Interactive comment on “State-space representation of a bucket-type rainfall-runoff model: a case study with State-Space GR4 (version 1.0)” by Léonard Santos et al.

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Answer to the review comments of Reviewer #2

We would like to thank the reviewer for his analysis and suggestions on the article which are, in our opinion, complementary to those made by the other reviewer, Dr Barry Croke.
1. In developing the state-space representation of their model, the authors introduce two changes. First, a different routing model is used (Nash cascade vs unit hydrograph). And second, the model is solved with a different numerical technique (implicit Euler with adaptive time stepping vs operator splitting approach with fixed time step). It would be preferable to introduce these two changes separately rather than together, so as to separate the effects of these two changes.

We thank the reviewer for this remark. We made additional tests to investigate this, we replaced the unit hydrograph by a Nash Cascade but integrated it using operator-splitting. This replacement does not change the performances and, when using hourly time step, the parameters values are similar for the two operator-splitted models. However, at the daily time-step, the $x_4$ parameter values of the Nash Cascade are higher than the ones of the unit hydrograph at daily time-step. It tends to prove that the insensitivity of the $x_4$ parameter values to temporal resolution (highlighted in the section 4.2 of the article) is not due to the replacement of the unit hydrograph by a Nash Cascade. These remarks will be taken into consideration in the revised version of the article.

2. Run times are longer with the new model compared to the original implementation due to the use of implicit Euler with adaptive time stepping. Have you considered using a single-step implicit Euler integration? This may be faster without losing the benefits of the new implementation.

Even if it is not mentioned in the article, we tested the Implicit Euler method with increasing sub-steps number from 1 to 100. The number of sub-steps seems to have an influence, particularly in high flow periods. To illustrate the impact of using a single-step, we compare (see Fig. 1 below) the boxplots of performance for an adaptive sub-step number implicit integration and a single-step Euler implicit one. The GR4 parameters used for this comparison are the ones obtained by the GR4 calibration on KGE’ calculated on square rooted streamflows that is presented in the article. The boxplots show a decrease of performances. This tends
to show that, even if single-step implicit Euler does not face instabilities when solving the equations, it can increase errors. This can be linked to the second comment made by Dr Barry Croke.

Another important disadvantage of not using sub-stepping is that it does not solve the parameter time instability issue. To prove it, we calibrated the continuous state-space model at the daily and hourly time-steps using a single-step implicit Euler method without sub-steps. In the Fig. 2, we plotted the resulting parameter scatter plots comparison (the same way that is used in Fig. 9 in the article). Unlike with adaptive time-step, the $x_4$ parameters show differences between daily and hourly time-steps. This result tends to confirm Barry Croke’s third remark in which he argues that increasing the sub-step number can help to approach a continuous time model.

3. Please provide some details/examples of the actual time steps and number of non-linear iterations in your model, for example for one specific basin.

If we take the example of the River Azergue at Chatillon catchment (the example catchment chosen in the article) on the validation period, the mean number of used sub-steps is 2 for hourly simulation and 22 for daily simulation. Figure 3 graph shows the cumulative appearance frequency of the different numbers of sub-steps. It is, in majority, one or two sub-steps for the hourly time-step but it is more variable in the case of daily simulation.

At the hourly time-step, we found out that the number of sub-steps increases when the rainfall amount increases. In the case of daily time-step it is not clear, possibly because the number of sub-steps is correlated with a combination of rainfall and the stores levels. We can notice that the average daily sub-step value (which approximately corresponds to 1 hour) is higher than the average hourly sub-step value (approximately 0.5 hour). This is probably due to the fact that the maximum sub-step value for the hourly simulation is limited to 1 hour. We will make a comment on this observation in the article.
4. Questions about the state-space formulation, Eq. 1:

- **Why not include water balance of the interception store as an additional differential equation?**
  In the current version of GR4, the interception is not calculated with a store but it is a simple difference between rainfall and potential evapotranspiration. Only one input (which is the difference between the larger and the smaller of the two) is considered in the model, which is a difference with other bucket-type rainfall-runoff models. We decided not to change this input calculation in order not to include more differences between the two models. This answer will be added to the article.

- **Simulated discharge Q in Eq. 2 is defined as an instantaneous flow I assume? Observed discharge is however an integrated quantity (total over an hour or a day). Wouldn’t it be better to define simulated Q also as an integrated quantity? You could in fact add Eq. 2 to the ODE system in Eq. 1: dQ/dt = Qr + Qd. Note that you then would have to reset Q = 0 at the start of each forcing time interval.**
  You are right, the discharge presented by Eq. 2 is an instantaneous flux. The simulated flow is the integration of this equation over the time-step. In the code, the integration is calculated using the adaptive sub-step implicit approximation. It can be seen in the “GR4_STSP.f” script (in the internal fluxes calculation part) of provided model sources. To clarify this point in the manuscript, we will add at line 7 page 6 (before the Eq. 2) that the output equation is to calculate the instantaneous output flow \( q(t) \). After this equation (where we will replace \( Q \) by \( q(t) \)) we will add that the simulated output \( Q \) is the integration of \( q(t) \) over the time-step.

- **It would be good to explicitly point out in table 1 that the instantaneous flux equations are the same for the two models.**
  We agree and will point this out.
5. Section 2: The discrete form is contrasted with the state-space form of the model. Note that a state-space representation can be either discrete or continuous, so it may be better to explicitly call it continuous state-space formulation.

You are right, we will try to be more precise by writing, at least in section 2, that the state-space representation is continuous. However, because of the first point of the review, we will also mention a discrete (or operator-split) form of the state-space formulation.

6. Section 4.3: this section describes the relation between the unit hydrograph approach for routing in the old model and the Nash cascade representation in the new model; in my view this section really fits better in the methods section, for example following the text at the bottom of page 6. My suggestion is to move it there.

To be more comprehensive, we will try to add this in the section 2. Because of the first comment we will also mention the operator-split state-space model with the Nash Cascade and the continuous state-space formulation of this model in section 2.

7. Abstract: what do you mean by “resolution”?

By “resolution” we meant “solution”. It will be fixed.

8. These typo mistakes will be corrected.

Léonard Santos, on behalf of co-authors

Fig. 1. Performances comparisons between adaptive sub-step and single-step Implicit Euler methods
Fig. 2. Scatter plots representing the four parameters of the discrete (daily and hourly) GR4 with Nash Cascade models obtained by calibration with $\text{KGE'}(\sqrt{Q})$ as the objective function. The solid line represents $r$. 

The text continues...
Fig. 3. Cumulative appearance frequency of the number of sub-steps obtained with adaptive Implicit Euler resolution of the continue GR4 state-space model at the daily and hourly time-steps.