Dear Editor,

We are very grateful to you for accepting the successive deadline extensions and we apologized to the long-time takes to submit this revised version. This time has been helpful for us to largely improve the manuscript following the numerous reviewers’ comments and detailed concerns during the peer-review process.

Firstly, as requested by the executive editor Astrid Kerkweg, and following referees’ comments, we have slightly changed the title of the present paper by changing spin-up to initialisation and by adding the GRISLI model version: A rapidly converging initialisation method to simulate the present-day Greenland ice sheet using the GRISLI ice-sheet model (version 1.3)

Following the D. Pollard and S. Price reviews, we have substantially clarified sections describing the minimisation procedure used with the ice sheet model GRISLI. We have also conducted numerous additional experiments to explore the sensitivity of the minimisation procedure to model parameters such as enhancement factor and initial conditions.

As requested by the referees we also applied a new metric to determine optimal parameters for minimisation procedure, requiring us to redo all the figures to fit with the new result section 4 and 5.

Finally, we have deeply investigated and discussed the limitations of our minimisation procedure. We also made our best to improve the English language.

Best regards,

Sébastien Le clec’h (on behalf of all co-authors)
We would like to thank the reviewer Dave Pollard for the evaluation of our study. Please find below the reviewer’s comments in black font and the author’s response in blue font.

**Responses to David Pollard (Referee #2)**

**General comments:**

This paper applies a simple method of adjusting basal sliding coefficients to obtain realistic ice thicknesses in an ice sheet model of modern Greenland. Similarly to previous simple methods used for Antarctica, the paper shows how the iterative method converges towards basal coefficient maps ("beta") that yield best-fit ice distributions. The method requires relatively short integrations, making it feasible for more complex models. The analysis is detailed and substantial, showing that the method functions well and yields meaningful results, and the paper will be of considerable interest to the modeling community.

Thank you for this comment.

My main concern is that, as described in the paper, there are large interior regions where ice thickness errors cannot be corrected due to internal deformation flow being too large, which detracts from the primary results. Additional runs to correct this are suggested below.

Following your comment, we now explore extensively the role of the enhancement factor and show that we are indeed able to correct the error for the interior regions using a lower enhancement factor. To this aim we considerably increased the number of simulations shown in the revised manuscript with respect to the initial submission. In light of these new simulations we address your comments in the following.

**Main specific comment**

Much of the paper’s primary analysis in section 4 concerns the progress of the procedure as the overall length increases (increasing NBcycle). For given NBiter and NByear, the rms thickness error "dH" tends more or less monotonically to a minimum (Fig. 5), but total volume error "dV" overshoots zero and becomes more unrealistic again (Fig. 6a). The analysis (sections 4.2.3, 4.2.4, Table 1) is mainly concerned with finding values of NBcycle and associated NBiter,NByear, at which dH, dV (and dV/dt, but see below) are qualitatively the best (small) if the procedure is stopped at some point.

I think these results are not the most useful or meaningful, because there are substantial regions in the east-central Greenland interior where internal deformation flow is too large,
producing too small ice thicknesses even with zero basal sliding. This prevents the \(dH\) and/or \(dV\) metrics from both converging to zero together as the procedure is extended indefinitely, and causes the "overshoots" in Figs. 6. This is fully described in the paper's section 5, but only after the primary results of section 4 are presented.

It would be better to address and fix the problem from the start in section 4, which would yield more meaningful results. The existing results, regarding the particular \(NBiter/NByear/NBcycle\) values where the \(dV\) and \(dV/dt\) metrics cross the zero lines, just reflect the influence of the problem region with excessive internal flow.

Also they depend on the choice of initial \(beta(x,y)\), which is arbitrary (as shown nicely by Fig. 3c), but if chosen further from the final state, needs more \(NBcycle\) cycles to reach the same point of evolution.

The problem is fully recognized in the paper's section 5.1, and a possible solution is implied in section 5.2, by trying different values of the enhancement factor \(Ef\). My main suggestion is to repeat the procedure of section 4 for a range of \(Ef\) values, say \(Ef = 0.1, 0.5, 1, 1.5, 2, 2.5, 3\). Hopefully just one long procedure would be sufficient for each \(Ef\) value, with just one set of \(NBiter, NByear\) values, and a large \(NBcycle\) of 10 or 15 (see below).

I would anticipate that for the smaller \(Ef\) values, the persistent thickness errors in the Greenland interior can be corrected by adjusting local \(beta\)'s, so both metrics \(dH\) and \(dV\) (and \(dV/dt\)) will converge towards zero and not overshoot (but see "basal temperatures", below). The main outcomes of the new section 4 would be (i) the value of \(Ef\) below which this occurs, and (ii) how long the overall procedure needs to be continued (how many \(NBcycle\)'s) to reach acceptably small \(dH\) and \(dV\). (Possibly the rate of convergence may be quicker for different ratios of \(NBiter\) and \(NByear\), but I suspect not, and for the smaller \(Ef\), everything depends just on the total number of years \((NBiter+NByear)*NBcycle\). Note that if \(dH\) converges on zero, then \(dV\) and \(dV/dt\) must too.

This would of course require significant re-running of the model for the other \(Ef\)s, and reorganizing sections 4 and 5, but would yield more useful and less arbitrary results in my opinion. One encouraging sign that it will work is how much better Fig. 10 looks (\(Ef=1\)) compared to Fig. 7b (\(Ef=3\)). (Much the same adjustment of \(Ef\) was done in Appendix B of Pollard and DeConto, The Cryo, 2012, called PDC12 here, but was not as important because their main results used a relatively low \(Ef\)).

Thanks for the in-depth analysis of our results. We fully agree with your comment and this is why we performed additional experiments varying the enhancement factor from 0.5 to 5 for a given set of \(Nb_{inv}, Nb_{free}, Nb_{cycle}\) values (former \(Nb_{iter}, Nb_{year}, Nb_{cycle}\)). As a result, Sections 4 and 5 have been completely reorganized. The results of these new simulations (with \(Ef\) ranging from 0.5 to 5) are now presented in Section 4 before discussing (Section 5) the sensitivity to the initialisation procedure coefficients \(Nb_{inv}, Nb_{free}\) (former \(Nb_{iter}, Nb_{year}\)). As you suggest in your comment, we are able to show that the enhancement factor can be used to correct the ice thickness error where deformation due to vertical shearing is predominant (e.g. interior region). In particular we show that for \(Ef \geq 2\), a larger \(Ef\) value leads systematically to a larger ice thickness RMSE. For lower \(Ef\) values (\(Ef < 2\)), we obtain minimum RMSE for \(Ef\) between 1 and 1.5. For \(Ef = 0.5\), the ice
thickness RMSE is slightly higher (with respect to that obtained for $E_f$ between 1 and 1.5 and we still have positive ice thickness anomalies (w.r.t. to observations) in the ice-sheet interior due, in that case, to a too slow ice flow related to vertical shearing. These results are discussed in Section 4.2.1 of the revised manuscript.

In the new section 5, we investigate the sensitivity of the method performance to the $N_{b_{inv}}$ and $N_{b_{free}}$ parameters. As suggested, for each ($N_{b_{inv}}, N_{b_{free}}$) combination, $N_{b_{cycle}}$ simulations have been performed with $N_{b_{cycle}} = 15$. We show that there is a strong decrease of the ice thickness RMSE after one cycle ($N_{b_{cycle}} = 1$) but only little improvement when using $N_{b_{cycle}} \geq 6$. These results are discussed in details in Section 5.3. As also mentioned in our response to your comment pg. 10, Fig. 6, and pg. 11 line 10 to top of pg. 12, the critical duration to obtain a good performance is defined by $N_{b_{inv}}*N_{b_{free}}$ because the initial condition for the different cycles is systematically the same: only the initial basal drag coefficient for step 1 is different (see Section 3). Finally, we have also to mention that in the revised paper, the ice thickness RMSE is the key parameter to assess the performance of our method. Moreover, the ice volume trend is no longer considered. Rather, we introduce a new metric that can be considered as the ice thickness change root mean square. This allows the compensatory biases to be circumvented (see Section 4.2.2).

**Related to main comment**

One complication involves the basal temperature field, i.e., frozen vs. thawed basal areas. Where the base is frozen, the procedure of adjusting beta is ineffective in reducing ice thickness errors of course. This is mentioned in the paper (pg. 11, lines 1-3), but because of its importance, I suggest showing a map of modeled basal temperatures $T_b(x,y)$, perhaps near the top of pg. 11 where basal temperatures are discussed, and assessing it versus other established Greenland $T_b$ maps (such as the recent modeling synthesis in MacGregor et al., JGR-Earth Surface, 2016).

Such a figure is shown in the revised manuscript (Fig. 1c). In Section 3, we also provide a brief comparison between our simulated distribution of frozen/thawed bed areas (inferred from the simulated basal temperatures) and the reconstructions of MacGregor et al. (2016): “The resulting basal temperature after this long integration, presented as a difference with respect to the pressure melting point, is shown in Fig. 1c. It shows areas with temperature largely below the pressure melting point, associated with frozen bed, and areas with temperature at the pressure melting point (red colors), associated with thawed bed. Compared to the recent synthesis of GrIS basal temperatures (see Fig. 11 in MacGregor et al., 2016), our initial basal temperature agrees generally well with the reconstructions in the northwestern and northeastern parts of the GrIS but are probably overestimated, with a too large thawed bed area, in the eastern and central parts of the GrIS (not shown). The impact of ice temperature on the minimisation procedure is discussed in Sect. 5.1”.

Also, it would help to mention this point in the description of the procedure itself on pg. 7. In the suggested new runs above, the model’s basal frozen areas will prevent the beta-adjustment procedure from fully reducing the metric $dH$ to zero (and $dV$). This can be
assessed in the new results.

The importance of basal temperature is explicitly presented in the description of the method (step 1): “Owing to its design, the method is only able to correct for the ice thickness mismatch where sliding occurs, i.e. where the base of the ice sheet is at the pressure melting point.”

It is also fully discussed in the results section (Sec. 4.2), when showing the results for the different enhancement factors.

With simple adjustment procedures (as here, and in PDC12), there is a valid concern that the problem is under-determined, i.e., there are more adjustable parameters than observed constraints, so errors due to one parameter may cancel errors in another parameter or in the model physics. Multiple combinations of $E_f$ and $\beta(x,y)$ can produce the correct ice thickness $H$ at a given point, and this is compounded by possible errors in model ice temperatures, both basal and internal (which affect ice rheology). One alternative for this study would be to fix all ice temperatures at some best-fit or at least modern spun-up state. That would (i) reduce total integration times for the procedure because of slowly varying ice temperatures, and (ii) somewhat alleviate concerns of under-determinedness.

In the experiments presented in this revised paper, the temperature equilibrium is done only once, using a fixed topography. For this kind of simulation, the time step can be greater than that used for a free-evolving simulation because the mass conservation equation is not solved. As a result the temperature equilibrium computation is not particularly computationally expensive. During the iterations, the temperature is allowed to evolve though it could have indeed been fixed. However, because the simulations are not very long we do not think that this would have changed significantly the minimisation results.

On a related matter, we acknowledge that our simulated temperature at the end of our fixed topography spin-up does not necessarily perfectly match the observations. Tuning the initial ice temperature is not an easy task because of the limited existing constraints (which mostly consist in basal temperature) and because of various degrees of freedom for such a tuning (paleo temperature, ice flow parameters and geothermal heat flux). It is true nonetheless that if our confidence in the simulated temperature field was increased, the under-determinedness aspect of the minimisation procedure would be reduced, it would not disappear. In Section 6, we added a discussion related to the uncertainty associated with the GRLS thermal state:

“[…] the overall performance of the method is critically dependent on the basal thermal state and points out that the finding of appropriate initial conditions with a simple adjustment procedure remains an undetermined issue. Actually, multiple combinations of the enhancement factor and the basal drag coefficient can produce a simulated ice thickness close the observed one, but this cannot discard the possibility of errors in modelled basal and vertical temperatures. However, we have shown that our minimisation procedure is able to reduce the ice thickness mismatch regardless of the initial temperature profile. This offers the possibility to tune the thermal state to be as close as possible to the observations (inferred basal temperature as in MacGregor et al.
(2016), or vertical profiles at ice core locations) before running the iterative minimisation procedure. Increasing our confidence in the vertical temperature profile would therefore increase our confidence in the choice of Ef and β values”.

Another possible way to improve the underdetermined aspects would be to quantitatively compare with observed surface velocities (as done qualitatively in Fig. 8 and pg. 12, lines 12-17, see comment below), and somehow combine that comparison automatically into the adjustment procedure for β(x,y) and Ef. This is just a suggestion for future work (not for this paper!), and connections could be made with other optimization techniques that fit to observed velocities (pg. 2, lines 24-25). Another step for future work could be to add a regularization term for β(x,y) (Pattyn, The Cryo, 2017).

These two aspects are now fully discussed in the discussion section (Section 6). In particular, we suggest the possibility of including an additional metric related to surface ice velocities:

“Finally, we have shown in this paper that the iterative adjustment of β produces modelled surface velocities that compare well with the observed ones. This suggests that future work could include an additional metric related to surface ice velocities so as to further reduce the uncertainties associated with the choice of model parameters and variables”.

Concerning the regularization term, please see our response to your comment referred to as p11, Fig.7.

Other specific comments:

pg. 2, line 10, regarding "Three main classes of initialization techniques have been developed:". Some of the text on this page blurs the distinction between initial conditions (model variables at start of integration) and boundary conditions (externally prescribed quantities).

We have substantially reshaped the text here and we are now more specific on initial conditions with respect to boundary conditions. We clarify what the initialisation procedure for ice sheet model is at the beginning of this paragraph:

“Reliable simulations of the GrIS require a proper ice sheet model initialisation procedure to avoid an unphysical model drift which can be caused by inconsistencies between the ice-sheet model initial conditions and the boundary conditions (external forcing fields). These initialisation procedures consist in finding the initial physical state of the ice sheet (such as the internal temperature), the model parameters, and sometimes the boundary conditions, that best reproduce the observations with a minimal model drift.”

