.5 climate change scenario (RCP8.5, Moss et al., 2010), as presented in Slangen et al. (2014). It is assumed that the mass change ratios between the different glacier regions on Earth remain the same throughout the 20th and 21st century, which is a valid assumption as long as none of the glacier regions “finishes” (which is not expected to happen in the next century). For the Greenland and Antarctic ice sheets, it is assumed that ice melt takes place uniformly over the ice sheet surface. Within the BRICK model structure, users may define a latitude and longitude to obtain regional sea level change.
4 Model experiments

4.1 Model calibration

We calibrate the model through a coupled physical-statistical calibration framework. The relatively simple physical modeling framework of BRICK is motivated by our epistemic modeling values (Sect. 2.1). This efficient model permits the use of a sophisticated model calibration technique. The calibration uses a robust adaptive Markov chain Monte Carlo (MCMC) approach (Vihola, 2012). The specifics of how it is applied to the BRICK model as well as a demonstration of calibrated BRICK model hindcast skill are documented in Bakker et al. (2016b).

The vastly different time scales and characterizations of uncertainty in the Antarctic paleoclimatic calibration period and the modern period (1850 to present) lead to two separate sets of calibration parameters: (i) DAIS parameters, calibrated using paleoclimatic data, and (ii) DOECLIM, GIC, GIS, and TE parameters, jointly calibrated using modern data. The paleoclimatic calibration is done using four parallel MCMC chains of 500,000 iterations each. The first 120,000 iterations of each chain are removed for burn-in. The paleoclimatic calibration requires about 10 hours on a laptop with a 2.9 GHz dual-core processor with 16 GB of RAM. The modern calibration is done using four parallel MCMC chains of $1 \times 10^6$ iterations each. The first 500,000 iterations of each chain are removed for burn-in. This requires less than one hour on the same machine as the paleoclimatic calibration. Convergence and burn-in lengths are assessed using Gelman and Rubin diagnostics (Gelman and Rubin, 1992).

We combine these two disjoint sets of parameters to form concomitant full BRICK model parameters sets, and calibrate these to global mean sea level data (Church and White, 2011) using rejection sampling (Votaw Jr. and Rafferty, 1951). Prior to rejection sampling, contributions from land water storage are estimated using trends from the IPCC (Church et al., 2013) and subtracted from global mean sea level. When projecting global mean sea-level rise, we estimate land water storage contributions by extrapolating using the 2003–2013 trend of $0.30 \pm 0.18$ mm y$^{-1}$ found by Dieng et al. (2015). This approximation may not hold in reality (Wada et al., 2012), but serves as a starting point for future model developments. The use of rejection sampling and the estimation of land water storage contributions to sea level are the two aspects in which our calibration approach differs from that of Bakker et al. (2016b). In this rejection sampling step, each full BRICK parameter set is constructed by parsing a random draw from the calibrated DAIS parameter sets with a random draw from the DOECLIM-GIC-GIS-TE calibrated parameter sets. This full BRICK model has the major components of global mean sea-level rise represented, so only at this stage is calibration using global mean sea level data possible. The calibration to global sea level data initially proposes 125,000 full BRICK model parameter sets. We use a joint Gaussian normal likelihood function centered at the time series of the global mean sea level data, with standard deviation given by the observational uncertainty of
the sea level data (corrected to account for land water storage). For rejection sampling, the enveloping distribution is this likelihood function evaluated at the observed sea level time series itself. Thus, no model simulation can yield a realization of the likelihood function that exceeds this value. Rejection sampling accepts each model simulation with probability equal to the ratio of the likelihood function evaluated at the selected model simulation to the maximal value of the likelihood function. 10,671 ensemble members remain after the calibration to global mean sea level data. These model realizations serve as the control ensemble for analysis. The entire analysis for the control model, including paleoclimatic simulations and the risk assessment presented in Sect. 4.4, requires about 4 hours on a modern laptop, but constructing smaller ensembles is much faster (an ensemble of about 600 members requires less than 10 minutes).

In the spirit of our epistemic values, calibration routines are provided with the available BRICK source code. These routines use modern methods readily available in R. It is our aim that the interested user can easily substitute their own likelihood function (as physical scientific knowledge progresses), a new calibration method (as the statistical state-of-the-art progresses), or both. To this end, we provide a sub-routinized likelihood function, called from an R-packaged calibration method (Vihola, 2012). We also provide individual likelihood functions and calibration scripts for each sub-model of BRICK individually, to enable interested users to perform experiments using stand-alone sub-models or pre-calibration (Edwards et al., 2011).

In the interest of accessibility and transparency, with the available BRICK source code we also provide the sets of calibrated model parameters for all experiments presented here. The purpose of this is twofold. First, it greatly enhances the reproducibility of these results. Second, these data sets enable users who would like to run their own ensembles and make projections of local sea levels to do so. This supports our goal of accessibility. The calibrated parameter sets are provided in netCDF format, with ensemble member as the “unlimited” dimension. This permits concatenating multiple data sets by using netCDF operators (NCO) such “ncrcat” (Zender, 2008). These are freely available tools for manipulating data stored in netCDF format.

