



# Simulation of the Performance and Scalability of MPI Communications of Atmospheric Models running on Exascale Supercomputers

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## Abstract

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In this study, we identify the key MPI operations required in atmospheric modelling; then, we use a skeleton program and a simulation framework (based on SST/macro simulation package) to simulate these MPI operations (transposition, halo exchange, and allreduce), with the perspective of future exascale machines in mind. The experimental results show that the choice of the collective algorithm has a great impact on the performance of communications, in particular we find that the generalized ring-k algorithm for the alltoallv operation and the generalized recursive-k algorithm for the allreduce operation perform the best. In addition, we observe that the impacts of interconnect topologies and routing algorithms on the performance and scalability of transpositions, halo exchange, and allreduce operations are significant. However, ~~that~~ the routing algorithm has a negligible impact on the performance of allreduce operations because of its small message size. It is impossible to infinitely grow bandwidth and reduce latency due to hardware limitations. Thus, congestion may occur and limit the continuous improvement of the performance of communications. The experiments show that the performance of communications can be improved when congestion is mitigated by a proper configuration of the topology and routing algorithm, which uniformly distribute the congestion over

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18 the interconnect network to avoid the hotspots and bottlenecks caused by congestion. It  
19 is generally believed that the transpositions seriously limit the scalability of the spectral  
20 models. The experiments show that ~~although~~ the communication time of the transposi-  
21 tion is larger than those of the wide halo exchange for the Semi-Lagrangian method and  
22 the allreduce in the GCR iterative solver for the Semi-Implicit method below  $2 \times 10^5$  MPI  
23 processes. ~~The~~ transposition ~~whose~~ communication time decreases quickly ~~as the~~ number  
24 of MPI processes ~~is~~ ~~and~~ ~~es~~ demonstrates strong scalability in the case of very large grids  
25 and moderate latencies. ~~The~~ halo exchange whose communication time decreases more  
26 slowly than that of transposition ~~as the~~ number of MPI processes ~~increases~~ reveals its  
27 weak scalability. ~~In~~ contrast, the allreduce whose communication time increases ~~as the~~  
28 number of MPI processes ~~increases~~ does not scale well. From this point of view, the scal-  
29 ability of ~~the~~ spectral models could still be acceptable. ~~Therefore~~ it seems to be premature  
30 to conclude that the scalability of the grid-point models is better than that of spectral  
31 models at exascale, unless innovative methods are exploited to mitigate the problem of  
32 the scalability presented in the grid-point models.

33 **Keyword:** performance, scalability, MPI, communication, transposition, halo exchange,  
34 all reduce, topology, routing, bandwidth, latency

## 35 1 Introduction

36 Current high performance computing (HPC) systems have thousands of nodes and millions  
37 of cores. According to the 49th TOP500 list ([www.top500.org](http://www.top500.org)) published on June 20, 2017,  
38 the fastest machine (Sunway TaihuLight) had over than 10 million cores with a peak perfor-  
39 mance approximately 125 PFlops (1 PFlops= $10^{15}$  floating-point operations per second), and  
40 the second HPC (Tianhe-2) is made up of 16,000 nodes and has more than 3 million cores with  
41 a peak performance approximately 55 PFlops. It is estimated that in the near future, HPC  
42 systems will dramatically scale up in size. Next decade, it is envisaged that exascale HPC  
43 system with millions of nodes and thousands of cores per node, whose peak performance ap-  
44 proaches to or is beyond 1 EFlops (1 EFlops= $10^3$  PFlops), will become available (Engelmann,  
45 2014; Lagadapati et al., 2016). Exascale HPC poses several challenges in terms of power con-  
46 sumption, performance, scalability, programmability, and resilience. The interconnect net-  
47 work of exascale HPC system becomes larger and more complex, and its performance which  
48 largely determines the overall performance of the HPC system is crucial to the performance



49 of distributed applications. Designing energy-efficient cost-scalable interconnect networks and  
50 communication-efficient scalable distributed applications is an important component of HPC  
51 hardware/software co-design to address these challenges. Thus, evaluating and predicting the  
52 communication behaviour of distributed applications is obligatory; it is only feasible by mod-  
53 elling the communications and the underlying interconnect network, especially for the future  
54 supercomputer.

55 Investigating the performance of distributed applications on future architectures and the  
56 impact of different architectures on the performance by simulation is a hardware/software  
57 co-design approach for paving the way to exascale HPCs. Analytical interconnect network sim-  
58 ulation based on an analytical conceptual model is fast and scalable, but comes at the cost of  
59 accuracy owing to its unrealistic simplification (Hoeffler et al., 2010). Discrete event simulation  
60 (DES) is often used to simulate the interconnect network, and it provides high fidelity since the  
61 communication is simulated in more detailed level (e.g., flit, packet, or flow levels) to take into  
62 account congestion (Janssen et al., 2010; Böhm and Engelmann, 2011; Dechev and Ahn, 2013;  
63 Acun et al., 2015; Jain et al., 2016; Wolfe et al., 2016; Degomme et al., 2017; Mubarak et al.,  
64 2017). Sequential DES lacks scalability owing to its large memory footprints and long exe-  
65 cution time (Degomme et al., 2017). Parallel DES (PDES) is scalable since it can reduce the  
66 memory required per node, but its parallel efficiency is not very good because of frequent  
67 global synchronization of conservative PDES (Janssen et al., 2010) or high rollback overhead of  
68 optimistic PDES (Acun et al., 2015; Jain et al., 2016; Wolfe et al., 2016). Generally, the simu-  
69 lation of distributed applications can be divided into two complementary categories: offline and  
70 online simulations. Offline simulation replays the communication traces from the application  
71 running on a current HPC system. It is sufficient to understand the performance and dis-  
72 cover the bottleneck of full distributed applications on the available HPC system (Tikir et al.,  
73 2009; Noeth et al., 2009; Núñez et al., 2010; Dechev and Ahn, 2013; Casanova et al., 2015;  
74 Acun et al., 2015; Jain et al., 2016; Lagadapati et al., 2016); however, is not very scalable be-  
75 cause of the huge traces for numerous processes and limited extrapolation to future architecture  
76 (Hoeffler et al., 2010; Núñez et al., 2010). Online simulation has full scalability to future system  
77 by running the skeleton program on the top of simulators (Zheng et al., 2004; Janssen et al.,  
78 2010; Engelmann, 2014; Degomme et al., 2017), but has the challenge of developing a skele-  
79 ton program from a complex distributed application. Most simulations in the aforementioned  
80 literatures have demonstrated the scalability of simulators. The simulator xSim (Engelmann,



81 2014) simulated a very simple MPI program, which only calls MPI\_Init and MPI\_Finalize with-  
82 out any communication and computation, up to  $2^{27}$  processes. For collective MPI operations,  
83 Hoefler et al. (2010) obtained an MPI\_Allreduce simulation of 8 million processes without con-  
84 sideration of congestion using LogGOPSim, Engelmann (2014) achieved an MPI\_Reduce simula-  
85 tion of  $2^{24}$  processes, and Degomme et al. (2017) demonstrated an MPI\_Allreduce simulation of  
86 65536 processes using SimGrid. For simulations at application level, Jain et al. (2016) used the  
87 TraceR simulator based on CODES and ROSS to replay  $4.6 \times 10^4$  process traces of several com-  
88 munication patterns that are used in a wide range of applications. In addition, Mubarak et al.  
89 (2017) presented a  $1.1 \times 10^5$  process simulations of two multigrid applications. However, to  
90 the best of our knowledge, there is no exascale simulation of complex communication patterns  
91 such as the MPI transposition (Multiple simultaneous MPI\_Alltoallv) for the spectral method  
92 and the wide halo exchange (the width of a halo may be greater than the subdomain size of its  
93 direct neighbours) for the Semi-Lagrangian method used in atmospheric models.

94 With the rapid development of increasingly powerful supercomputers in recent years, numer-  
95 ical weather prediction (NWP) models have increasingly sophisticated physical and dynamical  
96 processes, and their resolution is getting higher and higher. Nowadays, the horizontal resolution  
97 of global NWP model is in the order of 10 kilometres. Many operational global spectral NWP  
98 models such as IFS at ECMWF, ARPEGE at METEO-FRANCE, and GFS at NCEP are based  
99 on the spherical harmonics transform method that includes Fourier transforms in the zonal di-  
100 rection and Legendre transforms in the meridional direction (Ehrendorfer, 2012). Moreover,  
101 some regional spectral models such as AROME at METEO-FRANCE (Seity et al., 2011) and  
102 RSM at NCEP (Juang et al., 1997) use the Bi-Fourier transform method. The Fourier trans-  
103 forms can be computed efficiently by fast Fourier transform (FFT) (Temperton, 1983). Even  
104 with the introduction of fast Legendre transform (FLT) to reduce the growing computational  
105 cost of increasing resolution of global spectral models (Wedi et al., 2013), it is believed that  
106 global spectral method is prohibitively expensive for very high resolution (Wedi, 2014).