Techniques #1 and #2 discussed on this page are intrinsically concerned with initialization, but I would argue that beta is a boundary condition, and procedures to adjust it are a distinct type from #1 and #2. (For instance, #3 could first be used to produce a map of beta, and then #1 or #2 could be used with that map to produce an initial model state).

We agree with this comment. This has also been pointed out by S. Price (referee) and we acknowledge that the initial version was not clear. The aim of the initialisation procedure
is to find: the physical state of the ice sheet and the model parameter and/or the boundary conditions that reproduce the observations and allow for a minimal model drift for prognostic experiments. The three methods discussed in the first version of the paper aim at answering this but they are not mutually exclusive. This part has been substantially rewritten with clarity in mind.

pg. 2, line 30, or elsewhere: Note that, as well as PDC12, Pattyn (The Cryo., 2017) applied the method in his Antarctic model, using it both with Weertman sliding (as here) and Coulomb friction laws. Also note that linear sliding (n=1, Eq. 2 here) is not a requirement, and the procedure can be applied essentially as is to non-linear sliding (n>=2), as in the above papers).

Thank you for this information. We have thus added reference to Pattyn (2017) and specified the possibilities of applying the method using both linear or non-linear sliding laws: “Here, we present a new iterative minimisation procedure that relies on the same basic principles as those developed by Pollard and DeConto (2012) (referred to as PDC12 in the following) and applied by Pattyn (2017) for the Antarctic ice sheet using linear and non-linear sliding laws.”

pg. 2, line 29-30. The preceding text on this page mentions disadvantages of methods # 1 and 2. Disadvantages of the simple inverse method could also be mentioned here:
A) there are (probably) cancelling errors in the model physics hidden by errors in the basal coefficient map, and
B) the method as in sections 3 and 4 cannot fix ice thickness errors where the bed is frozen.

We agree with this. We added: “However, methods that choose to invert the basal drag coefficient only are not able to correct ice thickness errors in regions where there is no sliding (i.e. where bed is frozen). Moreover, while inverse methods are designed to produce an ice sheet state close to observations, the inferred basal drag coefficient may cancel errors coming from erroneous simulated basal temperatures and/or model physics shortcomings. Yet, as outlined by Pollard and DeConto (2012), the risk of cancelling errors is of lesser importance compared to those related to inconsistencies between internal conditions and surface properties that will likely to be considerably reduced with expected future improvements in ice-sheet models and better observations of basal conditions”.

pg. 4, line 5: In most places, beta is appropriately called a "basal drag coefficient", i.e., larger for stickier beds, smaller for slipperier beds. Here it is called a "basal sliding coefficient" which suggests the opposite sign. To help readers, check that "drag" is used throughout.

As recommended, we now call β the “basal drag coefficient” throughout the revised paper.

Pg. 7, Eq. 5: ... + U˜sli is in error, I think, should be ... + U˜def.
Thanks for noticing, the error is now corrected.

pg. 7, Eqs. 3-7, and Fig. 4: After careful reading, I think I understand the procedure details, but am not sure. First, it would help to state earlier whether NBiter, NByear and NBcycle are years, or number of iterations (on pg. 7 around line 18; it is done at top of pg. 8, but earlier would help). As a suggestion, a numbered list of sentences might help to communicate the procedure, something like:

1) Eqs. 3-7 are applied at the end of every model timestep, adjusting beta iteratively for the next timestep. The model is run in this way through NBiter years.

2) The model is then run in "free" mode, i.e., with beta unchanged from its state at the end of (1), through NByear years.

3) Steps (1) and (2) are repeated NBcycle times.

4) Finally the model is run for an additional 200 years in "free" mode with beta unchanged.

I am not sure if all the above is correct, especially step (4). Possibly the extra 200 years is run after every cycle of (1) and (2), i.e., as part of every NBcycle cycle. That seems to be implied by Fig. 4, because the upper black arrow for the NByear cycle includes everything including the 200-year integration. But if that were the case, it would be puzzling because it would be the same as tacking 200 years onto every NByear integration (my step (2)), i.e., just increasing the value of NByear by 200 and having no final step (4).

We acknowledge that the description of the procedure was not clear. Actually, it is based on points (1) to (3) you mention. We have substantially rewritten the description of the minimisation procedure with clarity in mind. In particular we have also added a bullet-point summary as you suggested. We have also modified the schematic representation of the iterative procedure.

pg. 7, Eqs. 6 and 7: What if Ucorr'sli in Eq. 6 is zero or negative, and so yields infinite or negative beta’s in Eq. 7? Physically this would occur when the internal deformation velocity alone is greater than the required total velocity, so the sliding velocity would have to be negative. This is presumably handled by imposing maximum limits on beta, as mentioned later on pg. 15, line 7 (occurring in the Greenland interior where Ef is too high). It would help to describe the use of maximum (and minimum?) limits on beta in section 3 as part of the procedure.

You are right, we effectively put limits on the value of the basal drag coefficient (from 1 to 5 \(10^5\) Pa yr m\(^{-1}\)). We added this precision in the revised manuscript: “It should be noted that \(U_{corr}^{sliv}\) can be lower or equal to 0, leading to infinite or negative basal drag coefficient. This can happen when the velocity due to vertical shearing \(U_{def}\) is greater or equal to \(U_{corr}\). In this case we artificially impose a no-slip condition by assigning to the basal drag coefficient a maximum value set to 5 \(10^5\) Pa yr m\(^{-1}\). On the other hand, in case of too small \(U_{def}\) velocity, \(\beta\) may be as low as 1 Pa yr m\(^{-1}\) to facilitate ice sliding”.

pg. 10, Fig. 6, and pg. 11 line 10 to top of pg. 12. In my opinion the ice volume trend \(dV/dt\)
is not fundamental. In the new suggested runs with lower Ef values (see main point above),
I think the convergence of dV and dV/dt towards zero would be smooth, and the size of
dV/dt would just indicate how far along (how many NBcycle’s) the procedure has been
run. If that is true (bearing in mind the caveat related to basal frozen areas above), then
the final dV/dt can be made as small as needed simply by continuing the procedure longer
(for instance to provide a near-equilibrated initial ice-sheet model state for subsequent
experiments).

The problem with dV/dt is that there are compensatory biases that can lead to a near zero
dV/dt while the ice sheet is far from equilibrium. You are right nonetheless: the longer
the model runs, the smaller dV/dt is. However, the initial condition for the different cycles
is systematically the same, only the initial basal drag coefficient for step 1 is different. As
such, considering more cycles does not mean necessarily getting closer to the ice sheet
equilibrium and the critical duration for convergence is only defined by N^{b_{inv}} \cdot N^{b_{free}}.

In the revised version of the manuscript, the total ice volume is no longer considered as a
criterion of the method performance, and its evolution for the different enhancement
factors is only discussed to introduce the idea of compensatory biases. To circumvent the
problem of compensatory biases and, to assess the model drift, we compute a new metric
(instead of dV/dt in the initial version of the manuscript) defined as the root mean square
ice thickness change:

\[ \xi(t) = \left( \langle (H(t) - H(t-1))^2 \rangle \right)^{1/2} \]

pg. 12, lines 12-17. Regarding Fig. 8, it might be worth pointing out that if ice thicknesses
are correct, and if the surface mass balance is realistic, then for an ice sheet in equilibrium,
total velocities must be correct. So a comparison with surface velocities is, in principle, just
a test of the model’s split between total and surface velocities.

We agree with this comment. This is now explicitly mentioned in the description of the
method (Sec. 3) and when presenting the ability of the model to simulate realistic ice
velocity for different enhancement factor (Sec. 4.2.3).

Section 3: “Our method does not use the observed surface velocity as a constraint. However,
at the end of the minimisation procedure (e.g. minimal thickness error and
minimal drift), the simulated velocity tends nonetheless to approximate the balance velocity, that is the depth-averaged velocity required to maintain the steady-state of the
ice sheet”.

Section 4.2.3: “Our iterative minimisation procedure aims at simulating an ice thickness
as close as possible to observations. Hence, the observed ice velocity is not used as a target
by the model. However, because our procedure generates an ice sheet at quasi-
equilibrium (trend \( \xi \) close to 0), the simulated velocities are close to the balance velocities,
which in turn are supposedly close to present-day observations”.

pg. 11, Fig. 7: The narrow (red) bands with too thick ice around southern and central
margins, where flow is in deep valleys and fjords through coastal mountains, are similar
to errors in PDC12 over the Transantarctics. The discussion there about under- resolved
bed temperatures may be relevant here, and a modified Tb based on sub-grid bed roughness may be a possible solution. (Related discussion is on pg. 17, lines 30-33).

This issue has been addressed in the Discussion section (see Section 6):

“Another limitation of the method may come from the model resolution. The succession of higher/lower ice thickness due to the succession of valleys/ridges in mountain areas may be poorly resolved. Owing to the insulation effect of the ice, this may lead to an erroneous representation of the basal temperature patterns, and SSA regions may be erroneously interpreted as frozen bed regions and vice versa (Pattyn, 2010). This drawback is clearly illustrated in our study in Figure 6 (Ef=1). Indeed, the simulated ice thickness obtained with the inversion procedure is generally less than 50 m in most GrIS areas, but can be greater than several hundred meters in coastal mountain ranges such the central eastern margin area where ice flow occurs in deep valleys. An alternative solution consists in correcting the basal temperature to account for bedrock roughness and, similarly to what was done in PDC12 to improve their inversion procedure in the Transantarctics”.

pg. 5, line 7: Maule et al. (2005) has geothermal heat flux maps only for Antarctica, not for Greenland, I think.

For the SEARISE project a geothermal heat flux for Greenland was provided by Mike Purucker (co-author of the Fox Maule et al. (2005)) and colleagues. Because it has remained unpublished, they recommended at the time to cite Fox Maule et al. (2005) when using this data. Here is the link to the data:

pg. 5, line 10: Should be Fig. 3a, not Fig. 2a.

You are right, although Fig. 2 is now the one in which we show the basal drag coefficient, so it actually is Fig. 2a in the revised manuscript.

pg. 6, Fig. 3a. Just for interest, where do the finely spaced N-S lineations in basal drag coefficients in western Greenland come from, in the GRISL1 ice2Sea simulations?

We did not investigate specifically this. In fact, these lineations are present in all our inversion results, even if they are sometimes less visible. We guess that it could be an artefact related to the interpolation of the original ice thickness from Bamber et al. (2013) to the GRISLI grid at 5km.

pg. 7, line 19: change to "let the model freely evolve".

This has been rephrased as: “The second step consists in running a new free-evolving simulation but this time using a time constant (but spatially varying) basal drag coefficient, i.e. the last inferred basal drag coefficient of the first step”.

pg. 9, Fig. 5: To be consistent with pg. 8, line 3, the labels in the key in the top right hand corner should be "NBiter^20 - NByear^50", "NBiter_20 - NByear_100", etc., (where ^ means superscript). Same for Figs. 6 and 7. Also, for consistency throughout, use either NB... or
We no longer use this notation in the revised version of the manuscript.

*pg. 9, line 11:* \( \sim 10000 \text{ Gt} \): It looks more like \(-12000\) to \(-13000\) in Fig. 6a.

This number no longer appears in the revised manuscript.

*pg. 10, Fig. 6 caption:* It seems a bit confusing to have total volume in Gt, and total ice volume trend in mm yr\(^{-1}\). (Presumably the latter is an average over all ice surfaces). It may be clearer to have the latter in Gt yr\(^{-1}\).

As mentioned earlier, we no longer present the trend in ice volume. Our new metric, the root mean square ice thickness change, is expressed in cm yr\(^{-1}\).

*pg. 11, Fig. 7 caption, last line:* \( N_{bcyle}^4 \) should be \( N_{bcyle}^5 \) or \( N_{bcyle}^7 \), I think, from Fig. 5.

True. This figure does no longer appear in the revised manuscript though.

*pg. 14, Fig. 8 caption:* Is there a reference for this RADARSAT surface ice velocity map?

In the first version of the paper, we used the surface ice velocity map from Joughin at al. (2010). This dataset has been updated in the revised manuscript and we now use data taken from Joughin et al. (2018). This reference has been added in the Fig. 10 caption (former Fig. 8).

*pg. 16, line 4:* Change to "allows us to...", or "allows the deformation to decrease and thus..."

This sentence has been moved to Section 2 in the description of the GRISLI model when introducing the role of the enhancement factor. It has been changed to: “Lower enhancement factors lead to lower deformation rates and as such to slower ice velocities”.

*Nd...*
We would like to thank the reviewer Stephen Price for the evaluation of our study. Please find below the reviewer’s comments in black font and the author’s response in blue font.

Responses to Stephen Price (Referee #2)

SUMMARY
This paper presents a detailed study of a proposed method for providing optimized initial conditions for ice sheet models. The method attempts to formalize ad hoc approaches proposed and applied in a number of previous studies. Because the method does not use a formal PDE constrained optimization framework (hence the description as “ad hoc”), it can be expected to be applicable to, and potentially used by, a wider range of ice sheet models (e.g., adjoint-based methods are not required for calculating gradients and minimizing cost functions).
In the manuscript, the authors do a generally good job of 1) carefully explaining the method (although some confusions remain in parts – see below), 2) interpreting how and why the method works, 3) demonstrating the overall success of the method as applied to a realistic Greenland ice sheet application, and 4) exploring the sensitivity to various aspects of the method. Overall, the method shows promising results and the authors are honest about its shortcomings.
While I have some possibly significant points for the authors to consider and address in revision (noted below in more detail), overall this paper is interesting, well written, presents significant and useful findings, and clearly falls within the scope of GMD.
Thank you for your positive evaluation. We hope that we address your concerns in the following.