### 4.2 Testing alternative model components: a sea-level rise module intercomparison

#### 4.2.1 Experimental description

We achieve the accessibility, transparency, and computational efficiency of the BRICK modeling framework through use of simple models written in a simple programming environment (R, R Core Team, 2016). It remains to be demonstrated that this framework is flexible and efficient in post-processing.
We demonstrate BRICK’s flexibility and efficiency by implementing and switching in an alternative formulation for the global mean sea level, \( S(t) \). We exchange the more detailed model configuration for global mean sea level (the BRICK control, see Fig. 1) for the simple emulator described by Rahmstorf (2007). This is

\[
\frac{dS}{dt}(t) = a_{GMSL}(T_g(t) - T_{eq,GMSL}),
\]

where \( t \) is time (years), \( S \) is the global mean sea level (m), \( a_{GMSL} \) is a sensitivity constant \( \text{m °C}^{-1} \text{yr}^{-1} \), \( T_g \) is the global mean surface temperature anomaly \( \text{(°C)} \), and \( T_{eq,GMSL} \) is the theoretical temperature at which the global sea level is steady \( \text{(°C)} \). The parameters \( a_{GMSL} \) and \( T_{eq,GMSL} \), as well as the statistical parameters \( \rho_{GMSL} \) (the first-order lag) and \( \sigma_{GMSL} \) (the homoscedastic component of the innovation variance), are calibrated using the same global mean sea level data set as the full BRICK sea-level rise module (Church and White, 2011). The “BRICK-GMSL” model performance using Eq. (1) for the sea-level rise module (while still coupled to DEOCLIM as the climate module) is compared against the full BRICK model configuration. This BRICK-GMSL model configuration is calibrated using four parallel MCMC chains of 100,000 iterations each. The first 50,000 iterations are removed for burn-in, as determined using Gelman and Rubin diagnostics (Gelman and Rubin, 1992). We randomly sample from the resulting posterior distribution to form an ensemble for analysis of 10,671 model realizations. This ensemble size is chosen to be consistent with the BRICK control model ensemble size. The prior ranges and posterior medians, 5, and 95% quantiles for the BRICK-GMSL parameters are provided in Appendix A.

Note that the Rahmstorf (2007) emulator is arguably not the state-of-the-art anymore (Grinsted et al., 2010; Kopp et al., 2016). However, it serves here the purpose of demonstrating the ease with which alternative model formulations can be tested. This greatly simplifies, for example, model intercomparisons and improvements. Some advantages of a simple emulator such as this include fewer parameters to estimate and a transparent analysis. Disadvantages of such a model include the inability to resolve individual contributions to global mean sea level. This disables the use of sea level fingerprinting to obtain regional sea-level patterns. Thus, the choice of model should not only be motivated by goodness-of-fit metrics, but also by applications.

### 4.2.2 Metrics for model-data comparison

Many goodness-of-fit metrics are available for the comparison of models and data. We focus on three metrics that are motivated by the heavily-parameterized full BRICK model framework. There are 39 free parameters in the coupled climate/sea-level rise model. By contrast, BRICK-GMSL has 13 free parameters. We use the global mean sea level time series of Church and White (2011) for the model-data comparisons in skill hindcasting global mean sea level.
Root-mean-squared-error (RMSE) is a commonly-used error metric, so we employ it here. For consistency with other error criteria defined below, we define the RMSE for a model as the RMSE of the model ensemble member that maximizes the likelihood function.

Akaike Information Criterion (AIC) is a measure of the relative goodness-of-fit between two potential models for the same data (Akaike, 1974).

$$AIC = -2 \ln(L_{\text{max}}) + 2N$$  \hspace{1cm} (12)

In Eq. (12), $L_{\text{max}}$ is the maximum value of the likelihood function and $N$ is the number of model parameters. Lower values of AIC provide a better match between model output and data, and consider a penalty for over-parameterizing a model.

Bayesian Information Criterion (BIC) is formulated similarly to AIC, but enacts a different penalty for over-parameterization (Schwarz, 1978).

$$BIC = -2 \ln(L_{\text{max}}) + N \ln (N_{\text{obs}})$$  \hspace{1cm} (13)

In Eq. (13), $N_{\text{obs}}$ is the number of observational data points used in the model-data comparison. Thus, for $N_{\text{obs}} > e^2$, the BIC metric penalizes over-parameterization more harshly than does AIC.

4.2.3 Experimental results: sea-level rise module intercomparison

The full BRICK sea-level rise module (Fig. 1) performs better than the GMSL emulator (Eq. 11) according to RMSE; the full sea-level rise module has RMSE of 0.0059 m, which is about half the GMSL emulator RMSE of 0.0125 m (Fig. 2). These hindcasts are presented as sea level relative to 1961-1990 global mean sea level. This is of course expected, because the number of free model parameters in the full BRICK model is 39, while the GMSL emulator contains only 13 free parameters. The BIC metric gives the expected result for this disparity in model complexity. The BIC for the full BRICK model with respect to the sea level data is 61.6 higher than the BIC for the GMSL emulator. The AIC is actually lower (by 12.5) for the full BRICK model than for the BRICK-GMSL emulator. These mixed results for the model comparison metrics indicate that the full BRICK sea-level rise module is not unreasonably over-parameterized; if the full BRICK model were obviously over-parameterized, we would expect the AIC for the GMSL emulator experiment to be lower than for the full BRICK model.

These results also show that the sea level hindcast in the full BRICK model smooths much of the year-to-year variability in sea-level rise. This can be seen by contrasting the full BRICK maximum likelihood ensemble member (solid blue line) in Fig. 2a with the BRICK-GMSL emulator maximum likelihood ensemble member in Fig. 2b. The full BRICK simulation does not capture the annual variation in global mean sea level that the BRICK-GMSL simulation successfully captures. This is attributed to the smoothing effect of averaging over the model ensemble the four major contributions to global mean sea level, as opposed to calibrating the BRICK-GMSL simulations.
directly to global mean sea level data. This does not affect ensemble statistics, however, which can be seen from the shaded envelopes around the model simulations in Fig. 2. The BRICK model has been developed with efficiency and large ensemble simulations in mind, so missing annual variability is of little concern.

This demonstrates the ease with which model intercomparisons may be undertaken using BRICK. Deactivating the glaciers and ice caps, thermal expansion, and Greenland and Antarctic ice sheet components and integrating the GMSL emulator into BRICK involves low overhead in computer code. GMSL is the main output of the BRICK physical model. As such, it is our aim to provide a framework in which users can easily integrate new processes and models into the climate and sea-level rise modules as the scientific forefront progresses.