107 A global (regional) spectral model performs FFT and FLT (FFT) in the zonal direction and  
108 the meridional direction, respectively. Because both transforms require all values in the corre-  
109 sponding directions, the parallelization of spectral method in global (regional) model is usually  
110 conducted to exploit the horizontal domain decomposition only in the zonal direction and merid-  
111 ional directions for FFT and FLT (FFT), respectively (Barros et al., 1995; Kanamitsu et al.,  
112 2005). Owing to the horizontal domain decomposition in a single horizontal direction for the

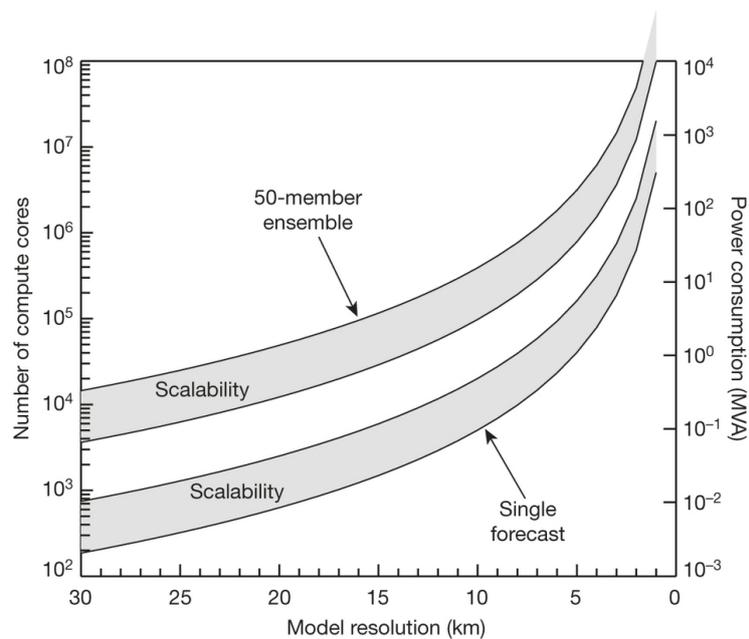


Fig. 1: CPU and power requirements as a function of NWP model resolution, adapted from Bauer et al. (2015). The left and right y axes are the number of cores and the power (in megavolt amps), respectively, required for a single 10-day model forecast (the lower shaded area including its bounds) and a 50-member ensemble forecast (the upper shaded area including its bounds) as a function of model resolution, respectively, based on current model code and compute technology. The lower and upper bounds of each shaded area indicate perfect scaling and inefficient scaling, respectively.

113 parallelization of spectral transforms, there is a transposition between the spectral transforms  
114 in the zonal direction and meridional directions. MPI (Message Passing Interface) transposition  
115 is an all-to-all personalized communication which can cause significant congestion over inter-  
116 connect network when the number of MPI tasks and the amount of exchanged data are large,  
117 and results in severe communication delay. Bauer et al. (2015) estimated that a global NWP  
118 model with a two-kilometre horizontal resolution requires one million compute cores for a single  
119 10-day forecast (Fig. 1). With one million compute cores, the performance and scalability of  
120 the MPI transposition become of paramount importance for a high resolution global spectral  
121 model. Thus, evaluating and predicting the performance and scalability of MPI transposition  
122 at exascale is one of the foremost subjects of this study.

123 The Semi-Lagrangian (SL) method is a highly efficient technique for the transport of mo-  
124 mentum, heat and mass in the NWP model because of its unconditional stability which permits  
125 a long time step (Staniforth and Côté, 1991; Hortal, 2002). However, it is known that the MPI



126 exchange of wide halo required for the interpolation at the departure point of high wind-speed  
127 particles near the boundary of the subdomain causes significant communication overhead as  
128 resolution increases towards kilometres scale and the HPC systems move towards exascale.  
129 This communication overhead could reduce the efficiency of the SL method; thus, modelling  
130 the performance and scalability of wide halo exchange at exascale is essential and is another  
131 subject of this study.

132 With consideration of the efficiency of the Legendre transform and the scalability of MPI  
133 transposition that may arise in the global spectral model on exascale HPC systems, a cou-  
134 ple of global grid-point models have recently been developed (Lin, 2004; Satoh et al., 2008;  
135 Qaddouri and Lee, 2011; Skamarock et al., 2012; Dubos et al., 2015; Zangl et al., 2015; ~~Kuhle~~ *et al.*  
136 2017). Since spherical harmonics are eigenfunctions of the Helmholtz operator, the Semi-  
137 Implicit (SI) method is usually adopted in order to implicitly handle the fast waves in the  
138 global spectral model to allow stable integration with a large time step (Robert et al., 1972;  
139 Hoskins and Simmons, 1975). However, for a grid-point model, the three-dimensional Helmholtz  
140 equation is usually solved using Krylov subspace methods such as the generalized conjugate  
141 residual (GCR) method (Eisenstat et al., 1983), and a global synchronization for the inner  
142 product in Krylov subspace methods may become the bottleneck at exascale (Li et al., 2013;  
143 Sanan et al., 2016). As it is not clear whether the three-dimensional Helmholtz equation can  
144 be solved efficiently in a scalable manner, most of the aforementioned models use a horizontally  
145 explicit vertically implicit (HEVI) scheme. The HEVI scheme typically requires some damping  
146 for numerical stability (Satoh et al., 2008; Skamarock et al., 2012; Zangl et al., 2015), and its  
147 time step is smaller than that of the SI method (Sandbach et al., 2015). Therefore, it is de-  
148 sirable to know whether the SI method is viable or even advantageous for very high resolution  
149 grid-point models running on exascale HPC systems. Thus, it is valuable to explore the per-  
150 formance and scalability of global synchronization in solving the three-dimensional Helmholtz  
151 equation using Krylov subspace methods; this forms the third subject of this study.

152 In this paper, we present the application of SST/macro 7.1, a coarse-grained parallel discrete  
153 event simulator, to investigate the communication performance and scalability of atmospheric  
154 models for future exascale supercomputers. The remainder of the paper is organized as fol-  
155 lows. Section 2 introduces the simulation environment, the SST/macro simulator, and our  
156 optimizations for reducing the memory footprint and accelerating the simulations. Section 3  
157 reviews three key MPI operations used in the atmospheric models. Section 4 presents and

Probably better to cite instead Smolarkiewicz et al., 2016 : A finite-volume module for simulating global all-scale atmospheric flows, *J. Comput. Phys.*, 314, pp. 287–304, doi: 10.1016/j.jcp.2016.03.015



158 analyses the experimental results of the modelling communication of the atmospheric model  
159 using SST/macro. Finally, we summarize the conclusions and discuss future work in section 5.

## 160 **2 Simulation Environment**

### 161 *2.1 Parallel Discrete Event Simulation*

162 Modelling application performance on exascale HPC systems with millions of nodes and a  
163 complex interconnect network requires that the simulation can be decomposed into small tasks  
164 that efficiently run in parallel to overcome the problem of large memory footprint and long  
165 simulation time. PDES is such an approach for exascale simulation. Each worker in PDES is  
166 a logical process (LP) that models a specific component such as a node, a switch, or an MPI  
167 process of the simulated MPI application. These LPs are mapped to the physical processing  
168 elements (PEs) that actually run the simulator. An event is an action such as sending an MPI  
169 message or executing a computation between consecutive communications. Each event has its  
170 start and stop times, so the events must be processed without violating their time ordering.  
171 To model the performance of an application, PDES captures time duration and advances the  
172 virtual time of the application by sending timestamped events between LPs.

173 PDES usually adopts conservative or optimistic parallelized strategies. The conservative  
174 approach maintains the time ordering of events by synchronization to guarantee that no early  
175 events arrive after the current event. Frequent synchronization is time-consuming so the effi-  
176 ciency of the conservative approach is highly dependent on the look ahead time; a larger look  
177 ahead time (that means less synchronization) allows a much greater parallelism. The optimistic  
178 approach allows LPs to run events at the risk of time-ordering violations. Events must be rolled  
179 back when time-ordering violations occurs. Rollback not only induces significant overhead, but  
180 also requires extra storage for the event list. Rollback presents special challenges for online  
181 simulation, so SST/macro adopts a conservative approach (Wike and Kenny, 2014).

### 182 *2.2 SST/macro Simulator*

183 Considering that the offline trace-driven simulation does not provide an easy way for extrap-  
184 olating to future architectures, the online simulator SST/macro is selected here to model the  
185 communications of the atmospheric models for future exascale HPC systems. SST/macro is a



186 coarse-grained parallel discrete event simulator which provides the best cost/accuracy trade-off  
187 simulation for large-scale distributed applications (Janssen et al., 2010). SST/macro is driven  
188 by either a trace file or a skeleton application. A skeleton application can be constructed from  
189 scratch, or from an existing application manually or automatically by source-to-source trans-  
190 lation tools. SST/macro intercepts the communications issued from the skeleton program to  
191 estimate their time rather than actually execute it by linking the skeleton application to the  
192 SST/macro library instead of the real MPI library. Since the purpose of this study is to investi-  
193 gate the performance and scalability of communications in an atmospheric model, we construct  
194 the communication-only skeleton program from scratch by identifying the key MPI operations  
195 taking place in the atmospheric models.