MAJOR COMMENTS
Where applicable, page and line numbers in comments below are referred to as “x, y:“, where x = page number and y = line number.
1,11: “spin-up parameters” – this terminology, “spin-up” and “parameters”, is confusing, and used throughout the paper. “Spin-up” is first referred to as an existing, standard method for initializing and ice sheet model (on p.2), then later it is used interchangeably to describe the new method described here. I think the two should be clearly distinguished throughout the paper. Similarly, “parameters”, unless clearly distinguished, are generally going to be thought of as belonging to the dynamic ice sheet model (e.g., the sliding coefficient is often referred to as a tunable “parameter”). The method proposed here is really more of a nested iteration, and some coefficients used to specify the number of iterations that take place in each loop (more comments on this below). Starting on p. 4, section 3, it seems like it might make sense to refer to this as something other than a “spin-
up” method, which has historical associations with your “free spin-up” description. Call it an iterative minimization, or something like that?

We agree that the terminology used to describe the method in the initial version of the manuscript was confusing. In the revised manuscript we use “spin-up” only for the long-term free evolving simulations as in Goelzer et al. (2018). Following your suggestion, we referred to our method as iterative minimisation procedure or minimisation procedure.

We still use the term “parameter” to refer to the coefficients of the model but following your advice, we systematically distinguish between ice-sheet model parameters and minimisation procedure parameters.

There was also some possible confusion with the terminology for the different parameters used in our procedure. $\text{Nb}_{\text{iter}}$ represents the duration of the period during which we compute the basal drag coefficient. During this period, the basal drag coefficient is updated at each model time step (i.e. one year in our case, specified in the revised version of the manuscript). The term “iter” for this parameter is misleading as this step corresponds to a unique continuous simulation without iterating/looping back to a previous state of the model. For this reason, we changed $\text{Nb}_{\text{iter}}$ to $\text{Nb}_{\text{inv}}$ in the revised version. For sake of clarity, $\text{Nb}_{\text{year}}$ is now referred as $\text{Nb}_{\text{free}}$, as it corresponds to the duration of the free-evolving simulation performed within the 2nd step of the procedure (see Section 3).

2,10-30: Here, methods 2 and 3 are discussed as distinct from one another. But in reality, does anyone ever do just 2, or do 3 without doing 2 first? It seems like these are most often combined into a single method: use a fixed topography to spin-up the temperature (and maybe also the velocity field, so that the temperature and velocity are internally consistent), and then use that temperature field along with an inverse method to calculate velocities that better match observations.

We agree with your comment. This has also been pointed out by D. Pollard (referee 1) and we acknowledge that the initial version was not clear. The aim of the initialisation procedure is to find: the physical state of the ice sheet and the model parameter and/or the boundary conditions that reproduce the observations and allow for a minimal model drift for prognostic experiments. The three methods discussed here aim at answering this but they are not mutually exclusive. This part has been substantially rewritten with clarity in mind (From P2 L18 to P3 L15).

4, section 3: Somewhere in here, you might discuss or mention the work of Perego et al. (2014, JGR Earth Surf., 119, p.1894), which has very similar overall goals to that discussed here, but using a formal minimization framework (e.g., your Figure 2b is analogous to their Figure 1, although the timescales are different).

Thank you for mentioning this omission. We now mention the study of Perego et al. (2014) in the introduction and in Section 3:

“While numerous studies are based on fitting the modelled ice velocities (e.g., Gudmundsson and Raymond, 2008; Arthern and Gudmundsson, 2010; Morlighem et al., 2010; Gillet-Chaulet et al., 2012; Perego et al., 2014), or both surface velocities and basal topography (Perego et al., 2014; Mosbeux et al., 2016), only few authors opted for fitting ice surface elevation (Pollard
and DeConto, 2012; Pattyn, 2017). Here, we decided to adjust the basal sliding velocities via the adjustment of the $\beta$ coefficient to fit the GrIS ice thickness to the observed one. Similarly to Perego et al. (2014), our choice is motivated by the need to refine the estimates of GrIS contribution to future sea-level rise without the sea-level rise signal being contaminated by unphysical transients from the initial condition. However, while Perego et al. (2014) adopted a formal minimisation approach (i.e. adjoint-based model) we suggest instead an ad hoc method potentially applicable to any ice sheet model.”

6, 4-5: “…performance in terms of trend and error in simulated ice volume compared to observations”. While you do somewhat address the mismatch between observed velocities and/or ice flux later in the paper, I think it would make more sense to bring it up here. Or even earlier, when you first discuss the metrics you are going to use here. I kept wanting to see some discussion on that and felt like it was being ignored. It would have helped if you had stated early on that you were going to look at this topic later on in the paper.

Our method is based on fitting the simulated ice thickness to the observation while the observed velocity is not used to constrain our results. At the end of the minimisation procedure (minimal thickness error and minimal model drift), the simulated velocities are close to the balance velocities, which are, in turn, expected to be close to the observed velocities. In the revised manuscript, this point is mentioned in Sec. 3 at the end of the description of the minimisation procedure:

“In the following, we also discuss the spatial patterns of ice thickness and ice velocity mismatches with respect to observations. Our method does not use the observed surface velocity as a constraint. However, at the end of the minimisation procedure (e.g. minimal thickness error and minimal drift), the simulated velocity tends nonetheless to approximate the balance velocity, that is the depth-averaged velocity required to maintain the steady-state of the ice sheet”.

We also dedicate a section on the simulated velocities for a range of enhancement factors in the revised manuscript (Sec. 4.2.2.c).

6, Figure 4: I found this figure a bit confusing. A couple of ways that might help to improve it include 1) tying it to the discussion in the text more clearly (and vice versa – refer to the steps in the figure when you are describing them in the text) and, 2) drawing it as a set of nested loops instead of a left-to-right flow chart. It seems to me like what you describe is two back-to-back loops (Nb_iter followed by Nb_year) that both sit inside of a larger, outer loop (Nb_cycle). A different figure might capture that better (it could still include parts of what you have here).

We have completely redesigned the schematic representation of the method (Fig. 3 in the revised manuscript). Compared to the previous version, the figure is largely simplified. It still consists mostly of a left-to-right flow chart because there is a temporal continuity between the different steps: the results of the basal drag coefficient computation (step 1) feed the free-evolving simulation (step 2). However, the outer loop in which the two steps are nested appears now more clearly. We also specifically refer to this schematic representation when needed in the description of the procedure.
7, steps 1 and 2: Note that what you describe here in steps 1 and 2 is essentially identical to the iteration described in Price et al. (2011; PNAS, 108(22) – see “methods” and SI for more details), except that they are using observed and modeled velocities rather than observed and modeled ice thickness to adjust the sliding coefficient. Also, it took me a while to figure out exactly what “Nb_iter” was. It’s not immediately clear why this is >1 (i.e., what are you iterating on?). Eventually, I guessed that you are allowing the new sliding coeff. and the model velocities to come into some sort of equilib. with one another. If that is true, you should state it explicitly!

It is true that the assumptions made to report the modification of the sliding velocity to the basal drag coefficient is essentially similar to those of Price et al. (2011). This is now acknowledged in the description of the method. However, in addition to the differences you mention, Price et al. (2011) also maintain a fixed geometry, which is not the case here. The fact that we systematically have a free-evolving ice elevation is now clearly stated in the revised version of the manuscript to avoid any confusion.

Nbiter (now Nbinv) is the duration of the period during which the basal drag coefficient is computed. It does not involve any iteration as it is simply a free-evolving simulation for which the basal drag is updated at each model time step. This is now better explained in the revised paper.

Figures 5 and 6: The labeling of the legend should be changed here to “Nb_year” rather than “Nb_iter”. It’s too easy to confuse what you are varying here as currently labeled. It takes careful reading to understand that Nb_iter is actually held fixed while you vary Nb_year. You could use Nb_year instead and just mention in caption that the value of Nb_iter is the same for all.

This notation is no longer used in the revised manuscript and the sensitivity to Nbfree (former Nb_year) and Nbinv (former NbIter) is assessed in a dedicated section (Sec. 5.3).

End of p.9 to start of p.11 – It took me a few readings to understand the explanation here. I think it could be written a bit more clearly. The point is that the volume metric needs to be used carefully because it cannot discern compensating errors (overall too thin in the interior and too thick at the margins cancels out and looks like a good match), and thus one either needs to look at the spatial pattern of thickness errors or include some other metrics.

This was indeed the idea behind this section. However, we now discuss this point when presenting the results for a range of enhancement factors. In doing so, the compensating errors appear more clearly as we show 2D maps of ice thickness mismatch. We would also like to draw your attention to the fact that the ice volume, as well as ice volume trend, are no longer used as metrics in the revised manuscript. This avoids artefacts related to compensating errors. Rather, we use the ice thickness root mean square error and the ice thickness changes root mean square error. The latter is a metric of the drift of geometry and is defined as (see Sec. 4.2.2.b):

\[ \xi(t) = \sqrt{\langle (H(t) - H(t-1))^2 \rangle} \]

12, 12-17: This discussion of the model fit to observed velocities is appreciated. I think it
would make sense to mention much earlier in the paper that you are going to look at this. The lack of discussion of the importance of getting both the thickness AND velocity state and trends correct (and hence the flux correct) early on in the paper made me wonder how useful the method could be. At the same time, while the fit to observed vels looks good by eye, I think it would be appropriate to give a slightly more quantitative measure for how well the final initial condition matches observed velocities (e.g., RSME of speed). I don’t think a relatively poorer match to the velocities (relative to the thickness) really speaks poorly of the method as there are times when having a near steady-state initial condition might be more important than matching the velocities better. But overall, it would be good to know how easily a good match to velocities follows a good match to the thickness / volume.

As mentioned above (see our response to your comment referred to as 6, 4-5), we added the following at the end of the method description (Sec. 3): “Our method does not use the observed surface velocity as a constraint. However, at the end of the minimization procedure (e.g., minimal thickness error and minimal drift), the simulated velocity tends nonetheless to approximate the balance velocity, that is the depth-averaged velocity required to maintain the steady-state of the ice sheet”

We agree on the fact that a discussion about the ice velocity RMSE could have been included. However, from our experience, this would have been not very informative because of two main reasons:

- i) Ice velocities are highly spatially variable and present their maximum values at the ice sheet margins. This means that small errors in the simulated extent of the ice sheet lead to important discrepancies with observations. As such, marginal regions, which represent a small fraction of the ice sheet, have more weight for metrics such as the RMSE.
- ii) The ice streams have generally a very fine structure (~100 m), and the aggregation of this fast moving ice with neighboring slow moving ice is not necessarily meaningful at 5 km resolution.

We have nonetheless computed the RMSE of velocity for the different enhancement factors considered in this revised version. The evolution of the ice velocity RMSE as a function of the number of iterative cycles (Nb_cycle) is shown in the Supplementary Material (Fig. Supp. Mat. 1). This figure confirms the conclusions drawn from the 2D maps (Fig. 11): for large Ef values, the agreement with observations is poorer than for low Ef values. In addition, performing more cycles does not improve the RMSE. This conclusion is valuable for both ice thickness and ice velocities.

Section 4.2.4: Do you have any physical explanation for the lack of sensitivity to the value of Nb_iter, or why Nb_iter is better at smaller values?

Nb_inv (former Nb_iter) does play a similar role to Nb_free (former Nb_year) on the computed RMSE: a longer Nb_inv leads to a smaller RMSE. In the original version of the manuscript, we discarded the simulations with large Nb_inv because the volume difference w.r.t. observations was larger than for small Nb_inv. This was due to the use of an enhancement factor of 3 leading to too high deformation-driven velocities and thus to negative ice thickness biases in the interior of the ice sheet. We fully discussed this in the revised manuscript. Nb_inv has nonetheless a smaller impact than Nb_free, probably because of the
chosen values (\(N_{\text{free}}\) varies from 50 to 400 years while \(N_{\text{inv}}\) varies from 20 to 160 years) and also because of a greater change induced in \(\mathbf{U}_{\text{corr}}\) at each iteration for large \(N_{\text{free}}\) values.

*Figure 8:* I am actually quite surprised to see that this method somehow “gets” the NEGIS in the modeled velocity field. Can you confirm if this is still the case when you start the iteration from a uniform value of beta? It seems like it would be very hard for the iteration to form this subtle feature in the model without some direct connection between the sliding coefficient and the velocity field (the topography is too subtle and it doesn’t seem like the metrics being used could possibly discern the necessary variations in the sliding coefficient based on the subtle changes in ice thickness). I’m curious if it is somehow a “relict” feature that exists primarily because of the initial sliding coefficient field you started with (which, for ice2sea, may have been tuned somehow to reproduce the NEGIS).

Having a good representation of the NEGIS could indeed be a reminiscence of the initial 3D fields as the 30,000-yr temperature equilibrium has been computed using the Ice2Sea basal drag coefficient, which is itself derived from the inversion of ice velocities. However, it seems to be a robust feature of the minimisation procedure since the NEGIS is well reproduced even when starting from a homogeneous basal drag coefficient. We have added this discussion in the revised manuscript (Sec. 4.2.3):

“Interestingly, the extent of the NEGIS is particularly well represented, in particular for lower enhancement factors (Fig. Supp. Mat. 2). This can be a relic of the long temperature equilibrium performed with a time constant basal drag coefficient taken from Ice2Sea experiments (Edward et al., 2014), in which the NEGIS is well delimited (Fig. 2a). However, because this feature is still present when starting the iterations from a spatially homogeneous basal drag coefficient (see Sec. 5.2), it can also suggests that there is some topographic control of this feature as the adjustment of our local basal drag coefficient is very effective in reproducing the observed velocity in this area. Having a good representation of the NEGIS is an encouraging sign for the performance of our minimisation procedure, especially since most models fail to achieve this (Goelzer et al., 2018).”

16, 5.2: I was also glad to see this section, as it seemed like a logical next step given the limitations of the method for adjusting the ice speed and ice thickness in the interior. However, I was expecting at least maybe the suggestion that one could combine the method of tuning the sliding coefficient with a similar method for tuning \(E_f\) where the ice was determined to be frozen to the bed. It seems like the exact same method could be used to iterate on the value of \(E_f\) that is used to iterate on the value of the sliding coefficient. Have the authors thought of trying this? It seems relevant to at least speculate on, or comment on as a logical next step.