4.3 Interchanging BRICKs and sub-model intercomparisons

4.3.1 Experimental description

We conduct an experiment to demonstrate the flexibility of BRICK to permit easy exchanging of a single sub-model for one component of global sea-level rise. In the control BRICK model set-up, SIMPLE is used to emulate the sea-level rise contributions from the Greenland ice sheet (GIS) and GIC-MAGICC is used to emulate the contributions from glaciers and ice caps (GIC). In this model intercomparison experiment, a second version of SIMPLE is calibrated to represent the GIC component of sea-level rise. This experiment is motivated by potential structural shortcomings of the GIC-MAGICC model. In Eq. (2), the implied GIC volume equilibrium depends only on the current surface temperature relative to the fixed parameter $T_{eq,GIC}$. If the GIC volume is quite low (almost entirely melted), this structure potentially enables unphysically fast growth of GIC volume. The SIMPLE model (Eqs. 3–5) contains an arguably more realistic representation of the relaxation of ice sheet volume towards an equilibrium. In this formulation, the time-scale of the relaxation and the equilibrium itself both depend on the surface temperature state. This type of potential disagreement within the scientific community regarding model structure is precisely where the BRICK model framework can be useful. The flexibility of BRICK enables easy exchange of one component sub-model (GIC-MAGICC) for another (GIC-SIMPLE). This enables experiments examining the impacts of model structural choices.

This GIC-SIMPLE model configuration calibrates GIC-SIMPLE using the same observational data as the control GIC-MAGICC set-up. One key difference is that the prior distributions of the model parameters for GIC-SIMPLE were modified to be specific to the GIC conditions instead of the GIS. These prior distributions are given in Appendix A. The same calibration method and likelihood functions are used for the GIC-SIMPLE experiment as in the GIC-MAGICC control model. We use the same calibration approach as in the control ensemble, which yields an

Deleted: The full BRICK simulation does not capture the annual variation in global mean sea level that the BRICK-GMSL simulation successfully captures.
ensemble of 10,400 model realizations for analysis in the GIC-SIMPLE experiment. As in Sect. 4.2, we focus on RMSE, AIC, and BIC as model goodness-of-fit metrics. The GIC-MAGICC model has six model parameters (four physical model, two statistical) and the GIC-SIMPLE model has seven parameters (five physical model, two statistical).

4.3.2 Experimental results: glaciers and ice caps sub-model intercomparison

When the GIC-MAGICC model is used, RMSE, AIC, and BIC are all lower than when the GIC-SIMPLE model is used (Fig. 3). But the AIC and BIC are not drastically lower for GIC-MAGICC than for GIC-SIMPLE. This indicates that the addition of a model parameter (GIC-SIMPLE) may not be justified (Kass and Raftery, 1995). The GIC contribution to global sea level in Fig. 3 is presented relative to 1960 GIC sea-level rise. The median, 5, and 95% quantiles of the calibrated GIC-SIMPLE parameters are given in Appendix A.

The two models display similar levels of under-confidence, illustrated by the wide model ensemble envelope around the narrower range of observational data (Fig. 3) (Dyurgerov and Meier, 2005). That both models show under-confidence is often judged to be preferable to over-confidence, especially when physical models are linked of applications-oriented decision-making frameworks (Herman et al., 2015). This experiment demonstrates BRICK's flexibility, and ability to allow the user to isolate and examine any source of uncertainty or dissatisfaction in the modeling framework. These results also provide guidance for the use of the BRICK model framework for model intercomparison and selection experiments. At present we do not make any recommendations regarding which GIC sub-model to use. The GIC-MAGICC component has both strengths (e.g., fewer parameters and appropriate in melting regimes) and weaknesses (unphysical GIC growth, does not encourage growth beyond $V_{GIC}$, state-independent equilibrium).

4.4 Linking an impacts and decision-analysis module to BRICK

4.4.1 Experimental description

We demonstrate the ability of the BRICK framework to incorporate additional structure to link the physical model for surface temperature and sea-level rise (climate and sea level modules, Fig. 1) to socioeconomic implications (impacts module, Fig. 1). In this example application, we use the calibrated ensemble in the BRICK control configuration to obtain local sea level projections for New Orleans, Louisiana ($29^\circ 57' \ N, 90^\circ 4' \ W$). We use a common didactic model for coastal flood protection (Van Dantzig, 1956; Jonkman et al., 2009). In this flood risk model, the policy lever available to decision-makers is the amount by which to heighten the dikes protecting the coastal community. We consider a previously published simple analysis that focuses on the northern dike ring in
central New Orleans (Jonkman et al., 2009). We use this illustrative cost-benefit approach to calculate an economically-efficient dike-heightening by weighing the decrease in probable losses due to flooding achieved by building taller dikes against the increase in costs due to investments in construction.

The flood risk model implemented here follows a commonly used simple approach (Van Dantzig, 1956). The present implementation considers the current year as 2015 and a time horizon of 85 years (to 2100). We consider discrete dike heightenings in increments of 5 centimeters, between 0 and 10 meters. The average annual flood probability is calculated from the simulated local sea-level rise, the land subsidence rate (Dixon et al., 2006), and flood frequency parameters (Jonkman et al., 2009). We calculate the expected losses (US dollars) for each proposed dike heightening from the flood probabilities for each heightening, the value of goods protected by the dike ring, and the net discount rate (Jonkman et al., 2009). The total expected costs are the sum of the expected losses and the expected investments. In this simplified model, we assume that the investment costs only depend on dike heightening and are approximated by linear interpolation between data points provided by Jonkman et al. (2009) and linear extrapolation for dike heightenings outside this range. The expected losses are an exponentially decreasing function of dike height above mean sea level. The minimum total expected cost then is the economically-efficient dike heightening strategy in the framework of this simple illustrative model (Eq. 14 of Van Dantzig (1956)).