196 Congestion is a significant factor that affects the performance and scalability of MPI appli-  
197 cations running on exascale HPC systems. SST/macro has three network models: the analytical  
198 model transfers the whole message over the network from point-to-point without packetizing  
199 and estimates the time delay  $\Delta t$  predominantly based on the logP approximation

$$200 \quad \Delta t = \alpha + \beta N, \quad (1)$$

201 where  $\alpha$  is the communication latency,  $\beta$  is the inverse bandwidth in second per byte, and  $N$  is  
202 the message size in bytes; the packet-level model PISCES (Packet-flow Interconnect Simulation  
203 for Congestion at Extreme Scale) divides the message into packets and transfers the packets  
204 individually; the flow-level model will be deprecated in the future. Compared to the SimGrid  
205 simulator, the packet-level model of SST/macro produces almost identical results (figure omit-  
206 ted). Acun et al. (2015) also found that the SST/macro online simulation is very similar to  
207 the TraceR simulation. Thus, we adopt the PISCES model with a cut-through mechanism  
208 (SNL, 2017) to better account for the congestion. SST/macro provides three abstract machine  
209 models for nodes: the AMM1 model is the simplest one which grants exclusive access to the  
210 memory, the AMM2 model allows multiple CPUs or NICs (network interface controller) to  
211 share the memory bandwidth by defining the maximum memory bandwidth allocated for each  
212 component, the AMM3 model goes one further step to distinguish between the network link  
213 bandwidth and the switch bandwidth. In this paper, the AMM1 model with one single-core  
214 CPU per node is adopted since simulation of communications is the primary goal.

215 SST/macro provides several topologies of the interconnect network. In this study, three



216 types of topologies (Fig. 2) commonly used in current supercomputers, and their configurations  
217 are investigated. Torus topology has been used in many supercomputers (Ajima et al., 2009).  
218 In the torus network, messages hop along each dimension using the shortest path routing  
219 from the source to the destination (Fig. 2a), and its bisection bandwidth typically increases with  
220 increasing dimension size of the torus topology. The practical implementation of the fattree  
221 topology is an upside-down tree that typically employs all uniform commodity switches to  
222 provide high bandwidth at higher levels by grouping corresponding switches of the same colour  
223 (Fig. 2b). Fattree topology is widely adopted by many supercomputers for its scalability and  
224 high path diversity (Leiserson, 1985); it usually uses a D-mod-k routing algorithm (Zahavi et al.,  
225 2010) for desirable performance. A dragonfly network is a multi-level dense structure of which  
226 the high-radix routers are connected in a dense even all-to-all manner at each level (Kim et al.,  
227 2008). As shown in Fig. 2c, a typical dragonfly network consists of two levels: the routers at  
228 the first level are divided into groups and routers in each group form a two-dimension mesh  
229 of which each dimension is an all-to-all connected network; at the second level, the groups as  
230 virtual routers are connected in an all-to-all manner (Alverson et al., 2015). There are three  
231 available routing algorithms for dragonfly topology in SST/macro:

232 **minimal** transfers messages by the shortest path from the source to the destination. For  
233 example, messages travel from the blue router in group 0 to the red router in group 2 via  
234 the bottom-right corner in group 0 and the bottom-left corner in group 2 (Fig. 2c).

235 **valiant** randomly picks an intermediate router, and then uses a minimal routing algorithm to  
236 transfer messages from the source to the intermediate router and from the intermediate  
237 router to the destination. For example, the arrow path from the blue router in group 0  
238 to the red router in group 2 goes via the intermediate yellow node in group 1 in Fig. 2c.

239 **ugal** checks the congestion, and either switches to the valiant routing algorithm if congestion  
240 is too heavy, or otherwise uses the minimal routing algorithm.

241 Table 1 summarizes the network topology configurations used in this paper. Torus-M (torus-  
242 L) configuration is a 3D torus of 25x25x25 (75x25x25) size. Fattree-M (fattree-L) configuration  
243 has 4 layers: the last layer consists of nodes while the other layers consist of switches with 25 (33)  
244 descendant ports per switch. We tested four configurations of dragonfly topology. Dragonfly-  
245 MM configuration has a medium size of a group of a 25x25 mesh with 25 nodes per switch

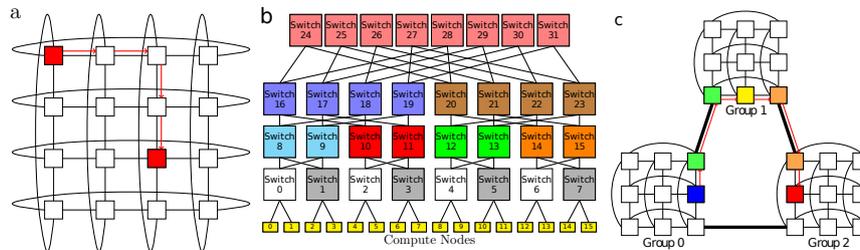


Fig. 2: Topology illustration: a, b, and c are the torus, fattree, and dragonfly topologies, respectively. Adapted from SNL (2017)

Table 1: Summary of the network topologies: the geometry of a torus topology specifies the size of each dimension; the first and second number in the geometry of a fattree topology are the number of layers and descendant ports per switch, respectively; the first two numbers and the last number in the geometry of a dragonfly topology indicate the group mesh size and the number of groups, respectively.

name	geometry	switches	nodes per switch	nodes	radix
torus-M	25,25,25	15625	25	390625	31
fattree-M	4,25	46875	25	390625	50
dragonfly-MM	25,25,25	15625	25	390625	97
dragonfly-SL	25,25,125	15625	5	390625	177
dragonfly-LS	125,125,5	15625	5	390625	257
torus-L	75,25,25	46875	25	1171875	31
fattree-L	4,33	107811	33	1185921	66
dragonfly-ML	25,25,75	46875	25	1171875	147

What is meant with radix?

246 and medium number (=25) of groups. Dragonfly-SL configuration has a small size of a group  
 247 of a 25x25 mesh with 5 nodes per switch and large number (=125) of groups. Dragonfly-LS  
 248 configuration has a large size of a group of a 125x125 mesh with 5 nodes per switch and small  
 249 number (=5) of groups. Dragonfly-ML configuration has a medium size of a group of a 25x25  
 250 mesh with 25 nodes per switch and large number (=75) of groups. The fattree configuration  
 251 has a significant larger number of switches than other topologies for the same number of nodes,  
 252 which indicates that fattree is not cost- or energy-efficient. All the configurations with 390625  
 253 nodes are used for simulating transposition for the spectral transform method. Torus-L, fattree-  
 254 L, and dragonfly-ML with more than one million nodes are used for the cases of halo exchange  
 255 and allreduce communication since we cannot finish the simulation of transposition for the  
 256 spectral transform method (multiple simultaneous all-to-all personalized communications) on  
 257 such large configuration within 24 hours (see Section 3 for three key MPI communications in  
 258 the atmospheric model).



### 259 *2.3 Reduce the Memory Footprint and Accelerate the Simulation*

260 Although SST/macro is a parallel discrete event simulator that can reduce the memory foot-  
261 print per node, its parallel efficiency degrades if more cores are used. Even with an MPI  
262 transposition of  $10^5$  processes, this all-to-all personalized communication has almost  $10^{10}$  dis-  
263 crete events, which consumes a considerable amount of memory and takes a very long time  
264 for simulation. Furthermore, almost every MPI program has a setup step to allocate memory  
265 for storing the setup information such as the parameters and the domain decomposition of all  
266 processes what each process must know in order to properly communicate with other processes,  
267 therefore, it needs to broadcast the parameters to and synchronize with all processes before  
268 actual communications and computation. Even if the setup information for a single process  
269 needs only  $10^2$  bytes memory, a simulation of  $10^5$  processes MPI transposition will need one  
270 terabyte ( $10^2 \times 10^5 \times 10^5 = 10^{12}$  bytes) memory, which is not easily available on current com-  
271 puters if the simulator runs on a single node. In addition, the MPI operations in the setup step  
272 not only are time-consuming, but also affect subsequent communications. A common way to  
273 eliminate this effect is to iterate many times to obtain a robust estimation of communication  
274 time; however, one iteration is already very time-consuming for simulation. To circumvent the  
275 issue of setup steps, we use an external auxiliary program to create a shared memory segment  
276 on each node running SST/macro and initialize this memory with the setup information of all  
277 the simulated MPI processes. Then, we modified SST/macro to create a global variable and  
278 attach the shared memory to this global variable; this method not only reduces the memory  
279 footprint and eliminates the side effect of communications in the setup step, but also avoids  
280 the problem of filling up the memory address space if each simulated process attaches to the  
281 shared memory.

282 Large-scale application needs a large amount of memory for computation; and in some  
283 cases, such as spectral model, the whole memory for computation is exchanged between all the  
284 processes. Even when computation is not considered, a large amount of memory for the message  
285 buffers is usually required for MPI communications. Fortunately, the simulator only needs  
286 message size, the source/destination, and the message tag to model the communication; thus,  
287 it is not necessary to allocate actual memory. Since SST/macro can operate with null buffers,  
288 the message buffer is set to null in the skeleton application, which significantly reduces the size  
289 of memory required by the simulation of communication of the high resolution atmospheric



290 model.