For the revised manuscript, we did not use an iterative method (similar than that applied to the basal drag coefficient) to adjust the enhancement factor, but we performed the minimisation procedure for various values of the enhancement factor ranging from 0.5 to 5 to examine the impact on deformation rates. The results are now presented in Section
4 (instead of Section 5 as in the initial version) and are also discussed in terms of basal thermal state (thawed vs frozen bed areas, see Section 4.2.2). Moreover, we have also addressed this point in the Discussion section (see Sect. 6).

17, 5-9: It would be interesting to see a 1:1 plot of the sliding coefficient values for the two different initial conditions. This would be a nice visual way of convincing the reader that there really is little sensitivity to the initial value of the sliding coefficient. As noted above, it would be very nice to see a comment here on whether or not the NEGIS is still an “emergent” feature when starting from a uniform sliding coefficient.

Such a figure is shown below (Fig. 1). It confirms that the final adjusted basal drag coefficients (obtained when starting from Ice2Sea and from $\beta=1$) are quite similar despite persisting local differences that make the plot to appear noisy. Note that the ice thickness RMSE and the ice thickness trend obtained with both initial basal drag coefficient are almost identical. Moreover the ice thickness and surface velocity differences remain very small (see Fig. S3b and Fig. S4b). These results have been presented in Section 5.2.:

“Using $\text{Nb}_{\text{inv}}=20$, $\text{Nb}_{\text{free}}=200$, and $\text{Nb}_{\text{cycle}}$ varying from 1 to 15 with $\text{Ef}=1$, we obtain a minimum ice thickness RMSE of 49.9 m and a trend $\xi$ of 15.1 cm yr$^{-1}$. While there are some minor spatial differences in terms of the inferred basal drag coefficient (Fig. 2c), the aggregated metric such as the RMSE and the trend are identical to the results presented in Tab. 1. In the same way, the simulated ice thickness and surface velocities obtained with $\beta = 1$ present very small differences with those obtained when starting from the Ice2Sea basal drag coefficient (Figs S3 and S4). This illustrates the robustness of the method and shows that it does not depend on the chosen initial distribution of the basal drag coefficient”.

Fig. S4 also shows that the NEGIS ice velocities differences are negligible, despite slightly higher in the $\beta = 1$ case, demonstrating that the NEGIS is still an emergent feature.
Figure 1: Basal drag coefficient ($\beta$) 1:1 scatter plot between uniform $\beta = 1$ and $\beta$ from Ice2Sea (Edwards et al., 2014) in Log10 Pa yr m$^{-1}$.

Summary and Conclusions:

There is the suggestion here that the method could work better at higher resolution. However, I don’t think this will actually be the case. This is because this method can only adjust the value of the sliding coefficient point-by-point; each grid point is adjusted independently of every other one. Once you get down to a grid spacing of a few ice thicknesses or less, this will cease to work very well, because the change in sliding coefficient at one grid point will lead to changes in ice speed at that point AND at neighboring points, via horizontal stress gradients. When this happens, the iteration ceases to make further improvements because it doesn’t have a way to avoid the “noise” that local adjustments cause at neighboring points (I have some experience with this problem, based on the similar iteration described in Price et al. (2011; PNAS paper prev. referenced). This is one reason that, at high resolution, it starts to become difficult to use ad hoc methods like this for very precise tuning and one may need to turn to more formal optimization methods.

Thank you for this comment. We addressed this issue in the Discussion section (Sec. 6):

“[...higher resolution models can also better account for the dynamics of small-scale outlet glaciers and for their interactions with floating ice that strongly influence the ice-sheet mass balance (e.g., Aschwanden et al., 2016). However, due to the elliptic character of the SSA equation (e.g., Quiquet et al. 2018), the local adjustment of the basal drag coefficient impact the ice velocity of neighbouring points. As a result increased resolution may increase the noise,
unless introducing a smoothing function that filters the high frequency noise (Pattyn, 2017)."

Some speculation on future directions would be appreciated. For example, could you also include a metric on ice velocity, so that your iteration was scored by the weighted mean of the fit to thickness AND the velocity? This would also be a good place to speculate on iterating on the value of Ef in areas where the bed is frozen.

These comments have also been raised by D. Pollard (Referee 1). In Section 6, we now suggest the possibility of including an additional metric on surface ice velocity:

“Finally, we have shown in this paper that the iterative adjustment of \( \beta \) produces modelled surface velocities that compare well with the observed ones. This suggests that future work could include an additional metric related to surface ice velocities so as to further reduce the uncertainties associated with the choice of model parameters and variables”.

Moreover, we have changed the structure of Section 4 and 5 in the revised manuscript, and we now investigate the impact of the enhancement factor for a wide range of values (from 0.5 to 5). Corresponding results are presented in Section 4.2.

**MINOR COMMENTS**

1, 6: “to infer reliable initial conditions of the ice sheet”. This is not really true. Most inverse methods applied to ice sheet models currently only really “work” well if you are only interested in a snapshot of the ice sheet velocity. Without other considerations, you might get a model snapshot that does a great job of mimicking observed velocities, but it will likely suffer very badly from the problem you aim to address here (that is, large, unphysical transients).

The abstract has been considerably modified to match with the new structure and content of the paper. The point you raise here has been addressed by including the following sentence in the new abstract: “Most often such approaches allow for a good representation of the mean present-day state of the ice sheet but are accompanied with unphysical trends”.

1, 11: “. . . to minimize errors in sea-level projections”. This is misleading, as it’s not really one of your criteria here. We can’t know that this will minimize errors in SLR projections can we?

This part of the abstract has been completely reformulated (along with the target criteria of the minimization procedure: “The quality of the method is assessed by computing the root mean square errors in ice thickness changes”).

2, 1: Be explicit – the “unrealistic evolution” you are talking about is large, unphysical transients in ice thickness.

We changed the text for: “Reliable simulations of the GrIS require a proper ice sheet model initialization procedure to avoid unphysical model drift which can be caused by
inconsistencies between the initial conditions of the ice-sheet model and the boundary conditions (external forcing fields)"

2,5: “GrIS characteristics” -> GrIS “state”?
Changed for “GrIS current state”.

2,5: “the major source of uncertainty” -> “a major source of uncertainty”
Corrected.

2,6: the vertical temperature profile is not part of the “basal properties”, as this sentence implies (probably just poorly written).

Following your comment, we have changed the sentence as follows to avoid any confusion: “... offer only a partial description of the GrIS current state and a major source of uncertainty lies in the poor knowledge of the basal properties (e.g. water content in the sediment or basal dragging) and of the internal thermomechanical conditions (e.g. temperature and deformation profile).”

2,15-18: “significant mismatch . . . topography”. I would use “state” here instead of topography, since it is much more than just the topography (velocity, flux, etc.). For “Such spin-up methods” it seems relevant to mention why only low cost models can do this, because the spin up is order 10,000-100,000 yrs long.

We agree with you. The sentence has been changed in:
“Even if model parameters can be chosen to reduce the mismatch between modelled and observed present-day ice sheet state (e.g. topography, velocity), this approach may lead to important errors. In addition, due to the long integrations needed (>10 000-100 000 year long), such spin-up methods can only be used with low computational cost models, which are often unable to properly capture fast ice flow processes.”

2,22: “inconsistencies between . . . “. You could be more explicit here. The problem is that the modeled flux divergence is nowhere close to being balanced by the sum of the surface and basal mass balance terms.

Thanks for clarifying this point. We have now explained in the revised paper why the fixed topography spin-up method could lead to an artificial drift when the free evolving topography is restored: “In this case, because the simulated ice flux divergence is generally far from being balanced by the net mass balance (i.e. surface and basal mass balance), an artificial drift arises when free evolving topography is restored (Goelzer et al., 2013).”

3, 10: Clarify that hybrid model refers to the momentum balance?
Since the velocity computation is described later in the text we prefer to remove the reference to the fact that GRISLI combines the SIA and SSA velocities in this sentence.

3,11: “velocity fields” -> “ice dynamics” ?
Changed.
3.15: and equation 1 – clarify that $U_{\text{bar}}$ is a 2d vector field?

In the revised manuscript, we use a bold font for the vector fields.

3,20-21: Clarify that the SIA and SSA solutions are summed heuristically, and point to a reference where you describe what that heuristic is?

We changed the text for: “In the model, the velocities are computed as the heuristic sum of the SSA and the SIA components, as in Bueler and Brown (2009) but with no-weighting function (Winkelmann et al., 2011).”

3,23: “linear till” -> “linear viscous till”; note that there’s a missing assumption here (in eq. 2) about the thickness of the till layer being uniform everywhere.

Thanks, we have added this additional information: “In the model version used in this study, we assume a linear viscous till with a uniform thickness”.

3,29: What is value of $E_f$ used here?

In the initial version, we used $E_f=3$, except in Sec. 5.2. Now we run a whole range of $E_f$ values (results discussed in Sect. 4.2) to assess the importance of this parameter.

3,32: The calving criterion is not clear as written. Do you mean that everywhere floating ice is <250 m is thickness it is assumed calved?

Floating ice at the front with a thickness < 250 m is calved, yes. We rephrased as follows: “Calving physics is not explicitly computed, but if a grid point at the ice-shelf front fails at maintaining a thickness threshold, it is automatically calved (Peyaud et al., 2007). The ice thickness cut-off threshold is set to 250 m.”

4,6: “either the simulated … velocities or the ice sheet geometry” … what above both? See comment above about Perego et al. (2014) paper

We have reformulated: “[...] in order to reduce the mismatch between the simulated surface ice velocities and/or the ice-sheet geometry and the observed ones.” More details are provided in the next paragraph of the revised manuscript.

5,2: “Our choice is motivated by . . . sea-level rise.” add, “without that sea-level rise signal being contaminated by unphysical transients from the initial condition.” (or some- thing to this effect)

Added, thank you for the suggestion.

5,9: It’s not clear if you hold the temperatures fixed during the iterative process dis- cussed here.

No, the temperature is allowed to change. It is now clarified in the revised manuscript. However, because the restart conditions used are systematically the same from one iteration to another, we do not think that the change in temperature can make a big difference. We have clarified this: “[...] GRISLI is run forward (free-evolving surface elevation and temperature) starting from the present-day observed ice thickness...”

8, 5: 4.1 “is the spin-up needed” – again, suggest using something else to describe this
(“iteration”? rather than spin-up, to avoid confusion with the common understanding of spin-up.)

This terminology has been avoided. This section is now entitled “The importance of the initialisation procedure”

12, 8: “RMSE” -> “thickness RMSE”

This paragraph has been removed in the revised manuscript.

Table 1: I assume the commas are analogous to periods in the numbers listed? Is this standard? Should periods be used instead?

Sorry for this misunderstanding: the commas within numbers are the French standard for a dot. The text editor made an automatic replacement of the numerical dots by commas. We have made sure that the numbers are correctly written in the revised version of the manuscript.

The paper is reasonably well organized (aside from some suggestions noted above) and written. There are a fair number of minor edits and corrections that could be made, related to English language use. I do not point those out here explicitly but instead suggest the authors enlist a native English speaker / writer to provide a careful editing before the submission of a revised version.

We apologize for English mistakes. In the revised manuscript, we made our best to correct them.
A rapidly converging spin-up-initialisation method for to simulate the present-day Greenland ice sheet using the GRISLI ice-sheet model (version 1.3)

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Abstract. Providing reliable projections of the ice-sheet contribution to future sea-level rise has become one of the main challenges of the ice-sheet modelling community. To increase confidence in future projections, a good knowledge of the present-day state of the ice flow dynamics, which is critically dependent on basal conditions, is strongly needed. The main difficulty is tied to the scarcity of observations at the ice-bed interface at the scale of the whole ice sheet, resulting in poorly constrained parameterisations in ice-sheet models. To circumvent this drawback, inverse modelling approaches can be developed and validated against available data to infer reliable initial conditions of the ice-sheet to infer initial conditions for ice sheet models that best reproduce available data. Most often such approaches allow for a good representation of the mean present-day state of the ice sheet but are accompanied with unphysical trends. Here, we present a spin-upinitialisation method for the Greenland ice sheet using the thermo-mechanical hybrid GRISLI ice-sheet model. Our approach is based on the adjustment of the basal drag coefficient that relates the sliding velocities at the ice-bed interface to basal shear stress in unfrozen bed areas. This method relies on an iterative process in which the basal drag is periodically adjusted in such a way that the simulated ice thickness matches the observed one. The process depends on three parameters controlling the duration and the number of iterations. The best spin-up parameters are chosen according to two criteria to minimize errors in sea-level projections: the final difference between the simulated and the observed Greenland ice volume as well as the final ice volume trend which must both be as low as possible. To increase confidence in the inferred parameters, we also make sure that the final ice thickness root mean square error from the observations is not greater than a few tens of meters. Our best results are obtained after only 420 years of simulation, highlighting quality of the method is assessed by computing the root mean square errors in ice thickness changes. Because the method is based on an adjustment of the sliding velocities only, the results are discussed in terms of varying ice flow enhancement factors that control the deformation rates. We show that this factor has a strong impact on the minimisation of ice thickness errors and has to be chosen as a function of the internal thermal state of the ice sheet (e.g. a low enhancement factor for a warm ice sheet). While the method performance slightly increases with the duration of the minimisation procedure, an ice thickness RMSE of 50.3 m is obtained in only 1320
model years. This highlights a rapid convergence and demonstrating that our demonstrates that the method can be used for computationally expensive ice sheet models.

1 Introduction

Recent observations provide evidence that the rate of mass loss of the Greenland ice sheet (GrIS) is continuously increasing. Simulating the GrIS response under future warm periods is therefore crucial to establish reliable projections of future sea-level rise at decade to century time scales, but also to investigate the effects of changes on the climate system. As a result, better constraining the GrIS evolution has become a key objective of the climate and ice sheet modelling communities.