The uncertain parameters considered in this cost-benefit analysis include the initial flood frequency with no heightening \( (y^0) \); the exponential flood frequency constant \( (m^-1) \); the value of goods protected by the dike ring (billion US dollars); the net discount rate (%); the uncertainty in investment costs (a unitless multiplicative factor); and the land subsidence rate \( (m \ y^-1) \) (Table 1). The central estimates for the exponential flood frequency constant \( (\alpha) \) and the initial flood frequency with no heightening \( (p_0) \) are taken from Van Dantzig (1956). The exponential flood frequency constant relates the increase in flood probability that results from an increase in sea level relative to the dike height. We make the assumption that this factor should scale (to first order) relatively well from Dutch case considered by Van Dantzig (1956) to the test case of New Orleans considered presently. The initial flood frequency with no heightening \( (p_0) \) may not translate directly between these two cases, but highlights our intent for this experiment to serve as an example of future applications of the BRICK model to inform decision analyses. The admittedly ad hoc distributions assumed for \( \alpha \) and \( p_0 \) were selected to sample tightly around the central estimates from Jonkman et al. (2009). A more detailed treatment of this risk management problem would include using methods from extreme value theory to address the risks posed by storm surges (Coles, 2001).
The investment uncertainty considered in the sensitivity tests of Jonkman et al. (2009) included a base case, 50% lower, and 100% higher than the base case. We use this range for the investment uncertainty, applied as a multiplicative factor ranging from 0.5 to 2. The range for the value of good protected by the dike ring is taken from Jonkman et al. (2009), where the lower bound is the lowest estimate of value of goods protected by the three dike rings considered in that work (US$5 billion), and the upper bound is the estimated combined value protected by all three dike rings (US$30 billion). The net discount rate range is centered at 4%, the estimate from Jonkman et al. (2009) accounting for inflation and interest rate. Those authors’ net discount rate is decreased to 2% due to economic growth (1%) and increased flooding probability due to sea-level rise (1%). Our demonstrative example endogenizes the effects of sea-level rise and accounts for parametric uncertainty in the value of good protected by the dike ring. Hence, we center our range for the net discount rate at 4% but allow for ±2% uncertain range. The rate of land subsidence is based on the estimates of Dixon et al. (2006), with mean 5.6 mm y\(^{-1}\) and standard deviation 2.5 mm y\(^{-1}\). We transform this to a log-normal distribution to disallow negative rates of land subsidence.

We sample the uncertainty in these parameters via Latin hypercube, where the population size is given by the number of sea-level rise ensemble members that are present (10,671 for the control BRICK ensemble). The distributions from which the economic model parameters are drawn are given in Table 1. Each realization of regional sea level is assigned a concomitant sample of flood risk model parameters. An economically-efficient dike heightening is calculated for each ensemble member. “Return periods” (years) correspond to the frequency of storms with the potential to overtop dikes with the corresponding dike height – essentially, the inverse of the annual flood probability. Return periods are a convenient and intuitive way to view the probabilities of flooding in this economic analysis.

We present results for the flood risk management experiment using sea level projections under RCP8.5. We note that many factors are not incorporated into this analysis and this simple illustration is not designed to be used for real decision making. For example, storm surge and structural failure are not considered (Grinsted et al., 2013; Moritz et al., 2015). The purpose of this illustration is to demonstrate the flexibility and transparency of the BRICK model framework. This experiment highlights the importance of transparency in particular when linking physical modeling results to the impacts on socioeconomic modeling and policy decision-making.

4.4.2 Experimental results: regional sea-level changes

In order to link projections of sea-level rise to problems of local coastal adaptation, regional sea level is projected to 2100 under the climate change scenarios of RCP2.6, 4.5, and 8.5 (Fig. 4). These projections use the control
configuration of the model, with GIC-MAGICC and the full sea-level rise sub-model set-up depicted in Fig. 1. The ensemble median projection is shown in Fig. 4. Sea level rises by 2100 globally by about 54 cm (42-69 cm), 75 cm (58-99 cm), and 120 cm (95-174 cm) under RCP2.6, 4.5, and 8.5, respectively (ensemble median and 5%-95% range in parentheses). The Arctic Ocean is an obvious exception to the rest of the ocean. Due to the Greenland ice mass loss, Arctic regional sea level will fall as a result of the loss of gravitational attraction. However, the addition of mass raises sea level in other parts of the ocean farther away. Arctic sea level (median sea level of all latitudes higher than 60°N) increases by 8 cm under RCP2.6, but falls by 1 cm under RCP4.5 and by 28 cm under RCP8.5. By contrast, the tropical sea level (median of all latitudes between 30°S and 30°N) rises by 56 cm, 80 cm, and 143 cm under RCP2.6, 4.5, and 8.5, respectively, which is greater than the global mean rise. Due to the asymptotically increasing gravitational effects in proximity to the melting Greenland ice sheet, sea-level fall below -1.5 m is cut off at -1.5 m.

4.4.3 Experimental results: Link to coastal defense strategies

We now focus on the regional sea-level projections for the gridcell containing New Orleans, Louisiana under RCP8.5 (Fig. 4c), to demonstrate the use of these sea-level projections in a common local flood risk management example. We find the economically-efficient (i.e., cost-minimizing) dike heightening to be 1.45 m (ensemble mean; 90% range is 0.75 to 1.95 m; Fig. 5). This heightening corresponds to a return period of about 790 years (ensemble mean; 90% range is roughly 200-3000 years; Fig. 5). The simple analysis presented here should not be used to inform on-the-ground decisions in New Orleans. This experiment is meant to demonstrate BRICK’s ability to contribute in risk assessment applications.

5 Conclusions

We present BRICK v0.2, a modeling framework for global and regional sea-level change. BRICK has been designed with four epistemic modeling goals: accessibility, transparency, efficiency, and flexibility. BRICK can skillfully match observational data for individual sea level contributions in hindcast (Bakker et al., 2016b). Here we focus on how BRICK achieves our epistemic values using a set of modeling experiments.