## 291 **3 Key MPI Operations in Atmospheric Models**

### 292 **3.1 Transposition for the Spectral Transform Method**

293 A global spectral model generally uses spherical harmonics transform on the horizontal with  
294 triangular truncation. The backward spherical harmonics transform is

$$295 \quad f(\theta, \lambda) = \sum_{m=-M}^M \left( e^{im\lambda} \sum_{n=|m|}^M f_n^m P_n^m(\cos \theta) \right), \quad (2)$$

296 where  $\theta$  and  $\lambda$  are the colatitude and longitude,  $f_n^m$  is the spectral coefficients of the field  $f$ , and  
297  $P_n^m$  is the associated Legendre polynomials of degree  $m$  and order  $n$ . Moreover, the forward  
298 spherical harmonics transform is

$$299 \quad f_n^m = \frac{1}{2} \int_{-1}^1 \left( P_n^m(\cos \theta) \frac{1}{2\pi} \int_0^{2\pi} f(\theta, \lambda) e^{-im\lambda} d\lambda \right) d \cos \theta, \quad (3)$$

300 In (2), the backward Legendre transform of each  $m$  can be computed independently; then,  
301 the same is for the backward Fourier transform of each  $\theta$ . Similar to (3), the forward Fourier  
302 transform of each  $\theta$  can be computed independently; then, the same is for the forward Legendre  
303 transform of each  $m$ . This leads to a natural way to parallelize the spectral transforms. If  
304 we start with the grid-point space (Fig. 3a), which is decomposed by  $cx/cy$  cores in the  $x/y$   
305 direction,  $cy$  simultaneous  $xz$  slab MPI transpositions lead to the partition (Fig. 3b) with  $cy/cx$   
306 cores in the  $y/z$  direction, and a spectral transform such as a forward FFT can be performed  
307 in parallel since data w.r.t.  $\lambda$  are local to each core. Then,  $cx$  simultaneous  $xy$  slab MPI  
308 transpositions lead to the partition (Fig. 3c) with  $cy/cx$  in the  $x/z$  direction, and a spectral  
309 transform such as a forward FLT can be computed in parallel because data w.r.t.  $\theta$  are now  
310 local to each core. Finally,  $cy$  simultaneous  $yz$  slab MPI transpositions lead to the spectral space  
311 (Fig. 3d) with  $cy/cx$  cores in the  $x/y$  direction, where the Semi-Implicit scheme can be easily  
312 computed because spectral coefficients belonging to the same column are now local to the same  
313 core. The backward transform is similar. It is of paramount importance that the partition of  
314 the four stages described in Fig. 3 must be consistent so that multiple slab MPI transpositions  
315 can be conducted simultaneously, which significantly reduces the communication time of MPI

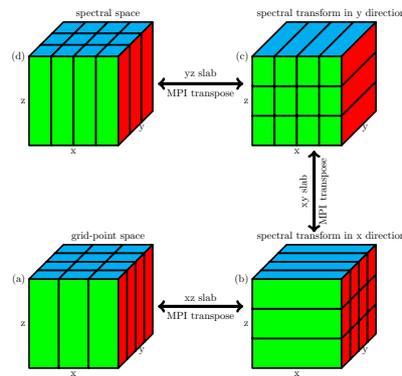


Fig. 3: Parallel scheme of regional spectral model: (a) 2D decomposition of 3D grid field with  $cx/cy$  cores in the  $x/y$  direction, (b) 2D decomposition of 3D grid field with  $cy/cx$  cores in the  $y/z$  direction, (c) 2D decomposition of 3D grid field with  $cy/cx$  cores in the  $x/z$  direction, and (d) 2D decomposition of 3D grid field with  $cy/cx$  cores in the  $x/y$  direction. Transposition between (a) and (b) can be conducted by  $cy$  independent  $xz$  slab MPI transpositions, transposition between (b) and (c) can be conducted by  $cx$  independent  $xy$  slab MPI transpositions, and transposition between (c) and (d) can be conducted by  $cy$  independent  $yz$  slab MPI transpositions.

316 transpositions from one stage to another. It is worth noting that the number of grid points in  
 317 one direction is not always a multiple of the number of cores in the corresponding direction;  
 318 thus, the partition shown in Fig. 3 can use as many as possible computed cores without any  
 319 limit on  $cx$  or  $cy$  provided  $cx \times cy = ncpu$ , and  $cx$  or  $cy$  is not greater than the number of grid  
 320 points in the corresponding direction. It is generally believed that the MPI transpositions from  
 321 one stage to another poses a great challenge to the scalability of spectral models because each  
 322 slab MPI transposition is an all-to-all personalized communications which is the most complex  
 323 and time-consuming all-to-all communication.

324 There are different algorithms for all-to-all personalized communication. Table 2 lists the  
 325 three algorithms for all-to-all personalized communication, whose performance and scalability  
 326 are investigated in this study. Algorithm ring-k is our proposal algorithm for all-to-all per-  
 327 sonalized communication which is a generalized ring alltoallv algorithm. In algorithm ring-k,  
 328 each process communicates with  $2k$  processes to reduce the stages of communications and make  
 329 efficient use of the available bandwidth, and thus reduces the total communication time.



Table 2: Three algorithms for all-to-all personalized communication.

name	description	stages
<b>burst</b>	Each process communicates with all other processes simultaneously by posting all non-block send and receive operations simultaneously. The burst messages cause significant congestion on the network. This algorithm is equivalent to the algorithm ring-k when k=n-1.	1
<b>bruck</b>	This algorithm is better for small message and a large latency since it has only $\lceil \log_2(n) \rceil$ stages of communications (Thakur et al., 2005). For $k^{th}$ stage, each process sends the messages whose destination process id has one at the $k^{th}$ bit (begin at Least Significant Bit) to process $i + 2^k$ .	$\lceil \log_2(n) \rceil$
<b>ring-k</b>	In the first stage, process $i$ sends to $i + 1, \dots, i + k$ and receive from $i - 1, \dots, i - k$ in a ring way (black arrows in Fig. 4a); in the second stage, process $i$ sends to $i + 1 + k, \dots, i + 2k$ and receive from $i - 1 - k, \dots, i - 2k$ in a ring way (blue arrows in Fig. 4a); this continues until all partners have been communicated with. This algorithm is a generalization of the ring algorithm and efficiently uses the available bandwidth by proper selection of radix $k$ .	$\lceil \frac{n-1}{k} \rceil$

### 3.2 Halo Exchange for Semi-Lagrangian Method

The SL method solves the transport equation:

$$\frac{D\phi}{Dt} = \frac{\partial\phi}{\partial t} + u\frac{\partial\phi}{\partial x} + v\frac{\partial\phi}{\partial y} + w\frac{\partial\phi}{\partial z} = 0, \quad (4)$$

where the scalar field  $\phi$  is advected by the 3D wind  $\mathbf{V} = (u, v, w)$ . In the SL method, the grid-point value of the scalar field  $\phi$  at next time step  $t + \Delta t$  can be found by integrating (4) along the trajectory of the fluid parcel (Staniforth and Côté, 1991; Hortal, 2002)

$$\int_t^{t+\Delta t} \frac{D\phi}{Dt} dt = 0 \rightarrow \phi^{t+\Delta t} = \phi_d^t, \quad (5)$$

where  $\phi^{t+\Delta t}$  is the value of the fluid parcel  $\phi$  arriving at any grid point at  $t + \Delta t$ , and  $\phi_d^t$  is the value of the same fluid parcel at its departure point  $d$  and departure time  $t$ . This means that the value of the scalar field  $\phi$  at any grid point at  $t + \Delta t$  is equal to its value at the departure point  $d$  and the departure time  $t$ . The departure point  $d$  usually does not coincide with any grid point, so the value of  $\phi_d^t$  is obtained by interpolation using the surrounding grid-point values  $\phi^t$  at time  $t$ . The departure point  $d$  is determined by iteratively solving the trajectory equation



343 (Staniforth and Côté, 1991; Hortal, 2002)

$$344 \quad \frac{D\mathbf{r}}{Dt} = \mathbf{V}(\mathbf{r}, t) \rightarrow \mathbf{r}^{t+\Delta} - \mathbf{r}_d^t = \int_t^{t+\Delta} \mathbf{V}(\mathbf{r}, t) dt, \quad (6)$$

345 where  $\mathbf{r}^{t+\Delta}$  and  $\mathbf{r}_d^t$  are the position of the arrival and the departure point, respectively. From  
346 (6), it is obvious that the departure point is far from its arrival point if the wind speed is large.  
347 Thus, the departure point of one fluid parcel at the boundary of the subdomain of an MPI task  
348 is far from its boundary if the wind speed is large and the wind blows from the outside. To  
349 facilitate calculation of the departure point and its interpolation, MPI parallelization adopts  
350 a “maximum wind” halo approach so that the halo is sufficiently large for each MPI task to  
351 perform its SL calculations in parallel after exchanging the halo. This “maximum wind” halo  
352 is named “wide halo” since its width is significantly larger than that of the thin halo of finite  
353 difference methods whose stencils have compact support. With numerous MPI tasks, the width  
354 of a wide halo may be larger than the subdomain size of its direct neighbour, which implies  
355 that the process needs to exchange the halo with its neighbours and its neighbours’ neighbours,  
356 which may result in a significant communication overhead which counteracts the efficiency of  
357 the favourite SL method, and pose a great challenge to the scalability of the SL method.

358 Fig. 4b demonstrates the halo exchange algorithm adopted in this paper. First, the al-  
359 gorithm posts the MPI non-block send and receive operations 1-4 simultaneously for the x-  
360 direction sweep. After the x-direction sweep, a y-direction sweep is performed in a similar way  
361 but the length of halo is extended to include the left and right haloes in the x-direction so that  
362 the four corners are exchanged properly. This algorithm needs two stages communications,  
363 but is simple to implement, especially for the wide halo exchange owing to its fixed regular  
364 communication pattern (Fig. 9d). In Fig. 9d, the pixels (near purple colour) tightly attached  
365 to the diagonal are due to the exchange in x-direction, the pixels of the same colour but off  
366 diagonal are due because of the periodicity in x-direction; the pixels (near orange or red colour)  
367 off diagonal are due to the exchange in y-direction, and the pixels of the same colour but far  
368 off diagonal are because of the periodicity in y-direction. This algorithm also applies to the  
369 thin halo exchange for finite difference methods which is extensively used in the grid-point  
370 models. The study emphasizes on the wide halo exchange, but the thin halo exchange is also  
371 investigated for comparison (see the red line in Fig. 9a).