Reliable simulations of the GrIS require a proper initialisation (i.e. spin-up) procedure to avoid an unrealistic evolution of the ice sheet unphysical model drift which can be caused by inconsistencies between the ice sheet model initial conditions and the initial conditions of the ice sheet model and the boundary conditions (external forcing fields). For short-term projections (next decades to next centuries) starting from the present-day ice sheet configuration, recent observations. These initialisation procedures consist in finding the initial physical state of the ice sheet (such as the internal temperature), the model parameters, and sometimes the boundary conditions, that best reproduce the observations with a minimal model drift. Recent observations, such as surface and bedrock topographies and horizontal surface velocity offer only a partial description of the GrIS characteristics and the current state and a major source of uncertainty lies in the poor knowledge of the basal properties, such as the (e.g., water content in the sediment and basal sliding, or basal dragging) and of the vertical temperature profile internal thermomechanical conditions (e.g., temperature and deformation profile). Indeed, the basal both the basal properties and the internal conditions have a strong impact on the ice motion and thus the simulated GrIS state. Optimizing the initial conditions of ice sheet initialisation procedure of ice sheet models is therefore an active area of research and a multidisciplinary effort. The initMIP project gives a recent example of this effort. Its goal is to compare different initialisation techniques and to assess their impact on the dynamic responses of the models. Three main classes of initialisation.

The goal of ice sheet model initialisation is to infer internal properties (e.g., temperature), some boundary conditions (e.g., basal drag) and model parameter values. To this aim, different techniques have been developed—

1. The free spin-up method allows the ice sheet topography. One approach is to allow the ice sheet model to evolve freely over a long enough time (ice sheet spin-up). This approach has long been the most commonly used technique to initialise ice sheet models (and other references in )

2. The free spin-up method allows the ice sheet topography. One approach is to allow the ice sheet model to evolve freely over a long enough time (ice sheet spin-up). This approach has long been the most commonly used technique to initialise ice sheet models (and other references in )

3. The free spin-up method allows the ice sheet topography. One approach is to allow the ice sheet model to evolve freely over a long enough time (ice sheet spin-up). This approach has long been the most commonly used technique to initialise ice sheet models (and other references in )

4. The free spin-up method allows the ice sheet topography. One approach is to allow the ice sheet model to evolve freely over a long enough time (ice sheet spin-up). This approach has long been the most commonly used technique to initialise ice sheet models (and other references in )

Three main classes of initialisation:
if model parameters can be chosen to reduce the mismatch between modelled and observed present-day ice sheet topography. Such ice sheet state (e.g., topography, velocity), this approach may lead to important errors. In addition, due to the long integrations needed (> 10 000 - 100 000 year long), such spin-up methods can only be used with low computational cost models, which are often unable to properly capture fast ice flow processes.

2. The fixed-topography spin-up method is similar to the free spin-up method except that during all the simulation the ice sheet topography is kept constant and equal to its present-day observed value. To compute the internal properties, an alternative approach is to keep the topography fixed, while vertical temperature fields, and possibly velocity fields, are allowed to freely evolve (e.g., ??). The disadvantage of this method is that (e.g., ??). In this case, because the simulated ice flux divergence is generally far from being balanced by the net mass balance (i.e., surface and basal mass balance), an artificial drift may arise when free evolving topography is restored due to inconsistencies between internal and surface ice sheet fields. (?).

3. The third kind of spin-up technique is based on an inverse method of the poorly known basal conditions in such a way that simulated surface velocities match the observed surface velocities (e.g., ??). However, a second category of initialisation methods relies on data assimilation techniques, whose goal is to infer model parameters or poorly known boundary conditions, are also used to minimise the mismatch between model variables (most often surface velocities) and observations (e.g., ??). However, this approach may lead to internal inconsistencies between the simulated internal conditions (temperature and velocities) and the actual ones. The inconsistencies within the different observational datasets (or between the simulated ice velocities and the observational datasets, such as surface and bedrock topography, velocities) can also have an impact on the results. These inconsistencies may have a strong impact on results of forward simulations. To circumvent this drawback, other authors (e.g., ??) developed a multi-parameter inversion technique to optimise both the sliding velocities and the bedrock topography in such a way that the modelled surface ice velocities match with the observed ones. This allows the set of initial conditions to be self-consistent. However, if not constrained by observed ice thickness, these methods may lead to unrealistic simulated topography. An alternative approach, which avoids the previously mentioned shortcomings, consists in considering the observed ice sheet-only the observed ice sheet geometry as the final target by finding appropriate basal conditions (generally the basal drag coefficient, see Sect. 2) that minimise the differences between observed and simulated ice thickness (??). However, methods that choose to invert the basal drag coefficient only are not able to correct ice thickness errors in regions where there is no sliding (i.e., where bed is frozen). Moreover, while inverse methods are designed to produce an ice sheet state close to observations, the inferred basal drag coefficient may cancel errors coming from erroneous simulated basal temperatures and/or model physics shortcomings. Yet, as outlined by ??, the risk of cancelling errors is of lesser importance compared to those related to inconsistencies between internal conditions and surface properties that will likely to be considerably reduced with expected future improvements in ice sheet models and better observations of basal conditions.

Here, we present a spin-up approach-new iterative initialisation procedure that relies on the same basic principles as those developed by ?? (referred to as PDC12 in the following) and applied by ?? for the Antarctic ice sheet using linear and non-linear
sliding laws. Similarly to PDC12, we compute the basal drag coefficient that minimises the error in the simulated ice thickness and relates basal stresses to basal velocities. However, while PDC12 requires long (multi-millennial) integrations for the method to converge, we suggest instead an iterative method of short (decadal to centennial) integrations starting from the observed ice thickness. Our iterative method ensures a more rapid convergence and is thus suitable for computationally expensive models.

The paper is organised as follows. In section 2 Sect. 2 we present the main characteristics of the GRISLI ice-sheet model used in this study. Section 3 describes the spin-up method. Section 4 describes the iterative minimisation procedure in detail. The main results are presented in section 4 Sect. 4 and sensitivity experiments in section 5 Sect. 5. These sections are followed by a discussion and the main conclusions of the present study (Sect. 6).

2 The ice-sheet model GRISLI

The GRISLI model was first designed to describe the Antarctic ice sheet (1-2) and further adapted to the northern hemisphere ice sheets (3,4). The version used in this study has been specifically developed for Greenland (5) with an (6) with a horizontal resolution of 5 km x 5 km (301 x 561 grid points) and 21 vertical unevenly-spaced levels, with the smallest grid spacing near the ice-bedrock interface to better resolve the basal motion. GRISLI is a hybrid model accounting evenly spaced vertical levels. GRISLI accounts for the coupled behaviour of temperature and velocity fields. It relies on basic principles of mass, heat and momentum conservation. The evolution of ice-sheet geometry is a function of surface mass balance, velocity fields and bedrock altitude. Since this study only deals with present-day steady-state simulations, the module describing the isostatic adjustment is not activated here. The evolution of the ice thickness is governed by the mass balance equation:

\[
\frac{\partial H}{\partial t} = -\nabla (\overline{U}^G H) + \nabla \cdot (M \text{SMB}) - b_{melt} \tag{1}
\]

where \(H\) is the ice thickness, \(\overline{U}^G\) is the depth-averaged velocity, \(M\) (2D vector), SMB is the surface mass balance and \(b_{melt}\) is the basal melting.

The ice flow velocity is derived from a simplified formulation of the Stokes equations (i.e. the stress balance) using the shallow-ice (7) and shallow-shelf (8,9) approximations. The shallow-ice approximation (SIA) assumes that, owing to the small ratio of vertical to horizontal dimensions of the ice sheet, longitudinal stresses can be neglected with respect to vertical shearing along the steepest slope. Conversely, in the shallow-shelf approximation (SSA), the horizontal strain rates become dominant and the horizontal velocities do not vary with depth. In the model, the velocities are computed as the heuristic sum of the SSA and the SIA components, as in (10) with (11 but with no-weighting function (12). In this case, the SSA velocity is used as the sliding velocity. We assume no-slip conditions for a frozen bed (i.e. basal temperature below the melting point), and in these conditions, the SSA velocity is set to 0. In the model version used in this study, we assume a linear till viscous till with a
uniform thickness, in which the basal shear stress ($\tau_b$) and basal velocity ($u_b$) are related via the following expression:

$$\tau_b = -\beta u_b$$

where $\beta$ is the basal drag coefficient and varies with space.

To describe the effect of ice rheology, the deformation rate and stresses are related via the Glen's flow law. As in other large scale ice-sheet models, GRISLI uses a flow enhancement factor ($Ef$) in the Glen's flow law to artificially account for the impact of ice anisotropy on the deformation rate. This enhancement factor ($Ef$) typically ranges from 1 to 5, depending on the stress regime (e.g., ?). Lower enhancement factors lead to lower deformation rates and as such to slower ice velocities. The grounding line position is defined according to a flotation criterion and floating points are treated following the SSA assumption only. Calving physics is not explicitly computed, but if a grid point at the ice-shelf front position is determined for a fails at maintaining a thickness threshold, it is automatically calved (?). The ice thickness cut-off criterion of threshold is set to 250 m (?). The amount of ice obeying this criterion (ice thickness > 250 m) is computed as the calving flux.

Since GRISLI is thermo-mechanically coupled, the ice temperature influences the ice velocity via the viscosity. The temperature is computed both in the ice and in the bedrock by solving a time-dependent heat equation. The temperature signal itself depends on ice deformation, surface conditions, and on basal temperatures, hence on the temperature forcing and geothermal heat flux.

3 The spin-up method

The basic principle of inverse modelling approaches for ice-sheet spin-up is to adjust the basal sliding coefficient ($\beta$) which varies spatially, in order to reduce the mismatch between either the simulated surface ice velocities or the ice-sheet geometry and the observed ones.

Climate forcing averaged over the 1979–2014 period simulated by the atmospheric regional model MAR (?) and interpolated on the GRISLI ice-sheet model grid (5 km x 5 km). a/ Mean surface mass balance (in Gt yr$^{-1}$). The black line represents the equilibrium line indicating the frontier between accumulation and ablation areas. b/ Mean annual surface temperature (in °C).

The white dashed lines represent the 5°C isocontours.

Numerous studies are based on fitting the modelled ice velocities (e.g., ????), while (?) or both surface velocities and basal topography (??), only few authors opted for fitting ice surface elevation (??). Here, we decided to adjust the basal sliding velocities via the adjustment of the $\beta$ coefficient to fit the GrIS ice thickness to the observed one. Our choice is motivated by the need to refine the estimates of GrIS contribution to future sea-level rise without the sea-level rise signal being contaminated by unphysical transients from the initial condition. However, while ? adopted a formal minimisation approach (i.e. adjoint-based model) we suggest instead an ad hoc method potentially applicable to any ice sheet model.
The GRISLI climate forcing is provided by the, i.e., surface mass balance and the surface air temperature simulated by the state of the art (Fig. 1), is provided by the regional atmospheric model MAR (??) forced at its boundary by the ERA-Interim reanalyses (??). Both (??). Both forcing fields are averaged over the 1979-2014 period (Fig. ??1979-2005 period (Figs. 1a and b). They are interpolated on the GRISLI grid (5 km x 5 km) and corrected for surface elevation differences between MAR and GRISLI by applying the method developed by (?). We use the reconstruction from ? for the geothermal heat flux. Using these boundary conditions, GRISLI is run forward starting from we use the data generated for the SEARISE project (?). Initial geometry consists in the present-day observed ice thickness (Fig. ??a), from which the ice volume is inferred ?, and from the bedrock elevation. Initial vertical temperature and velocity profiles as well as the initial map of the basal sliding and bedrock elevation taken from ?. To compute initial conditions consistent with the boundary conditions, we run a 30 000 year-integration of the model imposing a fixed topography. For this long experiment, similar to the fixed topography spin-up method, we assumed a perpetual present-day climate forcing (Figs. 1a and b) and we used a basal drag coefficient (Fig. ??a) are derived from previous GRISLI simulations carried out with boundary conditions close to those of the present study, and performed within-the-2a) coming from a previous simulation carried out within the Ice2Sea project, which aimed at reducing the uncertainties on future sea level rise projections ?. a/ Observed Greenland ice thickness (in m) from ? interpolated on the GRISLI grid. Grey areas represent non-ice covered areas. b/ Difference between the simulated and the observed ice thickness (in m) obtained at the end of a 200 year long simulation without spin-up procedure. The simulation has been carried out using the Ice2Sea initial conditions (see main text and ?) and the climate forcing simulated by MAR. project (?). The resulting basal temperature after this long integration, presented as a difference with respect to the pressure melting point, is shown in Fig. 1c. It shows areas with temperature largely below the pressure melting point, associated with frozen bed, and areas with temperature at the pressure melting point (red colours), associated with thawed bed. Compared to the recent synthesis of GrIS basal temperatures (see Fig. 11 in ?), our initial basal temperature agrees generally well with the reconstructions in the north-western and north-eastern parts of the GrIS but are probably overestimated, with a too large thawed bed area, in the eastern and central parts of the GrIS (not shown). The impact of ice temperature on the minimisation procedure is discussed in Sect. 5.1.

In order to avoid large-inconsistencies between the different datasets used as boundary and initial conditions, GRISLI is first run forward (free-evolving surface elevation and temperature) for 5 years. After this (relaxation step, blue box in Fig. 3). After this short relaxation period, we start the spin-up procedure–iterative minimisation procedure (red box in Fig. 3). This procedure is based on an iterative process set up to adjust the basal drag coefficient in such a way that the mismatch between observed and simulated ice thickness is reduced. At the end of the iterative process, we allow GRISLI to evolve freely for 200 years in order to assess the model performance in terms of trend and error in simulated ice volume compared to observations. The iterative process itself is: Instead of optimizing the basal drag coefficient every 5 000 years as in PDC12, here the optimization is done at every time step (which is set to one year for the present study), using an ice thickness ratio to correct the simulated sliding velocity with the help of a modification of the basal drag coefficient.
The iterative minimisation procedure itself consists in repeated cycles, each cycle being divided in two main steps (Fig. 4). Spatial distribution of the basal drag coefficient (in log10 Pa m$^{-1}$ s$^{-1}$) an for the initial condition as used in the GRISLI ice2Sea simulations, b/ obtained for the best fit Nb$^{20}_{\text{iter}}$, Nb$^{50}_{\text{year}}$, Nb$^{60}_{\text{cycle}}$ and c/ obtained at the end of a spin-up procedure using the same spin-up parameters as those inferred from the best fit but starting from a uniform spatial distribution of the basal drag coefficient ($\beta = 1$).