BRICK is coded in the widely available and simple coding language R (R Core Team, 2016) to achieve the goals of accessibility and transparency. The main physics (global mean temperature and sea-level rise) codes are also (redundantly) transcribed in Fortran 90, for more efficient simulations. BRICK is designed to be transparent, as well as efficient, by coupling previously published simple, mechanistically motivated models for the major contributors to global sea level. The efficient physical modeling approach provides the opportunity to incorporate a rigorous
statistical calibration framework as well, wherein various sources of uncertainty are incorporated into model projections (see Bakker et al. (2016b) for a more detailed discussion of this). Finally, the model comparison experiments in Sect. 4.2 and 4.3 demonstrate the flexibility of the BRICK modeling framework. These sections bring into focus the importance of these epistemic modeling values. A modeling framework that is (in particular) transparent and accessible can help to streamline the process of quantifying the local impacts of the physical model results, to link to decision-analytical models, and to communicate these results to stakeholders and decision-makers.

We hope that the accessibility and transparency of BRICK are helpful to others, and will stimulate the continuous peer-reviewing, challenging, and improving of the BRICK framework. Of course, although we tried to couple models that fit our epistemic model values as close as possible, we assume that others may prefer other models and may have different epistemic values. Our framework is designed in such a way that it is possible to plug in other model components to reflect these different values. For example, it would be very interesting to add the components models used for the semi-empirical model frameworks of Mengel et al. (2016) and Nauels et al. (2016).

We demonstrated the flexibility and transparency of BRICK in connecting projections from the physical model to the impacts on a local risk and decision-analysis problem. The simple probabilistic calibration method and cost-benefit analysis that we adopted for the simple demonstration can be expanded to incorporate aspects of deep uncertainties (Lempert et al., 2004; Weaver et al., 2013) as well as more complex decision-making frameworks (e.g., considering multiple objectives, beyond only expected total costs) (Kaspryzk et al., 2013; Lempert, 2014; Lempert and Collins, 2007). Climate change poses decision problems where strong connections across academic disciplines are critical. Further, the study of climate modeling relies on communal modeling efforts. The need for transparent communication among modelers and between disciplines is where the BRICK framework and the epistemic modeling values presented here can facilitate future developments. Above all, we hope that BRICK inspires the involved communities to pay careful attention to enhance flexibility, transparency, and accessibility of modelling frameworks.

**Code and Data Availability**

All BRICK v0.2 code is available at [https://github.com/scrim-network/BRICK](https://github.com/scrim-network/BRICK) under the GNU general public open source license. Large parameter files as well as model codes forked from the repository to reproduce this work (including the sea level projections) may be found at [https://download.scrim.psu.edu/Wong_etal_BRICK/](https://download.scrim.psu.edu/Wong_etal_BRICK/).
Appendix A: Prior probability distribution ranges for the sub-model parameters, and median, 5th, and 95th quantiles of the calibrated posterior parameter distributions.

Table A1. Prior probability distribution ranges for the DOECLIM climate model parameters, and median, 5th, and 95th quantiles of the calibrated posterior parameter distributions. The priors are all uniformly distributed.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Units</th>
<th>Lower bound</th>
<th>Upper bound</th>
<th>5%</th>
<th>Median</th>
<th>95%</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S$</td>
<td>°C</td>
<td>0.1</td>
<td>10</td>
<td>1.2</td>
<td>2.3</td>
<td>3.6</td>
</tr>
<tr>
<td>$\kappa_{DOECLIM}$</td>
<td>cm$^2$ s$^{-1}$</td>
<td>0.1</td>
<td>4</td>
<td>0.4</td>
<td>1.6</td>
<td>3.2</td>
</tr>
<tr>
<td>$\sigma_{DOECLIM}$</td>
<td>[-]</td>
<td>0</td>
<td>2</td>
<td>0.48</td>
<td>0.79</td>
<td>1.1</td>
</tr>
<tr>
<td>$T_D$</td>
<td>°C</td>
<td>-0.3</td>
<td>0.3</td>
<td>-0.084</td>
<td>-0.04</td>
<td>0.0052</td>
</tr>
<tr>
<td>$H_D$</td>
<td>10$^{22}$ J</td>
<td>-50</td>
<td>0</td>
<td>-42</td>
<td>-29</td>
<td>-16</td>
</tr>
<tr>
<td>$\sigma_T$</td>
<td>°C</td>
<td>0.05</td>
<td>5</td>
<td>0.069</td>
<td>0.080</td>
<td>0.092</td>
</tr>
<tr>
<td>$\rho_T$</td>
<td>10$^{22}$ J</td>
<td>0.1</td>
<td>10</td>
<td>0.19</td>
<td>1.1</td>
<td>2.9</td>
</tr>
<tr>
<td>$\rho_H$</td>
<td>[-]</td>
<td>0</td>
<td>0.999</td>
<td>0.054</td>
<td>0.5</td>
<td>0.95</td>
</tr>
<tr>
<td>$\rho_{TE}$</td>
<td>[-]</td>
<td>0</td>
<td>0.999</td>
<td>0.36</td>
<td>0.48</td>
<td>0.61</td>
</tr>
</tbody>
</table>