It's worth noting that this regularity is only possible for structured grids. Even then there are differences between regular and reduced grids. Unstructured grids would not have a preferred x or y sweeping, and must be done in a single sweep. Does the following analysis still hold then?



372 **3.3 Allreduce in Krylov Subspace Methods for the Semi-Implicit Method**

373 The three-dimensional SI method leads to a large linear system which can be solved by Krylov  
 374 subspace methods:

375 
$$\mathbf{Ax} = \mathbf{b}, \tag{7}$$

376 where  $\mathbf{A}$  is a non-symmetric sparse matrix. Krylov subspace methods find the approximation  
 377  $\mathbf{x}$  iteratively in a  $k$ -dimensional Krylov subspace:

378 
$$\mathcal{K} = \text{span}(\mathbf{r}, \mathbf{Ar}, \mathbf{A}^2\mathbf{r}, \dots, \mathbf{A}^{k-1}\mathbf{r}), \tag{8}$$

379 where  $\mathbf{r} = \mathbf{b} - \mathbf{Ax}$ . To accelerate the convergence, preconditioning is generally used:

380 
$$\mathbf{M}^{-1}\mathbf{Ax} = \mathbf{M}^{-1}\mathbf{b} \tag{9}$$

381 where  $\mathbf{M}$  approximates  $\mathbf{A}$  well so that  $\mathbf{M}^{-1}\mathbf{A}$  be conditioned better than  $\mathbf{A}$  and  $\mathbf{M}^{-1}$  can be  
 382 computed cheaply. The GCR method is a Krylov subspace method of easy implementation  
 383 and can be used with variable preconditioners. Algorithm 1 of GCR shows that there are two  
 384 allreduces operations using the sum operation for the inner product in each iteration, thus, it  
 385 has  $2N$  allreduce operations if the GCR iterative solver reaches convergence in  $N$  iterations.  
 386 Allreduce is an all-to-all communication and becomes expensive when the number of iterations  
 387 becomes larger in GCR solver with numerous MPI processes.

388 Fig. 4c demonstrates the recursive-k algorithm for the allreduce operation, which is a gen-  
 389 eralization of the recursive doubling algorithm. Let  $p = \lfloor \log_k(ncpu) \rfloor$ , this algorithm has  $2 + p$   
 390 stages of communications if the number of processes is not a power of radix  $k$ . In the first stage  
 391 with stage id  $j = 0$  (the first row in Fig. 4c), each remaining process whose id  $i \notin [0, k^p - 1]$   
 392 sends its data to process  $i - (ncpu - k^p)$  for the reduce operation. For the stage of stage id  
 393  $j \in [1, p]$  (rows between the first row and second last row in Fig. 4c), each process whose id  
 394  $i \in [0, k^p - 1]$  only reduces with the processes that are a distance of  $k^{j-1}$  apart from itself. In  
 395 the final stage with stage id  $j = 1 + p$  (the second last row in Fig. 4c), each process whose id  
 396  $i \notin [0, k^p - 1]$  receives its final result from process  $i - (ncpu - k^p)$ . The recursive-k algorithm  
 397 uses large radix  $k$  to reduce the stages of communications and the overall communication time.



**Algorithm 1** Preconditioned GCR returns the solution  $\mathbf{x}_i$  when convergence occurs where  $\mathbf{x}_0$  is the first guess solution and  $k$  is the number of iterations for restart.