Schematic representation of the spin-up method:

1st step: By using the red box in Fig. 3):

1st step: The first step consists in a free-evolving simulation (thickness and temperature) during which we adjust, at each model time step, the basal drag coefficient so that the ice thickness difference with respect to the observations becomes minimal. To this end, from the simulated vertically-averaged velocity $\overline{U^G}$ computed from the previous time step (or from the values obtained after the relaxation for the first iteration), we calculate compute a corrected vertically-averaged velocity field $(U^{corr} \overline{U^{corr}})$ as a function of the computed $(H^G H^G)$ and observed ice thickness $(H^{obs})$ deduced from $\beta$:

$$U^{corr} \overline{U^{corr}} = \frac{U^G \times H^G}{H^{obs}} \overline{U^G} \times \frac{H^G}{H^{obs}}$$

(3)

$U^{corr}$ can be seen as a the vertically-averaged velocity field corrected by a factor representing the difference between the observed and the simulated ice thicknesses.

As seen before (section 2 in Sect. 2), the mean velocity field $\overline{U^G}$ in GRISLI $\overline{U^G}$ is the sum of two velocity components: the sliding velocity $U^{sli}$ and the velocity $U^{def}$. $U^{sli}$ and the vertically-averaged velocity $\overline{U^{def}}$ due to vertical ice deformation:

$$\overline{U^G} = U^{sli} + U^{def}$$

(4)

Considering Assuming that the differences of velocity between $\overline{U^G}$ and $U^{corr}$, between $\overline{U^{corr}}$, the simulated vertically-averaged velocity field, and $\overline{U^{corr}}$, the idealised vertically-averaged velocity field, are only due changes of to changes in the sliding velocity $U^{sli}$, we can also write: write:

$$U^{corr} = U^{corr} + U^{sli}$$

(5)

Following Eqs. (4) and (5) we can deduce the corrected sliding velocity $(U^{corr})$ needed $U^{sli}$:

$$U^{sli} = \overline{U^{corr}} - \overline{U^G} + U^{sli}$$

(6)

$U^{sli}$ represents the corrected sliding velocity whose difference with $U^{sli}$ indicates how the simulated sliding velocity must change to reduce the difference between $H^G$ and $H^{obs}$.

$$U^{sli} = U^{corr} - \overline{U^G} + U^{sli}$$
The new value of the mismatch between \( H^G \) and \( H^{obs} \).

As such, we use the ratio between the simulated and the corrected sliding velocities \( \frac{U_{sli}}{U_{sli}^{corr}} \) to compute a new basal drag coefficient \( \beta_{new} \). This results in slowing down or speeding up the simulated sliding velocity and acts to reduce the gap between \( H^G \) and \( H^{obs} \) is deduced from the \( \beta_{old} \) value, inferred from the previous iteration and from the ratio between uncorrected and corrected sliding velocities:

\[
\beta_{new} = \beta_{old} \times \frac{U_{sli}}{U_{sli}^{corr}}
\]

with \( \beta_{new} \) calculated at each GRISLI Equation 7 is essentially identical to what is done in \( \beta_{old} \) except that they use observed and modelled velocities rather than observed and modelled ice thickness to adjust the basal drag coefficient. It should be noted that \( U_{sli}^{corr} \) can be lower or equal to 0, leading to infinite or negative basal drag coefficient. This can happen when the velocity due to vertical shearing \( \dot{U}_{def} \) is greater or equal to \( \dot{U}_{corr} \). In this case we artificially impose a no-slip condition by assigning to the basal drag coefficient a maximum value set to 5 \( 10^5 \) Pa yr m\(^{-1}\). On the other hand, in case of too small \( \dot{U}_{def} \) velocity, \( \beta \) may be as low as 1 Pa yr m\(^{-1}\) to facilitate ice sliding. Owing to its design, the method is only able to correct for the ice thickness mismatch where sliding occurs, i.e., where the base of the ice sheet is at the pressure melting point. Throughout this step, the basal drag coefficient is updated at each time step for each model grid point. \( H^G, U^G, U_{sli}^{corr} \) and \( \beta_{new} \) are updated during \( Nb_{corr} \) time steps. In the following, the duration of this step is referred to as \( Nb_{corr} \) and has typical value of a few decades.

2\(^{nd}\) step: With this new-Note that, using Eq. 3 and Eq. 4, we can show that Eq. 7 can be rewritten as:

\[
\frac{\beta_{old}}{\beta_{new}} = rH + \frac{\dot{U}_{def}}{U_{sli}}(rH - 1) \quad \text{where} \quad rH = \frac{H^G}{H^{obs}}
\]

As such, the adjustment of the basal drag coefficient is stronger in regions dominated by ice deformation.

2\(^{nd}\) step: The second step consists in running a new free-evolving simulation but this time using a time constant (but spatially varying) basal drag coefficient we let the model to freely evolve. After \( Nb_{year} \) of the free-evolving simulation, we obtain a new GrIS topography and new corrected velocity fields computed from the mismatch between the simulated ice thickness after \( Nb_{year} \) and, i.e., the last inferred basal drag coefficient of the first step. The duration of this second step, referred to as \( Nb_{free} \) in the following, is generally longer than that of the first step, typically a few decades to a few centuries. This
step aims at quantifying the model drift and the observations. With this, we can start a new cycle in which the 1st and 2nd steps are repeated. This new cycle uses the same set of spin-up parameters (Nb\textsubscript{iter} and Nb\textsubscript{year}) and an initial guess of \( \beta \) coming from the previous iteration. All the iterations use the same initial conditions presented previously – model mismatch with observations for the inferred basal drag coefficient. The simulated ice sheet velocity and topography at the end of this second step are used to compute a new \( \overline{U}_{\text{corr}} \) value in order to start a new cycle from the first step. The number of cycles carried out in this way is iterative cycles will be noted Nb\textsubscript{cycle}. For all the experiments presented in the following, we performed a maximum of nine cycles.

To assess the spin-up performance and the In summary, our iterative minimisation procedure consists in:

(i) Adjustment of the basal drag coefficient at each time step (each year) year for Nb\textsubscript{iter} years (1st step, Eqs 3 to 7).

(ii) Free-evolving simulation with the last inferred basal drag coefficient from (i) for Nb\textsubscript{free} years (2nd step).

(iii) The steps (i) and (ii) are repeated Nb\textsubscript{cycle} times.

In addition, to assess the performance of the minimisation procedure (i.e. the quality of the inferred \( \beta \) coefficient, we perform, basal drag coefficient), we compute some quality metrics at the end of each cycle (green box in Fig. 3). The metrics are computed at the year 200 of the free-evolving simulation of the second step, independently from its duration (i.e., Nb\textsubscript{free}). If Nb\textsubscript{free} is shorter than 200 years with a \( \beta \) coefficient fixed to the values computed during the last cycle. The best Nb\textsubscript{cycle} for a given set of Nb\textsubscript{iter} and Nb\textsubscript{year} will be the one that provides a final volume as close as possible to the observation and a minimal trend over the last ten years of this free-evolving simulation. In addition to these two criteria, we simply extend the simulation until 200 years. The quality metrics discussed in Sec. 4 include in particular the root mean square error (RMSE) of the simulated ice thickness with respect to the observations and the drift in geometry (integrated ice thickness changes). These metrics help to decide whether an additional cycle is required or not. In the following, we also take into account the ice thickness root mean square error from the observations. Once the best fit is obtained, steady-state or transient GrIS simulations can be performed with reliable initial conditions, as done in 7 and 8.

Four values of Nb\textsubscript{iter} (20, 40) discuss the spatial patterns of ice thickness and ice velocity mismatches with respect to observations. Our method does not use the observed surface velocity as a constraint. However, at the end of the minimisation procedure (e.g., minimal thickness error and minimal drift), 80, 160 years) and of Nb\textsubscript{year} (50, 100, 200 and 400 model years) have been tested with Nb\textsubscript{cycle} ranging from 1 to 9, giving a total of 144 combinations of the spin-up parameters. The corresponding simulations are referred to as Nb\textsuperscript{X}_{iter} Nb\textsuperscript{Y}_{year} Nb\textsuperscript{Z}_{cycle} where X, Y and Z stand respectively for the Nb\textsubscript{iter}, Nb\textsubscript{year} and Nb\textsubscript{cycle} values.

4 Results

3.1 Is the spin-up needed?
The annual mean climatological SMB for the 1979-2014 period integrated over the whole GrIS is 381 Gt yr$^{-1}$ (Fig. 2a), with strongly positive values in southeastern Greenland (up to 0.04 Gt yr$^{-1}$), and largely negative ones over the ablation zone at the edges, the simulated velocity tends nonetheless to approximate the balance velocity, that is the depth-averaged velocity required to maintain the steady-state of the ice sheet, with values reaching –0.10 Gt yr$^{-1}$ in the western area (Fig. 2a). The annual mean surface temperature is negative over the whole Greenland ice sheet, ranging from –29 °C in the highest altitude regions to –0.5 °C near the coast (Fig. 2b).

Once the optimal basal drag coefficient is found, it can be used to run prognostic forward simulations such as in ? and ?.

4 Results

4.1 The importance of the initialisation procedure

To illustrate the need for a spin-up an initialisation procedure, we performed a 200-year-long free-evolving simulation with this mean climatic forcing and with the initial conditions without any specific initialisation procedure using the mean 1979-2005 climatic forcing presented in Sec. 3. For this simulation, the initial internal condition correspond to the one obtained after the 5-year-long relaxation 30 000-year temperature equilibrium simulations (see Sect. 3), without any spin-up procedure (3) and the basal drag coefficient, coming from previous Ice2Sea simulations (?) is left unchanged (Fig. 2a).

The simulated GrIS ice volume obtained in for this experiment is smaller, by 20000 Gt, 1.4 % higher than the one estimated by ? from observations (i.e., 2.71 $10^6$ Gt) with. This overestimation is driven by large positive ice thickness differences (>200 m) with respect to observations in the margin regions (Fig. 4a). There are also negative ice thickness anomalies in particular in the central eastern region. On top of this geometry mismatch, this experiment also presents a drift at the end of the 200 years with a negative contribution to global sea level of 0.7 mm yr$^{-1}$ (i.e., 263 Gt yr$^{-1}$ ice mass gain). Compared to observations (?), the simulated ice velocity presents the same large-scale pattern but with important local differences (Fig. 4b). Moreover, the ice volume decrease contributes by 0.6 mm yr$^{-1}$ to the global sea level rise. Thus, despite an overall positive SMB, the model drift and the lack of spin up procedure result in a decrease of the GrIS volume as large as the present melting due to the global warming ?.

These results show the limitations of the simulated GrIS under constant climate forcing without appropriate initialisation procedure. In this specific case, the simulated model drift can potentially counterbalance the effect of climate warming expected in the future, leading to unrealistic projected Greenland melting contribution to global sea level rise. Therefore, the use of a spin-up method an initialisation procedure to minimise the model drift with a realistic simulated topography is not avoidable if the goal is to produce reliable sea-level projections.
4.2 Spin-up Iterative minimisation performance for a range of enhancement factor values

To assess the spin-up performance, we first examined the ice thickness root mean square error (RMSE). An increase (respectively a decrease) of the basal drag coefficient ($\beta$) slows down (resp. speeds up) the sliding velocity and thus the ice flow. Based on the adjustment of the sliding velocity, our iterative minimisation procedure, allows for a tuning of $\beta$ only in regions where the basal temperature is at the pressure melting point, i.e., where the ice can slide over the bedrock. Where the base is frozen, the tuning of the basal drag coefficient has no impact on the ice thickness minimisation, because no sliding occurs. In order to slow down or speed up the ice flow in such regions, the value of the enhancement factor, Ef (see Sec. 2), can be tuned. As explained in Sect. 2, this factor is used to increase (when $> 1$) or decrease (when $< 1$) the ice deformation velocity. The more the ice deformation is increased (respectively decreased), the more the ice flow in frozen base region speed-ups (resp. slow-downs) and thus decrease (resp. increase) the ice thickness.

The enhancement factor for the ice volume anomaly (computed—observed) and ice volume trend for each combination of the spin-up parameters. An illustrative example is given here for $Nb_{\text{iter}} = \text{SIA regime (slow ice flow)}$ is expected to have a large influence on shear-stress driven velocities (?). Generally set to 3 (?), the Ef can be chosen within a large range of values between 1 to 10 (?). In the following, we assess our iterative minimisation procedure for a range of Ef values: 0.1, 0.5, 1, 1.5, 2, 2.5, 4, 3, 3.5, 4, 4.5, 5. For this, we use a $Nb_{\text{spin}}$ of 20 and years and a $Nb_{\text{iter}}$ of 200 years and perform 15 iterative cycles ($Nb_{\text{year}}$ ranging from 50 to 400 years) with $Nb_{\text{cycle}}$ varying from 1 to 9 (Figs. ?? and ??). For the first cycle, i.e., $Nb_{\text{cycle}} = 1$, all the tests corresponding to different $Nb_{\text{year}}$ values—Ef experiments—start from the same identical initial conditions and the basal sliding velocity has not yet been updated with the new $\beta$ coefficient. Basal drag coefficient presented in Sec. 3.

Each of the 180 experiments (15 cycles for 12 enhancement factors) are evaluated after 200 years of the free-evolving simulation (2nd step, see Sect. 3) using 1D metrics (ice thickness RMSE, global ice volume, geometry drift) and 2D validation criteria (ice thickness differences).

