Interactive comment on “Implementation of higher-order vertical finite elements in ISSM v4.13. for improved ice sheet flow modeling over paleoclimate timescales” by Joshua K. Cuzzone et al.

Anonymous Referee #1

Received and published: 17 February 2018

This paper compares the performances of a finite element thermo-mechanical ice flow model, depending on the polynomial approximation in the vertical direction. The paper mainly focuses on the ability of the model to capture the sharp vertical gradients of the temperature near the ice-sheet base. They show that, for the same element size, convergence to a reference solution is faster with quadratic or cubic elements compared to linear elements. The conclusion is that, runtimes can be improved by one order of magnitude when using higher-order elements compared to linear elements, for a similar accuracy.

It is well known that the convergence of the finite element method with respect to the element size depends on the polynomial approximation, so that the results are not surprising. However, I think that it has never been discussed in the context of ice flow modelling, making this discussion relatively interesting for the ice flow modelling community.

Few points deserve more consideration:

- Abstract: the authors report runtimes 10 to 57 times faster. I think this is a bit misleading, as these numbers are when comparing the higher order elements with the reference 25 layers-P1 elements simulation. Improvement in runtimes are interesting if they allow to reach a similar accuracy. As the 25 layers-P1 elements simulation is used as a reference, the respective accuracy of the solutions is not known. However, it is shown that compared to the same reference, a 10 layers-P1 elements simulation falls within the same criteria of 1%. Improvement in runtimes is then only a factor 5. I think this is the more correct number to report in the abstract.

- Page 2, lines 95-96: it is said that the stress balance requires less vertical resolution. I’m not sure that this is a well established result, as for areas with high friction near the base, there is also very sharp gradient of the stresses and strain rates, also requiring higher resolution for the stress balance.

- Sec. 2.4. Figure 2 compares an exponential function captured by vertical elements with different polynomial interpolation. We understand that the figure is for 1 P3 element (i.e. 4 layers of nodes) or 3 P1 elements (i.e. 4 layers of nodes); but what is the corresponding number of P2 elements?

- Sec. 3.1: it is said that P1 elements are used for the stress balance. What is the default number of integration points. Does it allow to capture the temperature profile, affecting the viscosity, within the element?
• Sec. 3.1. All the introduction is about using higher order models for the stress balance, however most of the EISMINT comparison is done with the Shallow ice model, and we learn this very late in the results section. It should be said here that the 100 000 years experiment is done with the SIA, justifying the comparison with EISMINT2, and that the BP model is used only to do 100 years relaxations.

• Page 5, line 227. It is said that the elements are finer in areas of steep topography and ice flow gradients. I think that the refinement is based on the second derivatives, not the gradients, so elements are finer where changes in slope and ice flow gradients are high?