---

```

1: procedure GCR( $\mathbf{A}, \mathbf{M}, \mathbf{b}, \mathbf{x}_0, k$ )
2:    $\mathbf{r}_0 \leftarrow \mathbf{b} - \mathbf{A}\mathbf{x}_0$ 
3:    $\mathbf{u}_0 \leftarrow \mathbf{M}^{-1}\mathbf{r}_0$ 
4:    $\mathbf{p}_0 \leftarrow \mathbf{u}_0$ 
5:    $\mathbf{s}_0 \leftarrow \mathbf{A}\mathbf{p}_0$ 
6:    $\gamma_0 \leftarrow \langle \mathbf{u}_0, \mathbf{s}_0 \rangle, \eta_0 \leftarrow \langle \mathbf{s}_0, \mathbf{s}_0 \rangle$  ▷ Allreduce(sum) of two doubles
7:    $\alpha_0 \leftarrow \frac{\gamma_0}{\eta_0}$ 
8:   for  $i = 1, \dots$ , until convergence do
9:      $\mathbf{x}_i \leftarrow \mathbf{x}_{i-1} + \alpha_{i-1}\mathbf{p}_{i-1}$ 
10:     $\mathbf{r}_i \leftarrow \mathbf{r}_{i-1} - \alpha_{i-1}\mathbf{s}_{i-1}$ 
11:     $\mathbf{u}_i \leftarrow \mathbf{M}^{-1}\mathbf{r}_i$ 
12:    for  $j = \max(0, i - k), \dots, i - 1$  do
13:       $\beta_{i,j} \leftarrow \frac{-1}{\eta_j} \langle \mathbf{A}\mathbf{u}_i, \mathbf{s}_j \rangle$  ▷ Allreduce(sum) of min(i,k) doubles
14:       $\mathbf{p}_i \leftarrow \mathbf{u}_i + \sum_{j=\max(0,i-k)}^{i-1} \beta_{i,j}\mathbf{p}_j$ 
15:       $\mathbf{s}_i = \mathbf{A}\mathbf{p}_i$ 
16:       $\gamma_i \leftarrow \langle \mathbf{u}_i, \mathbf{s}_i \rangle, \eta_i \leftarrow \langle \mathbf{s}_i, \mathbf{s}_i \rangle$  ▷ Allreduce(sum) of two doubles
17:       $\alpha_i \leftarrow \frac{\gamma_i}{\eta_i}$ 
18:   return  $\mathbf{x}_i$ 

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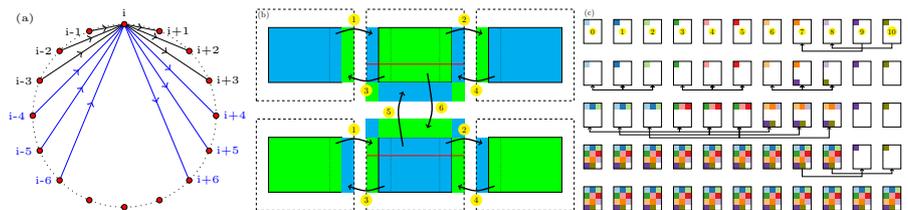


Fig. 4: Algorithms for three key MPI operations: (a) is the ring-k algorithm with  $k$  radix for all-to-all personalized communication generalized from ring alltoally algorithm, (b) is the halo exchange algorithm, and (c) is the recursive-k algorithm with  $k$  radix generalized from the recursive doubling algorithm.



Table 3: A three-dimensional grid for assessing the communication of the atmospheric model.  $\Delta x$  and  $\Delta y$  are given as if this grid is a uniform global longitude-latitude grid. In fact, this grid resembles the grid of a regional spectral atmospheric model or the uniform longitude-latitude grid used by some global models.

<b>nx</b>	<b>ny</b>	<b>nz</b>	<b><math>\Delta x</math></b>	<b><math>\Delta y</math></b>	<b>grid points</b>
28800	14400	256	0.0125°	0.0125°	> 100 billion
<b>memory size</b>			<b>max processes</b>		
> 800 GB per double field			3686400 for a 2D partition		

## 398 4 Experimental Results

### 399 4.1 Experiment Design

400 In the next decade, it is estimated the resolution of global NWP model will approach kilometre-  
 401 scale and the HPC will move towards exascale. What would the performance of a global NWP  
 402 model with a very high resolution on exascale HPC be? In this paper, we are especially  
 403 interested in the strong scaling of an atmospheric model, that is, how does the atmospheric  
 404 model with fixed resolution (such as the one presented in Table 3) behave as the number of  
 405 processes increases? Table 3 presents a summary of the three-dimensional grid for assessing  
 406 the communication of the kilometre-scale atmospheric model. The number of grid points of  
 407 this grid is beyond 100 billion, and one field of double precision variable for this grid requires  
 408 more than 800 gigabytes of memory. Only with such a large grid, is it possible to perform a  
 409 2D domain decomposition for a spectral model with more than one million processes so that  
 410 modelling the communication of the atmospheric model at exascale HPC become possible.

411 Besides the topology and its configuration, the routing algorithm, and the collective MPI  
 412 algorithm; the bandwidth and the latency of the interconnect network of an HPC system have  
 413 a great impact on the performance of communications. First, we simulate the transposition  
 414 for the spectral transform method in the simulator for three topologies (torus-M, fattree-M,  
 415 and dragonfly-MM in Table 1), three configurations of dragonfly topology (dragonfly-MM,  
 416 dragonfly-SL, and dragonfly-LS in Table 1), three routing algorithms (minimal, valiant, and  
 417 ugal), and three alltoally algorithms (Table 2). In addition, we compare the simulations of the  
 418 transposition for the spectral transform method between four interconnect bandwidths ( $10^0$ ,  
 419  $10^1$ ,  $10^2$ , and  $10^3$  GB/s) and between four interconnect latencies ( $10^1$ ,  $10^2$ ,  $10^3$ , and  $10^4$  ns).  
 420 After a thorough investigation of the transposition for the spectral transform method, we test



421 the halo exchange for the SL method with different halo widths (3, 10, 20, and 30 grid points),  
422 three topologies (torus-L, fattree-L, dragonfly-ML in Table 1), and three routing algorithms  
423 (minimal, valiant, and ugal). Finally, the allreduce operation in Krylov subspace methods for  
424 the SI method is evaluated on different topologies (torus-L, fattree-M, dragonfly-ML in Table  
425 1), and the statistics of the optimal radix of recursive-k algorithms for allreduce operations are  
426 presented.

#### 427 **4.2 Transposition for the Spectral Transform Method**

428 Fig. 5a shows that the communication times for the burst, bruck, ring-1, and ring-4 algorithms  
429 decrease as the number of MPI processes increases. The ring-1 and ring-4 algorithms are  
430 almost identical for less than  $5 \times 10^4$  MPI processes, but ring-4 performs better than ring-1 for  
431 more than  $10^5$  MPI processes. The burst and bruck algorithms perform worse than the ring-k  
432 algorithm. The SST/macro simulator cannot simulate the burst algorithm for more than  $2 \times 10^4$   
433 MPI processes because the burst messages result in huge events and large memory footprint.  
434 The communication time of the bruck algorithm is significantly larger than that of the ring-k  
435 algorithm for less than  $10^5$  MPI processes; however, for a greater number of processes, it is  
436 better than the ring-1 algorithm since the bruck algorithm is targeted for small messages, and  
437 the more processes, the smaller message for a fixed sized problem. The performance of these  
438 alltoallv algorithms is confirmed by actually running the skeleton program of transposition  
439 for the spectral transform method with  $10^4$  MPI processes on the research cluster of Météo  
440 France (Beaufix), which shows that the ring-4 algorithm is even better than the INTEL native  
441 MPI\_Alltoallv function (Fig. 6).

442 The differences in the communication times of the transpositions between the topology  
443 torus-M, fattree-M, and dragonfly-MM can be an order of magnitude (Fig. 5b). Messages have  
444 to travel a long distance in the topology torus-M which is a 3D torus, so its communication  
445 time is the largest. The best performance of the topology fattree-M can be attributed to its  
446 non-blocking D-mod-k routing algorithm, but its communication time gradually increases as  
447 the number of MPI processes increases beyond  $10^4$ . The performance of topology dragonfly-  
448 MM is between that of torus-M and fattree-M (Fig. 5b), it can achieve a better performance by  
449 tuning the configuration of the dragonfly topology (Fig. 5c). By comparing Fig. 5b and Fig. 5c,  
450 we can see that the topologies of dragonfly-SL and dragonfly-LS are still not as good as the



451 fattree-M, but their performance is very close to that of fattree-M and they lose less scalability  
452 than fattree-M for more than  $5 \times 10^4$  MPI processes.

453 The differences in communication time of the transpositions between the routing algorithms  
454 of minimal, valiant and ugal are also an order of magnitude (Fig. 5d), which indicates that the  
455 impact of routing algorithm on communication is significant. The valiant routing algorithm  
456 performs the best, but the communication time begins to increase when the number of MPI  
457 processes is larger than  $3 \times 10^4$ . The ugal routing algorithm performs the worst, and the  
458 performance of minimal routing algorithm is in between that of valiant and ugal routing al-  
459 gorithms. The valiant routing algorithm has the longest path for messages from the source to  
460 the destination with a randomly chosen intermediate node; thus, theoretically, its communica-  
461 tion time is larger. On the contrary, the minimal routing algorithm that moves the messages  
462 using the shortest path from the source to the destination has the smallest communication  
463 time. The congestion between processes in Fig. 7 shows that the valiant routing algorithm for  
464 the dragonfly-MM topology (Fig. 7b) and the minimal routing algorithm for the dragonfly-SL  
465 topology (Fig. 7d) are less congested and have a more uniform congestion, the minimal routing  
466 algorithm for the dragonfly-MM topology is moderately congested, but its congestion is not  
467 uniform (Fig. 7a), the congestion of the ugal routing algorithm for the dragonfly-MM topology  
468 is large and highly non-uniform (Fig. 7c). These congestions in Fig. 7 are consistent with the  
469 communication times in Fig. 5c and Fig. 5d, that is, the more uniform congestion, the lower  
470 communication time because the latter is determined by the longest delay event and uniform  
471 congestion can avoid the hotspot of the congestion with the longest delay event. Fig. 8 con-  
472 firms this that a high percentage of delay events has a delay time of less than 30 us using the  
473 valiant routing algorithm for the dragonfly-MM topology and the minimal routing algorithm  