4.2.1 Root mean square error

Ice thickness root mean square error w.r.t. observations from ?, in meters for $Nb_{\text{iter}} = 20$ and the four $Nb_{\text{year}}$ values (50, 100, 200, 400) as a function of the number of iterations ($Nb_{\text{cycle}}$). The RMSE behavior is approximately the same for all the $Nb_{\text{year}}$ values (Fig. ??). A strong decrease between the first two cycles is obtained meaning that the departure between simulated and observed ice thickness is rapidly reduced. This decrease is then followed by a stabilisation occurring for $Nb_{\text{year}}$. The ice thickness RMSE defined with respect to observations is displayed in Fig. 5 as a function of the number of cycles performed for the different enhancement factors. For a given Ef value, the RMSE quickly decreases during the first cycles and generally stabilises after $Nb_{\text{cycle}}$ between 4 and 6 depending on the $Nb_{\text{year}}$ value. Increasing $Nb_{\text{year}}$ results in lower RMSE values. For example, for $Nb_{\text{cycle}} = 6$, the RMSE decreases from 84.8 m ($Nb_{\text{year}} = 50$) to 57.4 m ($Nb_{\text{year}} = 400$). This can be explained by the fact that for longer free-evolving simulations, the basal velocity (computed through the previously determined $\beta$ coefficient) exerts a longer influence on the vertically averaged velocity, which in turn impacts the simulated ice thickness. This results in
larger differences between simulated and observed ice thickness. This implies that the corrections of the $\beta$ coefficient are more significant for the following cycle, and finally, that the method is more efficient to correct for the differences of the ice thickness $\approx 5-6$. This means that the procedure is very effective in reducing the ice thickness error for the first iterations but does not entirely correct the mismatch with observations. Depending on the enhancement factor considered, the overall improvement represents a reduction of about 20 to 40 m in ice thickness RMSE with respect to the observed one first iterative cycle.

4.2.2 Volume

Same as figure 5 for the GrIS ice volume anomaly (a) and the ice volume trend (b) represented in Gt and mm yr$^{-1}$ respectively.

The ice volume anomalies ($\Delta$Vol) obtained for the same set of spin-up parameters are displayed in Figure 6a. A strong $\Delta$Vol decrease is obtained, starting from a highly positive value ($\Delta$Vol $\approx$ 28,000 Gt for $\Delta$H) for the RMSE is largely different for the different enhancement factors. For $\text{Ef} \geq 2$, we have systematically a larger RMSE for a larger Ef value regardless of the number of iterative cycles performed. This is no longer the case for smaller Ef since the experiment (i.e., the Ef value) providing the lowest RMSE is different for the $\text{Nb}_{\text{cycle}}$ considered. Note that for $\text{Ef} = 1$ and reaching negative values when 0.5 the RMSE value is often larger than that obtained with Ef = 2.5 even with increasing $\text{Nb}_{\text{cycle}}$ increases. This illustrates that our method tends to underestimate the ice volume with respect to observations. This underestimation is also more pronounced for higher $\text{Nb}_{\text{year}}$ values. As a consequence, the combination of spin-up parameters providing the lowest RMSE values are also those for which the ice volume anomalies are the largest ones. For example, for $\text{Nb}_{\text{cycle}}$. Indeed, $\text{Ef} = 6$, $\Delta$Vol equals $\approx$ 10,000 Gt for $\text{Nb}_{\text{year}} = 0.5$ implies a too small deformation rate that leads to a too slow ice flow velocity. As such, the departure from the observations is mainly characterized by positive ice thickness anomalies at the edges and in the half southern part of the ice sheet. The simulations with Ef varying from 1 to 2 have very similar RMSE even if 1.5 has a slightly lower RMSE in most cases. If the lower RMSE value (49.8 m) is obtained for Ef = 400, while it is two order of magnitude below ($\Delta$Vol 1.5 after 9 cycles (Table 1), RMSE values below 55 m are nonetheless obtained after 4 cycles for Ef varying from 1 to 2. Considering that after one cycle the error is greater than 80 m, we are able to improve the RMSE by about 30 m in 880 years of simulations (4 x 220 years).

4.2.2 Model structural biases and consequence on total ice volume

a) Where are the errors: correction by deformation and basal sliding

In addition to the RMSE criterion, which is an integrated metric, the maps of the difference between the simulated and the observed ice thickness bring valuable information to understand the model structural biases. In Fig. 6, we can distinguish two main patterns. Except for Ef = 407 Gt) with $\text{Nb}_{\text{year}} = 50$.

This behaviour can be explained when examining the ice thickness anomaly ($\Delta$H, Fig. 6a). This anomaly tends to be negative in the central part and positive in coastal regions with 0.5, all the Ef experiments with a $\text{Nb}_{\text{cycle}}$ producing the minimum RMSE value (Fig. 6 and Table 1) are marked with an underestimation in ice thickness in the interior and an
overestimation at the edges of the GrIS. This overestimation can be slightly reduced using higher Ef values, the underestimation being nonetheless larger in this case.

As explained above, larger Ef values amplify ice deformation and therefore speed up the ice velocity, explaining the spread of the regions where the ice thickness is underestimated (Fig. 6). Some of these regions, such as a significant portion of the half central part of the ice sheet, are often associated in our model with thawed bed areas (i.e. basal temperature is over the pressure melting point, Fig. 1c) while frozen bed is expected (?). This may further enhance the ice flow acceleration by favouring basal sliding. On the other hand, when basal sliding occurs, our iterative minimisation procedure may counteract the ice flow acceleration by reducing the exception of the northwestern area which remains controlled by the SIA due to the high value of the basal sliding coefficient (Fig.2b). Since our method is based on the adjustment of the basal sliding coefficient, it only operates over non-frozen bed where the SSA is activated. This occurs mainly in the peripheral regions of the ice sheet or in ice-stream areas. In the central part, where the (i.e. increasing the basal drag coefficient). However, in some cases, the velocity due to deformation is too fast and the basal drag coefficient is set to its maximal value ($\beta_{\text{max}} = 5 \times 10^5$ Pa yr m$^{-1}$) so that the sliding velocity becomes virtually zero. This is visible in Fig. 7 where the area for which the basal drag coefficient is set to $\beta_{\text{max}}$ (dark red colour) is getting larger with increasing Ef. On the contrary, in Fig. 7, where the model overestimates the ice thickness (i.e. too slow ice flow) and where basal temperature is most often below the melting point, ice velocities are mainly governed by the SIA and are thus not corrected. Thus, during the first iterations, the ice velocities (and therefore the ice topography) have not been corrected so much and the regions where $\Delta H > 0$ are balanced by the central regions where $\Delta H < 0$, which are not impacted by the corrections. This compensating effect acts to reduce the ice volume anomaly. However, as $N_{\text{max}}$ increases, corrections of $\Delta H$ become more efficient in the peripheral areas. In these regions, the simulated ice thickness is improved (Fig. 2b) with respect to observations and the RMSE is lowered, but the compensating effect is reduced and the at the pressure melting point (i.e. sliding can occurs), the computed basal drag coefficient is weaker in order to increase basal sliding. Similar to the $\beta_{\text{max}}$ region, our iterative initialisation method could reach a minimum basal drag coefficient value (set to $\beta_{\min} = 1$ Pa yr m$^{-1}$) in regions where the sliding velocity must be as strong as allowed by the flow law equation (i.e. meaning no basal friction). Reducing the enhancement factor, and thus the ice deformation in these regions can locally increase the ice volume anomaly increases. Difference between the simulated and the observed (?) ice thickness (in meters) obtained for the spin-up parameters providing a/ the lowest ice volume anomaly and the lowest ice volume trend ($N_{\text{iter}}^{20}$, $N_{\text{year}}^{50}$, $N_{\text{cycle}}^{6}$) and b/ the lowest ice thickness RMSE ($N_{\text{iter}}^{20}$, $N_{\text{year}}^{100}$, $N_{\text{cycle}}^{4}$), thickness overestimation. Regions with $\beta_{\text{max}}$ or $\beta_{\min}$ values are an indication of the limit of our iterative ice thickness error minimisation procedure.

### 4.2.3 Ice volume trend

This analysis demonstrates that satisfying the ice volume anomaly criterion is in direct competition with the RMSE minimisation, and therefore that a compromise needs to be found. Since our main objective is to obtain a simulated ice volume as close as
possible to the observed one (?), the criterion related to the ice volume trend (IVT) must be also examined. Figure 2?b shows that this trend follows a behavior similar to the ice volume anomaly with increasing values of Nb\textit{year} and Nb\textit{cycle}. The key feature appearing in this figure is the strong decrease towards negative values (down to -0.5 mm yr\textsuperscript{-1}) for most of the spin-up parameter combinations. However small IVT values are obtained for three set of parameters: Nb\textit{year}. The ice thickness errors shown in Fig. 6 correspond to a median value ranging from +15 m to -99 m from the lowest (Ef = 400 and Nb\textit{cycle} = 0.5) to the highest (Ef = 3\textit{IVT}-5) enhancement factor. The decrease in the median of the error with increasing Ef values is mostly driven by the underestimation of the ice thickness in the interior regions. Our results show that the Ef = 0.04 mm yr\textsuperscript{-1}, Nb\textit{year} = 200 and Nb\textit{cycle} = 2 (IVT = 1 experiment produces the best ice thickness error pattern, ranging from +0.03 mm yr\textsuperscript{-1} and Nb\textit{year} = 50 and Nb\textit{cycle} = 133 m (5\textsuperscript{th} quantile) to -39 m (95\textsuperscript{th} quantile) and reaching a median error equal to +3 m.

b) Total ice volume and compensating biases

Because most of the Ef experiments have both positive ice thickness biases at the margins and negative biases over the central part (Fig. 6), the global ice sheet volume is not a good metric for model performance due to compensating biases. Figure 8 shows the total ice volume difference with respect to observations for varying enhancement factors as a function of the number of iterative cycle. Some specific experiments show a very small error in global ice volume with respect to observations for given Ef values even though they have a poor RMSE (Fig. 5). Also, for Nb\textit{cycle} = 6 (IVT = 6, RMSE value for Ef = 8.5\texttimes 10\textsuperscript{-3} mm yr\textsuperscript{-1}). While these two first combinations provide reasonable RMSE values (see Table 1), the 0.5 and Ef = 2.5 are identical (68.8 m) but ice volume anomalies are respectively 1079 and 4343 Gt. This must be compared to ΔVol drastically different with 57994 Gt and -38841 Gt respectively (Table 1). Thus, a small global ice sheet volume difference does not necessary mean a minimisation of the ice thickness difference.

For the same reasons, the trend in global ice volume is not a good metric for assessing the ice sheet drift because local changes in ice thickness can compensate for each other. As an illustration, using a range of enhancement factors, Fig. 9 shows for free-evolving simulations, the temporal evolution of the total ice volume difference with respect to observations, along with the evolution of the RMSE. This figure confirms that the GrIS volume equilibrium can be reached by biases compensation as we have a near-zero error in volume with Ef = 107 Gt obtained with Nb\textit{year} = 1.5 while the RMSE is very similar to that obtained with Ef = 50 and Nb\textit{cycle} = 1 and Ef = 6 which provide the smallest ΔVol and IVT values. Despite the corresponding RMSE being the highest one among all the tests which have been performed, it is less than 20 m higher than the lowest RMSE value. Thus the experiment Nb\textit{iter} = 20, Nb\textit{year} = 50 and Nb\textit{cycle} = 6 matches our two main criteria (i.e. minimum ΔVol and IVT values) and the corresponding spin-up parameters appear as good candidates for 2. For Ef ≥ 2, the negative biases in ice thickness dominate, with a decrease in ice volume as Ef increases. For Ef < 2, the positive biases in ice thickness dominate, leading to an increase of the overall spin-up procedure global ice volume.
Even if our approach is based on minimising the volume trend and fitting ice volume and ice thickness to the observed ones, the reliability of our method also depends on its capability to simulate ice velocities in good agreement with observations. We therefore compare our results to the surface ice velocity dataset provided by [2] (Fig. S3). The simulated results are slightly different from the observations especially in the central plateau where the region of low ice velocities is less extended in the simulations than in the observations. However, the overall patterns are in a good agreement, especially in regions of fast ice flow, providing confidence in our method. a/ Observed surface ice velocities coming from a compilation of interferometric synthetic aperture radar measurements obtained from RADARSAT data at different periods of the 2000s. b/ Simulated surface ice velocities obtained for our best fit (\( Nb_{\text{iter}}^{20} \text{ yr} \), \( Nb_{\text{year}}^{50} \text{ cycle} \), \( Nb_{\text{cycle}}^{6} \)). Values are given in log 10 m s\(^{-1}\).

### 4.2.3 Sensitivity to Nb\_\text{iter} values

Results obtained for other Nb\_\text{iter} values (40, 80 and 160) are reported in Table 1. None of these experiments fulfill both the \( \Delta V\text{ol} \) and IVT criteria. The \( Nb_{\text{iter}}^{50} \text{ yr} \), \( Nb_{\text{year}}^{100} \text{ cycle} \), \( Nb_{\text{cycle}}^{3} \) provides the lowest ice volumetrend (5.1 10 \(-3 \) mm yr\(^{-1}\)) but the simulated ice volume anomaly is more than 20 times larger than for \( Nb_{\text{iter}}^{20} \text{ yr} \), \( Nb_{\text{year}}^{50} \text{ cycle} \). Conversely, \( Nb_{\text{iter}}^{60} \text{ yr} \), \( Nb_{\text{year}}^{200} \text{ cycle} \) simulates a GrIS ice volume anomaly only 2.5 as large as for \( Nb_{\text{iter}}^{20} \text{ yr} \), \( Nb_{\text{year}}^{50} \text{ cycle} \) but the ice volume trend (0.041 mm sheet drift and in order to avoid the biases compensation, we compute the geometry trend as the root mean square ice thickness change (\( \xi_t \) expressed in cm yr\(^{-1}\)) is four times larger. Moreover, the duration of the spin up procedure for \( Nb_{\text{iter}}^{160} \text{ yr} \), \( Nb_{\text{year}}^{200} \text{ cycle} \) is 1080 model years while it is only 420 years for \( Nb_{\text{iter}}^{20} \text{ yr} \), \( Nb_{\text{year}}^{50} \text{ cycle} \). This confirms that \( Nb_{\text{iter}}^{20} \text{ yr} \), \( Nb_{\text{year}}^{50} \text{ cycle} \) appears as the best protocol in terms \( \Delta V\text{ol} \) and IVT criteria and that it is also designed to ensure a more rapid convergence.