Table A2. Prior probability distribution ranges for the thermal expansion model parameters, and median, 5th, and 95th quantiles of the calibrated posterior parameter distributions. The prior distribution for $1/\tau_E$ is a gamma distribution (see main text). The other priors are all uniformly distributed.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Units</th>
<th>Lower bound</th>
<th>Upper bound</th>
<th>5%</th>
<th>Median</th>
<th>95%</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_{TE}$</td>
<td>m °C$^{-1}$</td>
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<td>0.8595</td>
<td>0.11</td>
<td>0.43</td>
<td>0.81</td>
</tr>
<tr>
<td>$b_{TE}$</td>
<td>m</td>
<td>0</td>
<td>2.193</td>
<td>0.033</td>
<td>0.31</td>
<td>1.5</td>
</tr>
<tr>
<td>$1/\tau_{TE}$</td>
<td>y$^{-3}$</td>
<td>0</td>
<td>1</td>
<td>0.00048</td>
<td>0.0015</td>
<td>0.0053</td>
</tr>
<tr>
<td>$S_{TE}$</td>
<td>m</td>
<td>-0.0484</td>
<td>0.0484</td>
<td>-0.043</td>
<td>0.005</td>
<td>0.044</td>
</tr>
</tbody>
</table>
Table A3. Prior probability distribution ranges for the GIS-SIMPLE Greenland ice sheet model parameters, and median, 5th, and 95th quantiles of the calibrated posterior parameter distributions. The priors are all uniformly distributed. Due to convergence issues, $\rho_{GIS}$ is held fixed at a value calculated from a preliminary optimized model simulation (see main text).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Units</th>
<th>Lower bound</th>
<th>Upper bound</th>
<th>5%</th>
<th>Median</th>
<th>95%</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_{GIS}$</td>
<td>m °C$^{-1}$</td>
<td>-4</td>
<td>-0.001</td>
<td>-3.9</td>
<td>-3.0</td>
<td>-1.6</td>
</tr>
<tr>
<td>$b_{GIS}$</td>
<td>m</td>
<td>5.888</td>
<td>8.832</td>
<td>7.4</td>
<td>7.8</td>
<td>8.1</td>
</tr>
<tr>
<td>$\alpha_{GIS}$</td>
<td>°C$^{-1}$</td>
<td>0</td>
<td>0.001</td>
<td>0.00032</td>
<td>0.00074</td>
<td>0.00092</td>
</tr>
<tr>
<td>$\beta_{GIS}$</td>
<td>y$^{-1}$</td>
<td>0</td>
<td>0.001</td>
<td>2.4x10$^{-5}$</td>
<td>0.00013</td>
<td>0.00041</td>
</tr>
<tr>
<td>$V_{GIS}$</td>
<td>m</td>
<td>7.16</td>
<td>7.56</td>
<td>7.2</td>
<td>7.4</td>
<td>7.5</td>
</tr>
<tr>
<td>$\sigma_{GIS}$</td>
<td>m</td>
<td>0</td>
<td>0.002</td>
<td>0.00017</td>
<td>2.0x10$^{-4}$</td>
<td>0.00025</td>
</tr>
<tr>
<td>$\rho_{GIS}$</td>
<td>[-]</td>
<td>[-]</td>
<td>[-]</td>
<td>0.92</td>
<td>[-]</td>
<td>[-]</td>
</tr>
</tbody>
</table>

Table A4. Prior probability distribution ranges for the DAIS Antarctic ice sheet model parameters, and median, 5th, and 95th quantiles of the calibrated posterior parameter distributions. An inverse gamma prior distribution is used for $\sigma_{DAIS}^2$ (see Ruckert et al. (2017)). All other prior distributions are uniform.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Units</th>
<th>Lower bound</th>
<th>Upper bound</th>
<th>5%</th>
<th>Median</th>
<th>95%</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_{ANTO}$</td>
<td>°C °C$^{-1}$</td>
<td>0</td>
<td>1</td>
<td>0.03</td>
<td>0.4</td>
<td>0.95</td>
</tr>
<tr>
<td>$b_{ANTO}$</td>
<td>°C</td>
<td>0</td>
<td>2</td>
<td>0.1</td>
<td>1.0</td>
<td>1.9</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>[-]</td>
<td>0.5</td>
<td>4.25</td>
<td>1.4</td>
<td>3.1</td>
<td>4.2</td>
</tr>
<tr>
<td>$\alpha_{DAIS}$</td>
<td>[-]</td>
<td>0</td>
<td>1</td>
<td>0.039</td>
<td>0.36</td>
<td>0.79</td>
</tr>
<tr>
<td>$\mu$</td>
<td>m$^{-2}$</td>
<td>7.05</td>
<td>13.65</td>
<td>7.5</td>
<td>10</td>
<td>13</td>
</tr>
<tr>
<td>$\nu$</td>
<td>m$^2$ y$^{-1/2}$</td>
<td>0.003</td>
<td>0.015</td>
<td>0.0037</td>
<td>0.0089</td>
<td>0.014</td>
</tr>
<tr>
<td>$P_0$</td>
<td>m y$^{-2}$</td>
<td>0.026</td>
<td>1.5</td>
<td>0.13</td>
<td>0.51</td>
<td>1.3</td>
</tr>
<tr>
<td>$\kappa_{DAIS}$</td>
<td>°C$^{-1}$</td>
<td>0.025</td>
<td>0.085</td>
<td>0.029</td>
<td>0.057</td>
<td>0.082</td>
</tr>
<tr>
<td>$f_0$</td>
<td>m y$^{-2}$</td>
<td>0.6</td>
<td>1.8</td>
<td>0.7</td>
<td>1.3</td>
<td>1.8</td>
</tr>
<tr>
<td>$h_0$</td>
<td>m</td>
<td>735.5</td>
<td>2206.5</td>
<td>1100</td>
<td>1700</td>
<td>2200</td>
</tr>
<tr>
<td>$C$</td>
<td>m °C$^{-1}$</td>
<td>47.5</td>
<td>142.5</td>
<td>51</td>
<td>80</td>
<td>120</td>
</tr>
<tr>
<td>$b_y$</td>
<td>m</td>
<td>740</td>
<td>820</td>
<td>740</td>
<td>780</td>
<td>820</td>
</tr>
<tr>
<td>slope</td>
<td>[-]</td>
<td>0.00045</td>
<td>0.00075</td>
<td>0.00055</td>
<td>0.00065</td>
<td>0.00074</td>
</tr>
<tr>
<td>$\sigma_{DAIS}$</td>
<td>m$^2$ SLE</td>
<td>0</td>
<td>[-]</td>
<td>0.19</td>
<td>0.51</td>
<td>2.2</td>
</tr>
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</table>
Table A5. Prior probability distribution ranges for the GIC-MAGICC glaciers and ice caps model parameters, and median, 5th, and 95th quantiles of the calibrated posterior parameter distributions. The priors are all uniformly distributed.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Units</th>
<th>Lower bound</th>
<th>Upper bound</th>
<th>5%</th>
<th>Median</th>
<th>95%</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_{GIC}$</td>
<td>m y^{-1}/°C</td>
<td>0</td>
<td>0.041</td>
<td>0.00054</td>
<td>0.00090</td>
<td>0.0012</td>
</tr>
<tr>
<td>$V_{GIC}$</td>
<td>m</td>
<td>0.3</td>
<td>0.5</td>
<td>0.31</td>
<td>0.40</td>
<td>0.49</td>
</tr>
<tr>
<td>$N$</td>
<td>[-]</td>
<td>0.55</td>
<td>1</td>
<td>0.57</td>
<td>0.72</td>
<td>0.98</td>
</tr>
<tr>
<td>$S_{b,GIC}$</td>
<td>m</td>
<td>-0.0041</td>
<td>0.0041</td>
<td>-0.0036</td>
<td>-3.1x10^{-5}</td>
<td>0.0037</td>
</tr>
<tr>
<td>$\sigma_{GIC}$</td>
<td>m</td>
<td>0</td>
<td>0.0015</td>
<td>2.0x10^{-5}</td>
<td>0.0022</td>
<td>0.00065</td>
</tr>
<tr>
<td>$\rho_{GIC}$</td>
<td>[-]</td>
<td>-0.999</td>
<td>0.999</td>
<td>0.27</td>
<td>0.87</td>
<td>0.99</td>
</tr>
</tbody>
</table>