474 for the dragonfly-SL topology; however the minimal routing algorithm for the dragonfly-MM  
475 topology has a significant percentage of events that delays by more than 50 us, especially there  
476 are a large number of events delayed by more than 100 us using the ugal routing algorithm  
477 for the dragonfly-MM topology. Thus, the configuration of the interconnect network and the  
478 design of its routing algorithm should make the congestion as uniform as possible if congestion  
479 is inevitable.

480 Although the communication time with a bandwidth of  $10^0$  GB/s is apparently separated  
481 from those with bandwidths of  $10^1$ ,  $10^2$ , and  $10^3$  GB/s, the curves describing the communication  
482 times with bandwidths of  $10^1$ ,  $10^2$ , and  $10^3$  GB/s overlap (Fig. 5e). The communication times



483 with latencies of  $10^1$  and  $10^2$  ns are almost identical; that with a latency of  $10^3$  ( $10^4$ ) ns is  
484 slightly (apparently) different from those with latencies of  $10^1$  and  $10^2$  ns (Fig. 5f). Equation  
485 (1) indicates that the communication time stops decreasing only when  $\alpha$  ( $\beta$ ) approaches zero and  
486  $\beta$  ( $\alpha$ ) is constant. Neither  $\alpha$  in Fig. 5e nor  $\beta$  in Fig. 5f approaches zero, but the communication  
487 time stops decreasing. The inability of the analytical model (1) to explain this suggests that  
488 other dominant factors such as congestion contribute to the communication time. Latency  
489 is the amount of time required to travel the path from one location to another. Bandwidth  
490 determines how ~~many~~ **much** data per second can be moved in parallel along that path, and limits the  
491 maximum number of packets travelling in parallel. Because both  $\alpha$  and  $\beta$  are greater than zero,  
492 congestion occurs when data arrives at a network interface at a rate faster than the media can  
493 service; when this occurs, packets must be placed in a queue to wait until earlier packets have  
494 been serviced. The longer the wait, the longer the delay and communication time. Fig. 8b and  
495 Fig. 8c show the distributions of the delay caused by congestion for different bandwidths and  
496 different latencies, respectively. In Fig. 8b, the distributions of the delay for bandwidths of  $10^1$ ,  
497  $10^2$ , and  $10^3$  GB/s are almost identical, which explains their overlapped communication times  
498 in Fig. 5e; and the distribution of the delay for a bandwidth of  $10^0$  GB/s is distinct from the  
499 rest since near 20 percent of events are delayed by less than 10 us but a significant percentage  
500 of events are delayed more than 100 us, which accounts for its largest communication time in  
501 Fig. 5e. In Fig. 8c, the distributions of the delay for latencies of  $10^1$  and  $10^2$  ns are the same;  
502 the distributions of the delay for a latency of  $10^3$  ns is slightly different from the formers; but  
503 the distributions of the delay for a latency of  $10^4$  ns has a large percentage of events in the  
504 right tail which resulted in the longest communication time; these are consistent with their  
505 communication times in Fig. 5f.

506 In summary, the alltoallv algorithm, the topology and its configuration, the routing al-  
507 gorithm, the bandwidth, and the latency have great impacts on the communication time of  
508 transpositions. In addition, the communication time of transpositions decreases as the number  
509 of MPI processes increases in most cases; however, this strong scalability is not applicable for  
510 the fattree-M topology (the red line in Fig. 5b), the dragonfly-SL and dragonfly-LS topologies  
511 (red and black lines in Fig. 5c), and the valiant routing algorithm (the red line in Fig. 5d) when  
512 the number of MPI processes is large. Thus, the topology of the interconnect network and its  
513 routing algorithm have a great impact on the scalability of transpositions for the spectral trans-  
514 form method. Since the transposition for spectral transform method is a multiple simultaneous

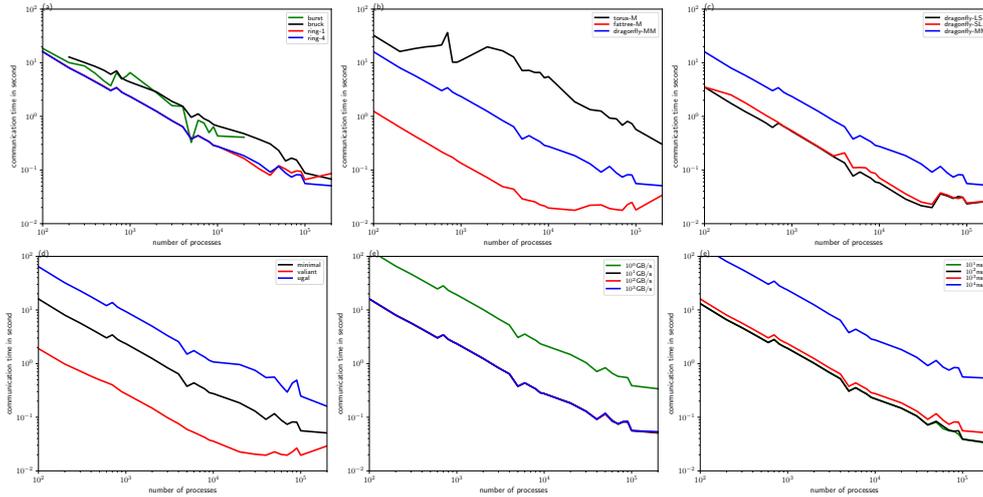


Fig. 5: Communication times of transposition for (a) alltoallv algorithms, (b) topologies, (c) configurations of the dragonfly topology, (d) routing algorithms for the dragonfly topology, (e) bandwidth, and (f) latency.

515 all-to-all personalized communication, congestion has a great impact on its performance.

### 516 4.3 Halo Exchange for the Semi-Lagrangian Method

517 The most common application of the wide halo exchange is the SL method. For the resolution  
 518 of 0.0125° in Table 3 and a time step of 30 seconds, the departure is approximately 5 grid  
 519 points away from its arrival if the maximum wind speed is 200 m/s; therefore, the width of the  
 520 halo is at least 7 grid points using the ECMWF quasi-cubic scheme (Ritchie, 1995); there are  
 521 more grid points if a higher order scheme such as the SLICE-3D (Zerroukat and Allen, 2012)

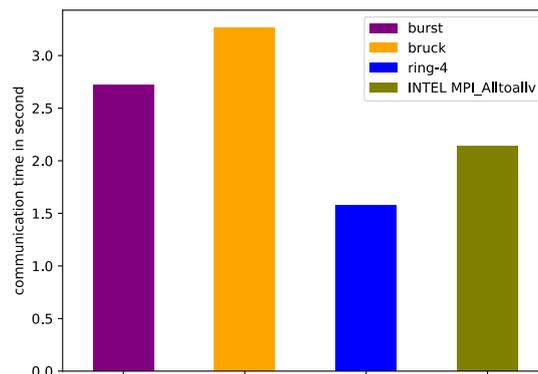


Fig. 6: Actual communication time of transposition for the spectral transform method with 10<sup>4</sup> MPI processes run on beaufix cluster in Météo France.

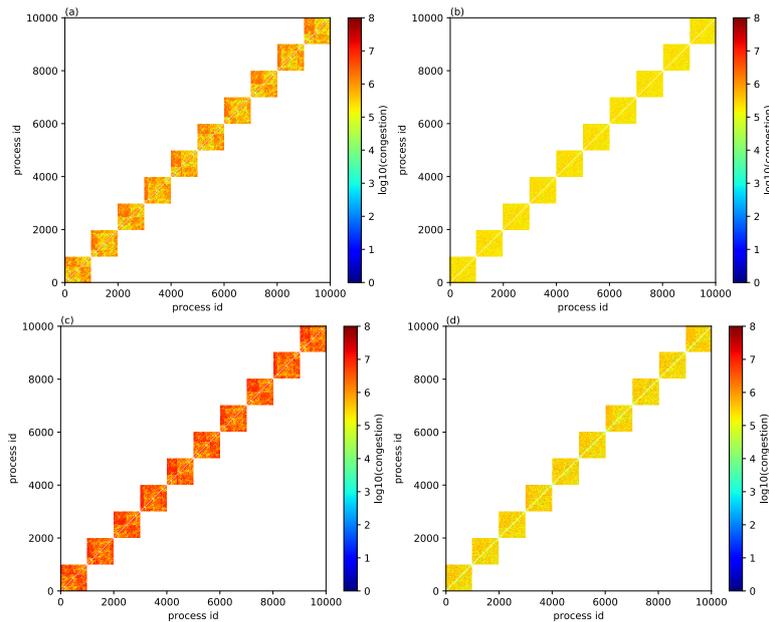


Fig. 7: Congestion of transposition using (a) minimal routing algorithm for the dragonfly-MM topology, (b) valiant routing algorithm for the dragonfly-MM topology, (c) ugal routing algorithm for the dragonfly-MM topology, and (d) minimal routing algorithm for the dragonfly-SL topology.

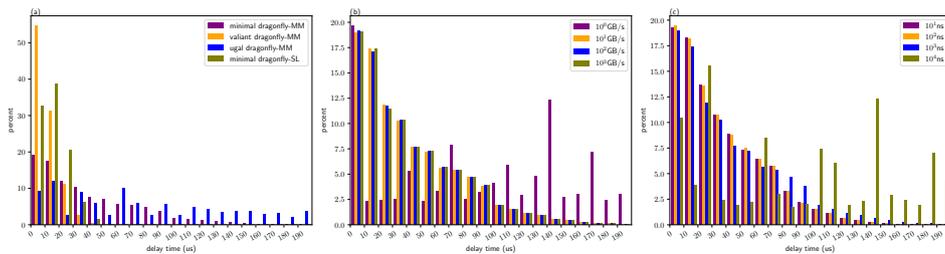


Fig. 8: Distribution of delayed events of transposition for the spectral transform method with  $10^4$  MPI processes using (a) different routing algorithms and topology configurations, (b) different bandwidths, and (c) different latencies, simulated by SST/macro.



522 is used. In Fig. 9a, the communication time of the halo exchange decreases more slowly with increasing ~~as the~~  
523 number of processes ~~increases~~ than that of transposition for the spectral transform method.  
524 This is because the message size decreases more slowly than that of transposition owing to  
525 the fixed width of the halo (figure omitted). If the communication time of the transposition  
526 (halo exchange) continues its decreasing (increasing) trend in Fig. 9a, they meet at certain  
527 number of MPI processes; then, the communication time of the halo exchange is larger than  
528 that of the transposition. In addition, it can be seen that the wider the halo, the longer the  
529 communication time. The halo exchange of a thin halo of 3 grid points, for such as the 6th  
530 order central difference  $F'_i = \frac{-F_{i-3} + 9F_{i-2} - 45F_{i-1} + 45F_{i+1} - 9F_{i+2} + F_{i+3}}{60\Delta}$  (the red line in Fig. 9a), is  
531 significantly faster than that of wide halo for the SL method (green and blue lines in Fig. 9a).  
532 Thus, the efficiency of the SL method is counteracted by the overhead of the wide halo exchange  
533 where the width of the halo is determined by the maximum wind speed. Wide halo exchange  
534 for the SL method is expensive at exascale, especially for the atmospheric chemistry models  
535 where a large number of tracers need to be transported. On-demand exchange is a way to  
536 reduce the communication of halo exchange for the SL method, and will be investigated in a  
537 future study.

538 Significant differences in the communication times of the wide halo exchange of 20 grid  
539 points for topology torus-L, fattree-L, and dragonfly-ML are shown in Fig. 9b. It can be  
540 seen that topology torus-L performs the worst, fattree-L is the best, and the performance of  
541 dragonfly-ML is between that of torus-L and fattree-L. The communication time of the wide  
542 halo exchange of 20 grid points for the topology torus-L abruptly increases at approximately  $10^3$   
543 MPI processes, and then gradually decreases when the number of MPI tasks becomes larger  
544 than  $3 \times 10^3$  MPI processes. The impact of the routing algorithm on the communication time  
545 of the wide halo exchange of 20 grid points (Fig. 9c) is the same as on that of transposition  
546 (Fig. 5d): the routing algorithm valiant performs the best, the routing algorithm ugal performs  
547 the worst, and the routing algorithm minimal is between valiant and ugal.

#### 548 **4.4 Allreduce in Krylov Subspace Methods for the Semi-Implicit Method**

549 If, in average, the GCR with a restart number  $k = 3$  is convergent with  $N = 25$  iterations, the  
550 number of allreduce calls is  $2 \times N = 50$ . The black and blue lines are the communication times  
551 of 50 allreduce operations using MPI\_Allreduce and the recursive-k algorithm, respectively;

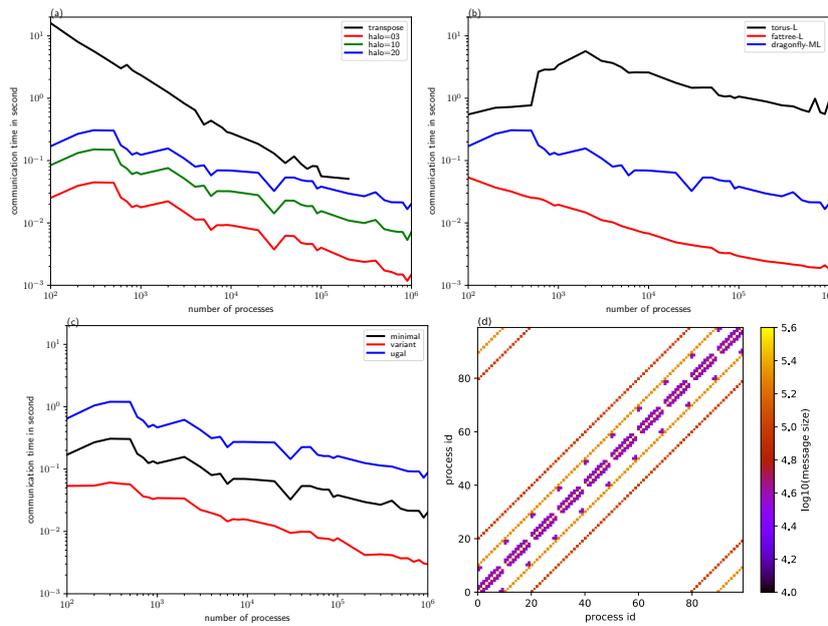


Fig. 9: (a) is the communication times of the halo exchange with a halo of 3 (red line), 10 (green line), and 20 (blue line) grid points, and the communication time of transposition for the spectral transform method is shown for comparison (black line). (b) is the communication times of the halo exchange with a halo of 20 grid points for the topology of torus-L (black line), fattree-L (red line), and dragonfly-ML (blue line). (c) is the communication times of the halo exchange with a halo of 20 grid points for the routing algorithm of minimal (black line), valiant (red line), and ugal (blue line). (d) illustrates the communication pattern of the halo exchange with a wide halo.



552 that is, the estimated communication time of one single GCR call (Fig. 10a). Contrary to that  
553 of transposition, the communication time of GCR increases as the number of MPI processes  
