Appendix S3 Checking the mass balance of soil CO2 for the DETECT and DETECT-SS models.

To mass balance of the DETECT and DETECT-SS models is theoretically guaranteed because equation 1 of the paper is actually the mass balance equation. The mass balance equation is defined as:

\[ \text{IN} + \text{PROD} = \text{OUT} + \text{ACC}, \]

where for our model, IN and OUT are the inputs and outputs of CO2 from the boxes below and above it in the soil profile, PROD is the production of CO2, and ACC is the accumulation of CO2 over time. We can rearrange this mass balance equation to put it in the form of equation 1 from the manuscript:

\[ \text{ACC} = (\text{IN} – \text{OUT}) + \text{PROD} \]

Where ACC is the dc/dt term from equation 1, (IN – OUT) is the d(Dgs*dC/dz)/dz term, and PROD is the S term. Similar comments can be made for the steady-state version of the DETECT model except that the ACC term in the above mass balance equation is equal to zero, i.e. there is no accumulation of soil CO2 over time (or the dC/dt term in equation 1 is set to zero).

As a practical check, we created a Matlab script which computes the total \{Rsoil + change in CO2 storage\} for both the DETECT and DETECT-SS models. Over the course of the year, \{Rsoil + change in CO2 storage\} was 497.1 gC/m^2 for the DETECT model and 497.1 gC/m^2 for the DETECT-SS model, under the control scenario.

Appendix S4 Alternative formulations of the functions that describe how soil CO2 production changes with soil water content

To test the robustness of the DETECT model, we try alternative formulations of the function \( f \) that describe the production of soil CO2 from root and microbial sources for different soil water content (\( \theta \)) values. The formulation used in the paper (equation 4a) is an exponential function that depends on current and past soil water content. An alternative formulation is one where soil CO2 production increases as \( \theta \) increases up to an optimum soil water content (\( \theta_{opt} \)) value. For values of \( \theta \) greater than \( \theta_{opt} \), soil CO2 production decreases. We represented this by a bell shaped curve:

\[
\begin{align*}
    f_R(\theta) &= \frac{0.9}{\sqrt{0.01 \pi}} \exp \left( -\frac{(\theta - \theta_{opt})^2}{0.01} \right) \\
    f_M(\theta) &= \frac{1}{\sqrt{0.0081 \pi}} \exp \left( -\frac{(\theta - \theta_{opt})^2}{0.0081} \right)
\end{align*}
\]

where \( f_R \) and \( f_M \) refers to the function used as part of the calculations for the soil CO2 production from roots (R) and microbial (M) sources, and where \( \theta_{opt} = 0.3 \).
The panel on the left of this figure shows the graphical representation of the equation that models RrBase as a function of θ, where RrBase is the base rate of Rr (production of root CO2) and θ is the soil water content. This equation is one of the equations used to calculate microbial CO2 production, where the other equations use this RrBase value to allow production to vary according the specific temperature and microbial C content of a particular depth. Here we show two options for modelling RrBase as a function of θ: (1) the exponential type function used in this analysis (see equation 4a of the paper); (2) an alternative to equation 4a, where RrBase increases as θ increases, but only up until a certain point given by θ_{opt}; for values of θ higher than θ_{opt}, RrBase decreases. For the Wyoming field site that we use to make measurements, θ never got high enough that resulted in ecosystem respiration CO2 rates to decrease. Hence, the graphical representation of this alternative RrBase function shows RrBase increasing for values of θ up to θ_{opt}. The description of the panel on the right of this figure is exactly the same as the left panel except that the y-axis shows RmBase (microbial production of CO2) instead of RrBase.
**Figure S8** The description for this figure is exactly the same as that of figure 2, except that the function used to simulate the production of soil CO2 from root and microbial sources is a bell shaped curve rather than an exponential function as used for the results of this analysis (see equation 4a of paper).

**Figure S9** Graphical representation of total production of CO2 from root and microbial sources (S, mg CO2 m-3 hr-1) as modelled by DETECT versus soil water content (θ, m3 m-3) at different soil depths.

Red=above 12°C
Black=between 4°C and 12°C.
Blue = below 4°C