Table A6. Prior probability distribution ranges for the GIC-SIMPLE model parameters, and median, 5th, and 95th quantiles of the calibrated posterior parameter distributions. The priors are all uniformly distributed.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Units</th>
<th>Lower bound</th>
<th>Upper bound</th>
<th>5%</th>
<th>Median</th>
<th>95%</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_{GIC}$</td>
<td>m °C</td>
<td>-4</td>
<td>-0.001</td>
<td>-3.20</td>
<td>-1.90</td>
<td>-0.72</td>
</tr>
<tr>
<td>$b_{GIC}$</td>
<td>m</td>
<td>0.3</td>
<td>0.5</td>
<td>0.31</td>
<td>0.39</td>
<td>0.49</td>
</tr>
<tr>
<td>$a_{GIC}$</td>
<td>°C^{-1} y^{-1}</td>
<td>0</td>
<td>0.001</td>
<td>4.3x10^{-5}</td>
<td>0.00042</td>
<td>0.00093</td>
</tr>
<tr>
<td>$\beta_{GIC}$</td>
<td>y^{-1}</td>
<td>0</td>
<td>0.001</td>
<td>8.9x10^{-5}</td>
<td>0.00046</td>
<td>0.00093</td>
</tr>
<tr>
<td>$V_{GIC}$</td>
<td>m</td>
<td>0.3</td>
<td>0.5</td>
<td>0.31</td>
<td>0.41</td>
<td>0.49</td>
</tr>
<tr>
<td>$\sigma_{GIC}$</td>
<td>m</td>
<td>0</td>
<td>0.0015</td>
<td>1.2x10^{-5}</td>
<td>0.00024</td>
<td>0.00064</td>
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<tr>
<td>$\rho_{GIC}$</td>
<td>[-]</td>
<td>-0.999</td>
<td>0.999</td>
<td>0.57</td>
<td>0.91</td>
<td>0.99</td>
</tr>
</tbody>
</table>

Table A7. Prior probability distribution ranges for the Rahmstorf (2007) global mean sea level model parameters, and median, 5th, and 95th quantiles of the calibrated posterior parameter distributions. The priors are all uniformly distributed.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Units</th>
<th>Lower bound</th>
<th>Upper bound</th>
<th>5%</th>
<th>Median</th>
<th>95%</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_{GMSL}$</td>
<td>m °C</td>
<td>0</td>
<td>0.0035</td>
<td>0.0012</td>
<td>0.0020</td>
<td>0.0031</td>
</tr>
<tr>
<td>$T_{eq,GMSL}$</td>
<td>m</td>
<td>-1.5</td>
<td>1.5</td>
<td>-1.2</td>
<td>-0.57</td>
<td>-0.28</td>
</tr>
<tr>
<td>$\sigma_{GMSL}$</td>
<td>m</td>
<td>0</td>
<td>0.05</td>
<td>6.2x10^{-5}</td>
<td>0.00069</td>
<td>0.0020</td>
</tr>
<tr>
<td>$\rho_{GMSL}$</td>
<td>[-]</td>
<td>0</td>
<td>0.999</td>
<td>0.36</td>
<td>0.63</td>
<td>0.89</td>
</tr>
</tbody>
</table>
Author Contributions

K. Keller initiated the study. A. Bakker and T. Wong designed the general framework and research. T. Wong and A. Bakker designed the initial figures and wrote the first draft. T. Wong, A. Bakker, K. Ruckert, and P. Applegate produced the major part of the coding and code testing. A. Slangen produced and interpreted the regional sea level fingerprinting data. All contributed to the final text.

Acknowledgments

We gratefully acknowledge Jared Oyler for guidance during code development. We thank Rob Nicholas, Chris and Bella Forest, Nancy Tuana, Robert Lempert, and Gary Shaffer for helpful contributions. This work was partially supported by the National Science Foundation through the Network for Sustainable Climate Risk Management (SCRIM) under NSF cooperative agreement GEO-1240507 as well as the Penn State Center for Climate Risk Management. Any conclusions or recommendations expressed in this material are those of the authors and do not necessarily reflect the views of the funding agencies. Any errors and opinions are, of course, those of the authors.
References


R Core Team: R: A Language and Environment for Statistical Computing, Vienna, Austria., 2016.