554 increases. Following the trend, the communication of a single GCR call may be similar to or  
555 even larger than that of a single transposition when the number of MPI processes approaches  
556 to or is beyond one million. Although it is believed that the spectral method does not scale  
557 well owing to its time-consuming transposition, it does not suffer from this expensive allreduce  
558 operation for the SI method because of its mathematical advantage that spherical harmonics are  
559 the eigenfunctions of Helmholtz operators. In this sense, a grid-point model with the SI method  
560 in which the three-dimensional Helmholtz equation is solved by Krylov subspace methods may  
561 also not scale well at exascale unless the overhead of allreduce communication can be mitigated  
562 by overlapping it with computation (Sanan et al., 2016).

563 Fig. 10b shows the communication times of allreduce operations using the recursive-k algo-  
564 rithm on the topologies of torus-L, fattree-L, and dragonfly-ML. The impact of topology on the  
565 communication performance of allreduce operations is obvious. The topology of torus-L has the  
566 best performance, but is similar to that of dragonfly-ML for more than  $5 \times 10^5$  MPI processes;  
567 and fattree-L has the worst performance. However, the impact of three routing algorithms  
568 (minima, valiant, and ugal) for the dragonfly-ML topology has a negligible impact on the com-  
569 munication performance of allreduce operations (figure omitted); this may be because of the  
570 tiny messages (only 3 doubles for the restart number  $k = 3$ ) communicated by the allreduce  
571 operation.

572 One advantage of the recursive-k algorithm of the allreduce operation is that the radix  $k$   
573 can be selected to reduce the stages of communication by making full use of the bandwidth  
574 of the underlying interconnect network. We repeat the experiment, whose configuration is  
575 as that of the blue line in Fig. 10a, for the proper radix  $k \in [2, 32]$ , and the optimal radix  
576 is that with the lowest communication time for a given number of MPI processes. For each  
577 number of MPI processes, there is an optimal radix. The statistics of all the optimal radices are  
578 shown in Fig. 10c. It can be seen that the minimum and maximum optimal radices are 5 and  
579 32, respectively. Thus, the recursive doubling algorithm that is equivalent to the recursive-k  
580 algorithm with radix  $k=2$  is not efficient since the optimal radix is at least 5. The median  
581 number of optimal radices is approximately 21, and the mean number is less than but very  
582 close to the median number. We cannot derive an analytic formula for the optimal radix since  
583 modelling the congestion is difficult in an analytic model. However, for a given resolution of

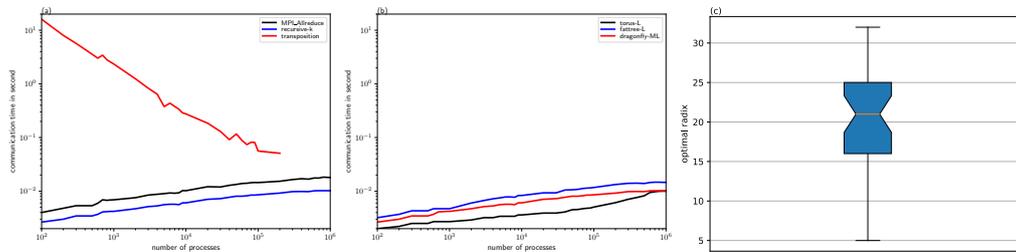


Fig. 10: (a) is the communication times of the allreduce operation using the MPI\_Allreduce (black line) and the recursive-k algorithm (blue line), and the communication time of transposition for the spectral transform method is shown for comparison (red line). (b) is the communication times of the allreduce operation using the recursive-k algorithm for the topology torus-L (black line), fattree-L (blue line), and dragonfly-ML (red line). (c) is the statistics of the optimal radices for the recursive-k algorithm.

584 NWP model and a given HPC system, fortunately, the number of processes, bandwidth, and  
585 latency are fixed; thus, it is easy to perform experiments to obtain the optimal radix.

## 586 5 Conclusion and Discussion

587 This work shows that it is possible to make simulations of the MPI patterns commonly used in  
588 NWP models using very large numbers of MPI tasks. This enables the possibility to examine  
589 and compare the impact of different factors such as latency, bandwidth, routing and network  
590 topology on response time. We have provided an assessment of the performance and scalability  
591 of three key MPI operations in an atmospheric model at exascale by simulating their skeleton  
592 programs on an SST/macro simulator. After optimization of the memory and efficiency of  
593 the SST/macro simulator and construction of the skeleton programs, a series of experiments  
594 was carried out to investigate the impacts of the collective algorithm, the topology and its  
595 configuration, the routing algorithm, the bandwidth, and the latency on the performance and  
596 scalability of transposition, halo exchange, and allreduce operations. The experimental results  
597 show that:

- 598 1. The collective algorithm is extremely important for the performance and scalability of  
599 key MPI operations in the atmospheric model at exascale because a good algorithm can  
600 make full use of the bandwidth and reduce the stages of communication. The generalized  
601 ring-k algorithm for the alltoallv operation and the generalized recursive-k algorithm for  
602 the allreduce operation proposed herein perform the best.



603 2. Topology, its configuration, and the routing algorithm have a considerable impact on the  
604 performance and scalability of communications. The fattree topology usually performs  
605 the best, but its scalability becomes weak with a large number of MPI processes. The  
606 dragonfly topology balances the performance and scalability well, and can maintain almost  
607 the same scalability with a large number of MPI processes. The configurations of the  
608 dragonfly topology indicate that a proper configuration can be used to avoid the hotspots  
609 of congestion and lead to good performance. The minimal routing algorithm is intuitive  
610 and performs well. However, the valiant routing algorithm (which randomly chooses an  
611 intermediate node to uniformly disperse the communication over the network to avoid  
612 the hotspot/bottleneck of congestion) performs much better for heavy congestion.

613 3. Although they have an important impact on communication, bandwidth and latency  
614 cannot be infinitely grown and reduced owing to the limitation of hardware, respectively.  
615 Thus, it is important to design innovative algorithms to make full use of the bandwidth  
616 and to reduce the effect of latency.

617 4. It is generally believed that the transposition for the spectral transform method, which is  
618 a multiple simultaneous all-to-all personalized communication, poses a great challenge to  
619 the scalability of the spectral model. This work shows that the scalability of the spectral  
620 model is still acceptable in terms of transposition. However, the wide halo exchange for  
621 the Semi-Lagrangian method and the allreduce operation in the GCR iterative solver for  
622 the Semi-Implicit method, both of which are often adopted by the grid-point model, also  
623 suffer the stringent challenge of scalability at exascale.

624 In summary, both software (algorithms) and hardware (characteristics and configuration)  
625 are of great importance to the performance and scalability of the atmospheric model at exascale.  
626 The software and hardware must be co-designed to address the challenge of the atmospheric  
627 model for exascale computing.

628 As shown previously, the communications of the wide halo exchange for the Semi-Lagrangian  
629 method and the allreduce operation in the GCR iterative solver for the Semi-Implicit method  
630 are expensive at exascale. The on-demand halo exchange for the Semi-Lagrangian and the  
631 pipeline technique to overlap the communication with the computation for the GCR iterative  
632 solver are not researched in this study and should be investigated. All the computed nodes in  
633 this work only contain one single-core CPU, which is good for assessing the communication of

Can MPI tasks be carefully pinned to cores using knowledge of the domain decomposition to reduce congestion?

Why "computed"? Do the authors mean "compute" nodes?



634 the interconnect network; however, **compute nodes contain multi-core CPUs or many-core accelerators.**  
635 **it is now very common for one CPU with multi-cores or even**  
636 **many-cores.** Multiple MPI processes per node may be good for the local pattern communication  
637 such as thin halo exchange since the shared memory communication is used, but may result  
638 in heavy congestion in the network interface controller for all-to-all communication. The more  
639 MPI processes, the less computation per node without limitation if there is only one single-core  
640 CPU per node, thus, computation is not considered in this paper. **However, the bandwidth**  
641 **of memory limits the performance and scalability of computation for multi-core or many-core**  
642 **systems.** The assessment of computation **is** currently underway and a detailed paper will be  
643 presented separately; the purpose of this subsequent study is to model the time response of a  
644 time step of a model such as the regional model (AROME) used by Météo-France.

Can the authors elaborate?

## 644 Code Availability

645 The code of the SST/macro simulator is publicly available at [https://github.com/sstsimulator/sst-](https://github.com/sstsimulator/sst-macro)  
646 [macro](https://github.com/sstsimulator/sst-macro). The skeleton programs, scripts, and our modified version of SST/macro 7.1.0 for the  
647 simulations presented the paper are available at <https://doi.org/10.5281/zenodo.1066934>.

## 648 Competing Interests

649 The authors declared no competing interests.

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