Table 1. Parameter descriptions and prior probability distributions for flood protection cost-benefit analysis.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_0$</td>
<td>Initial flood frequency (yr$^{-1}$) with zero heightening</td>
<td>log-N(log-µ=log(0.0038), log-σ=0.25)</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Exponential flood frequency constant (m$^{-1}$)</td>
<td>N(µ=2.6, σ=0.1)</td>
</tr>
<tr>
<td>$V$</td>
<td>Value of goods protected by dike ring (billion US$)</td>
<td>U(5, 30)</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Net discount rate (%)</td>
<td>U(0.02, 0.06)</td>
</tr>
<tr>
<td>$I_{unc}$</td>
<td>Investment uncertainty (%)</td>
<td>U(0.5, 2)</td>
</tr>
<tr>
<td>$r_{subs}$</td>
<td>Land subsidence rate (m yr$^{-1}$)</td>
<td>log-N(log-µ=log(0.0056), log-σ=0.4)</td>
</tr>
</tbody>
</table>
Figures

Figure 1. BRICK model structural diagram. Dashed connectors indicate couplings that are non-essential for projections of global mean sea level. These dashed couplings are required for projecting regional sea level and climate impacts. DOECLIM is the Diffusion-Ocean-Energy balance CLIMate model (Kriegler, 2005); GIC-MAGICC is the Glaciers and Ice Caps module from the climate model MAGICC (Meinshausen et al., 2011a); TE is the Thermal Expansion model (Grinsted et al., 2010; Mengel et al., 2016); SIMPLE is the Simple Ice-sheet Model for Projecting Large Ensembles (Bakker et al., 2016a); ANTO is the ANTarctic Ocean temperature model; DAIS is the Danish Center for Earth System Science Antarctic Ice Sheet model (Shaffer, 2014); regional sea level fingerprinting downscales from global sea-level contributions to regional (Slangen et al., 2014); and the model of Van Dantzig (1956) assesses flood risk.
Figure 2. Comparison of global mean sea-level rise hindcast skill relative to sea level data (Church and White, 2011), using (a) the full sub-model approach (GIC, GIS, TE, and AIS) and (b) the model for global mean sea-level rise of Rahmstorf (2007). Sea level is relative to 1961-1990 global mean sea level. Both model configurations use DOECLIM as the climate module. Lower values of Akaike Information Criterion (AIC), Bayesian Information Criterion (BIC), and root-mean-squared error (RMSE) indicate a better model fit to the data. These error metrics are all calculated using the maximum likelihood ensemble member, which is represented by the solid blue line. Green highlighting indicates the model structure suggested by each comparison metric.
Figure 3. Comparison of (a) GIC-MAGICC versus (b) GIC-SIMPLE model performance in hindcasting the glaciers and ice caps (GIC) contribution to sea-level rise. GIC sea-level rise is presented relative to 1960 GIC sea level contribution. Lower values of Akaike Information Criterion (AIC), Bayesian Information Criterion (BIC), and root-mean-squared error (RMSE) indicate a better model fit to the data (Dyurgerov and Meier, 2005). These error metrics are all calculated using the maximum likelihood ensemble member, which is represented by the solid blue line. Green highlighting indicates the model structure suggested by each comparison metric.
Figure 4. Regional projections of median sea-level changes under Representative Concentration Pathways (RCP) (a) 2.6, (b) 4.5, and (c) 8.5 in the year 2100. Sea-level rise is presented relative to 1986-2005 global mean sea level.
Figure 5. Illustrative cost-benefit analysis for the economically efficient dike heightening (lower horizontal axis) and return period (upper horizontal axis) for the north-central dike ring in New Orleans, Louisiana. The bold dot denotes the economically-efficient (i.e., cost-minimizing) solution. The shaded region gives the 90% ensemble range of trade-off curves and the bold line denotes the ensemble mean trade-off curve.
§ Now at: Royal Netherlands Institute for Sea Research (NIOZ), Department of Estuarine & Delta Systems (EDS), Yerseke, The Netherlands

is a linear mass balance between precipitation $P_{GIS}$, runoff of meltwater $Q_{GIS}$, and the dynamic outflow of ice $D_{GIS}$,

$$\frac{dM_{GIS}}{dt} = P_{GIS} - Q_{GIS} - D_{GIS},$$

and assumes height $H_{GIS}$, volume $V_{GIS}$, mass $M_{GIS}$, and slope $s_{GIS}$ of the ice sheet to vary proportionally. $P_{GIS}$ is often assumed to exponentially increase with temperature at mean sea-level $T_{GIS}$ by a rate of 5-7% K$^{-1}$ (e.g., Applegate et al., 2012). For a small temperature interval, this can be approximated by linearity,

$$P_{GIS}(t) = c_1 T_{GIS}(t) + c_2.$$  \quad (4)

In Eqs. (4) – (7), $c_j$ are all constants, $j=1,2,...,7$. Similarly, $Q_{GIS}$ depends on the mean temperature at the ice sheet surface $T_{GIS, surface}$

$$Q_{GIS}(t) = c_3 T_{GIS, surface}(t) + c_4.$$  \quad (5)

where the difference between $T_{GIS, surface}$ and $T_{GIS}$ depends linearly on $H_{GIS}$.

$$T_{GIS, surface}(t) = T_{GIS}(t) + c_5 H_{GIS}(t).$$  \quad (6)

The dynamic ice outflow is linearly dependent on the slope (thus, on the height), where the sensitivity is a function of temperature $T_{GIS}$.

$$D_{GIS}(t) = (c_6 T_{GIS} + c_7) H_{GIS}.$$  \quad (7)

SIMPLE (algebra) simplifies Eqs. (3) – (7) to Eqs. (8) and (9)