Ice volume trend (in mm yr\(^{-1}\)) and ice volume anomalies (simulated—observed) obtained for the 16 combinations of the NB\_\text{iter} and NB\_\text{year} parameters. The values of NB\_\text{cycle} correspond to the number of iterations providing the lowest IVT and \( \Delta V\text{ol} \) values. Corresponding ice thickness RMSE (w.r.t observations) are also indicated. Final GrIS Final volume RMSE of thickness:

\[
\xi_t = \sqrt{\langle (H_t - H_{t-1})^2 \rangle}^{0.5}
\]

where \( \langle (H_t - H_{t-1})^2 \rangle \) represents the averaged squared ice thickness change over the whole GrIS, compared to Obs. NB\_\text{cycle} (mm yr\(^{-1}\)) (Gt) (m) 50 6 0.0085 107 84.8 100 3 0.0368 1079 77.0

Values of \( \xi \) computed from the last 5 years of the 200 yr free-evolving simulation in the 2002–01 4343 75.9 400 3 0.1927 4908 65.5 50 6 0.0313 408 77.3 100 3 0.0054 2431 75.9, nd step (green box in Fig. 3) are reported in Table 1 for a given iteration and varying enhancement factors. The lowest values are generally obtained with the experiments that provide the lowest RMSE, which means that these simulated ice sheets are the closest to equilibrium. The minimal trends are about 15 cm yr\(^{-1}\) and are obtained with enhancement factors between 1 and 2.

c) Ice dynamics

400 2 0.0272 4430 75.2.

400 2 0.1323 4102 67.
Our iterative minimisation procedure aims at simulating an ice thickness as close as possible to observations. Hence, the observed ice velocity is not used as a target by the model. However, because our procedure generates an ice sheet at quasi-equilibrium (trend $\xi$ close to 0), the simulated velocities are close to the balance velocities, which in turn are supposedly close to present-day observations. As a result, our method simulates an ice flow pattern similar to the observations (Fig. 10).

The simulated velocity field is particularly sensitive to the choice of the enhancement factor (Fig. 10). In particular, for the highest Ef values, the simulated velocity is overestimated for the major ice streams where deformation due to vertical shearing is expected to be of lesser importance compared to basal sliding. For Ef = 1.5, the ice flow pattern in the margin regions is well reproduced compared to observations. Only some glaciers ice velocities can be faster (e.g. Jakobshavn or Kangerlussuaq) or slower (e.g. Petermann or NEGIS). While the best GrIS geometry (lowest RMSE) is obtained with Ef = 1.5, the experiments with Ef = 1 or Ef = 0.5 best reproduce the observed surface velocities (RMSE about 150 m yr$^{-1}$, Fig. S1).

5 Sensitivity to initial conditions and model parameters

4.1 Temperature equilibrium

In the work presented in section 4, the vertical temperature and ice velocities profiles taken as initial conditions came from previous experiments carried out with GRISLI in the Interestingly the extent of the NEGIS is particularly well represented, in particular for lower enhancement factors (Fig. S2). This can be a relic of the long temperature equilibrium performed with a time constant basal drag coefficient taken from Ice2Sea experiments (?), in which the NEGIS is well delimited (Fig. 2a).

However, because this feature is still present when starting the iterations from a spatially homogeneous basal drag coefficient (see Sec. 5.2), it can also suggests that there is some topographic control of this feature as the adjustment of our local basal drag coefficient is very effective in reproducing the observed velocity in this area. Having a good representation of the NEGIS is an encouraging sign for the performance of our minimisation procedure, especially since most models fail to achieve this (?)

5 Sensitivity of the method to the initial conditions and to the duration (Nb$_{inv}$ and Nb$_{free}$) of the minimisation procedure

5.1 Sensitivity to the initial temperature profiles

In Sec. 4.2 we have shown that the results of the minimisation are particularly impacted by the basal temperature. In particular, where the bed is frozen our iterative mininisation procedure is unable to correct for the ice thickness mismatch. This leads to a predominant role of the enhancement factor. The aim of this section is to investigate the sensitivity of our procedure to
the initial temperature profile. To this end, we followed the same methodology as in Sec. 4.2, and performed a new set of experiments for which we used an initial temperature profile coming from a previous simulation performed in the framework of the Ice2Sea project (7). These profiles have been chosen because they were assumed to reflect the present day conditions. However, they are not necessarily in equilibrium with the climatic forcing taken from the MAR simulations.

This temperature profile differs substantially from the one used in the previous section (black dashed line to be compared to the red line in Fig. ??). Indeed, at the ice sheet surface, the temperature obtained from MAR is about 12°C. The temperature profile taken from ? is not consistent with the MAR climatic forcing used for this work and the warmer climatic forcing used here leads to a warmer (about 5°C warmer than the Ice2Sea one) ice sheet compared to the one in ? (Fig. ??a). Therefore, we performed a 30,000 year long simulation to make the vertical temperature and velocity profiles consistent with the surface climatic forcing. This new experiment has been carried out with the surface topography fixed to the observed one (7) and the basal sliding coefficient deduced from the spin up procedure (i.e., Nb20 iter, Nb50 year, Nb6 cycle). As illustrated in figure 9a for a region located in the central part of the ice sheet (73.74.5 °N, 40.43 °W), the ice sheet becomes progressively warmer as the result of inconsistencies between the initial vertical temperature profile and the surface climate. a/ Vertical profile of temperature (in °C) in a central region of Greenland (73.74.5 °N, 40.43 °W) taken as initial condition from the Ice2Sea project (7), see black dashed line) at the end of the spin-up procedure (Nb20 iter, Nb50 year, Nb6 cycle) and at different periods of the temperature equilibrium experiment. b/ Difference between simulated and observed Ice thickness (in m) obtained after the temperature equilibrium and a new spin-up procedure performed with the spin-up parameters providing the best fit fit in terms of ice volume and ice volume trend.

New spin-up procedures have been undertaken with this temperature equilibrium and with Nb iter = 20 and 40 and the same Nb year (50, 100, 200, 400) and Nb cycle (1 to 9) as previously. These new spin-up tests reveal that the ice volume anomalies, 12). In the ice volume trends and the RMSE values are degraded compared to the results presented in section 4. For example, following, the temperature profile taken from ? is referred to as the non-equilibrated temperature as opposed to the lowest ∆Vol and IVT values are obtained for Nb20 iter, Nb50 year, Nb6 cycle with ∆Vol = 867 Gt and IVT = 0.90 mm yr⁻¹ but 30-kyr equilibrated temperature used in the ice thickness RMSE is 214.0 m. Conversely a lower RMSE value is reached for Nb20 iter, Nb50 year, Nb6 cycle, but for these parameters ∆Vol = 56,691 Gt—rest of the manuscript.

These results illustrate the limitations of our spin-up method, as also shown in Figure ??b which displays the ice thickness anomaly for Nb20 iter, Nb50 year, Nb6 cycle (∆Vol Figure 13 shows the evolution of the RMSE for 9 iterative cycles for the experiment performed with the non-equilibrated temperature profile with Ef = −56102 Gt, IVT 3 (dark blue dots). Similarly to what was shown in Sec. 4.2, the minimisation procedure reduces the RMSE from +76.0 m after Nb cycle = 118.3 m) after a 200 year free-evolving simulation. Actually, the warmer temperatures obtained after the 30,000 year long simulation induce higher ice velocities due to the thermo mechanical coupling. In the ice sheet interior (SIA areas), these velocities are not corrected by our spin-up approach as shown by the β coefficient which reaches maximum values in these regions (Fig. ??b). Thus, an increased ice flux takes place from the central part to the peripheral regions leading to amplified negative ice thickness anomalies (Fig. ??b).
5.2 Sensitivity to the enhancement factor

In the ice-sheet interior, the ice flow is mainly due to internal ice deformation which is controlled by the temperature and thus by the viscosity. A possibility to reduce errors in ice surface elevation in these locations is to adjust the enhancement factor of the Glen’s flow law, which relates viscosity to deformation rates. Lowering the Ef value allows to decrease the deformation and thus to slow down the ice flow velocities. Therefore we performed new sensitivity tests around +47 m. Figure 13 shows also the evolution of the RMSE for two experiments with Ef = 1 (instead of and Ef = 3 as in the experiments presented in section 4) with the same spin-up parameters used in the previous section.

As expected, for given Nb_{iter} and Nb_{year} values, the ice thickness RMSE is improved when but using the equilibrated temperature profile (cyan and orange dots in Fig. 13). If the pattern is essentially the same between the different experiments, the number of iterations (NbRMSE is higher when using the equilibrated temperature. For the same Ef value, the RMSE is 11.4 m higher (Nb_{cycle} ) increases. Contrary to previous tests (section 4), the parameters (Nb_{iter} = 20, Nb_{year} = 400, Nb_{cycle} = 8) providing the lowest RMSE value (55.9 m) are also those providing the lowest ice volume anomaly (∆Vol when using the equilibrated temperature. This is because the warmer equilibrated temperature with respect to the non-equilibrated one leads to higher velocities which ultimately favour the ice thickness underestimation in the central regions (shown in Fig. 6). Using a smaller enhancement factor with the equilibrated temperature reduces the gap (3.3 m for Nb_{cycle} = 5694 Gt) and ice-volume trend (IVT-8) and provides a closer response to that obtained for Ef = 0.03 mm yr^-3 with the non-equilibrated temperature.

If the RMSE is lower when the non-equilibrated temperature profile is used, the trend ξ is nonetheless largely higher (24.7 cm yr^{-1}). While the RMSE value is lower than the one obtained with for Nb_{cycle} = 6) compared to the experiments with an equilibrated temperature (16.5 cm yr^{-1} for Ef = 3 (Nb_{iter}^{20}, Nb_{year}^{50}, Nb_{cycle}^{6}), the ∆Vol and IVT values are about 50 and 4 times higher. Indeed, a lower enhancement factor reduces the mismatch between observed and simulated ice thickness in central areas and hence the compensating effects between ∆H < 0 16.3 cm yr^{-1} for Ef = 1, for Nb_{cycle} = 6 and ∆H = 0 regions (Fig. 13). Increasing Nb_{iter} does not improve the ∆Vol and IVT results (8 respectively). This is expected as there is an important thermal adjustment when using a profile that is not consistent with the climatic forcing.

Difference between simulated and observed ice thickness (in m) obtained for the spin-up parameters (Nb_{iter}^{20}, Nb_{year}^{50}, Nb_{cycle}^{8}) providing the best fit when Ef = 1. However, despite existing differences with results obtained with the equilibrated temperature profile, this shows that our minimisation procedure is able to reduce the mismatch between simulated and observed ice thickness independently from the initial temperature profile.

5.2 Sensitivity to the basal drag coefficient

To evaluate the sensitivity of our spin-up approach to the initial distribution of the.

5.2 Sensitivity to the initial basal drag coefficient
As explained in Sec. 3, the initial basal drag coefficient $\beta$ for the first iteration of the minimisation procedure is the one used in \(^?\) (shown in Fig. 2a). To assess the robustness of our iterative procedure to the choice of the initial basal drag coefficient $\beta$, we have performed a new series of experiments starting from a uniform $\beta$ equal to 1 instead of the one from \(^?\). In terms of ice volume anomaly and ice volume trend, the parameters providing the best fit are exactly the same as for the experiments presented in the section 4.

Using $\text{Nb}_{\text{inv}} = 20$, $\text{Nb}_{\text{free}} = 200$, and $\text{Nb}_{\text{cycle}}$ varying from 1 to 15 with $E \beta = 1$, we obtain a minimum ice thickness RMSE of 49.9 m and a trend $\xi$ of 15.1 cm yr\(^{-1}\). While there are some minor spatial differences in terms of the inferred basal drag coefficient (Fig. 2c), the aggregated metric such as the RMSE and the trend are identical to the results presented in Table 1. In the same way, the simulated ice thickness and surface velocities obtained with $\beta = 1$ present very small differences with those obtained when starting from the Ice2Sea basal drag coefficient (Figs S3 and S4). This illustrates the robustness of the method and shows that it does not depend on the chosen initial distribution of the basal drag coefficient.

### 5.3 Sensitivity to the duration ($\text{Nb}_{\text{inv}}$ and $\text{Nb}_{\text{free}}$) of the minimisation procedure

In this section we assess the sensitivity of the minimisation procedure to the coefficients $\text{Nb}_{\text{inv}}$ (i.e., $\text{Nb}_{\text{iter}}^{20}$, $\text{Nb}_{\text{year}}^{50}$, $\text{Nb}_{\text{cycle}}^{6}$), with $\Delta \text{Vol}$: the duration of the period during which the basal drag coefficient is iteratively computed — 1\(^{st}\) step (duration of the free-evolving simulations — 2\(^{nd}\) step). While in Sec. 4.2 we used $\text{Nb}_{\text{inv}} = 583 \text{ Gt} \text{ and IVT} = 0.018 \text{ mm yr}^{-1}$, although these values are not as low as those obtained in our reference experiment ($20 \text{ and } \text{Nb}_{\text{iter}}^{20}$, $\text{Nb}_{\text{year}}^{50}$, $\text{Nb}_{\text{cycle}}^{6}$)$_{\text{free}} = 200$, here we explore a range of combinations of these parameters exploring four values for $\text{Nb}_{\text{inv}}$ (20, they are still satisfactory, as the final $\Delta \text{Vol}$ value is only 0.02 \% that of the present-day ice volume. After $\text{Nb}_{\text{inv}} = 40, 80, 160$ yr and $\text{Nb}_{\text{free}} = 50, 100, 200$ and 400 yr). Using an enhancement factor of 1, we iterate on 15 cycles ($\text{Nb}_{\text{cycle}} = 6$, the new spatial distribution of the from 1 to 15). The initial conditions are the same as in Sec. 4.2.

Figure 14 shows the evolution of the RMSE as a function of the number of cycles performed for a range of $\text{Nb}_{\text{free}}$ values. As previously shown, there is a strong decrease in RMSE between the first two cycles and only a limited improvement when using more than 6 cycles. The response is very linear: using larger $\text{Nb}_{\text{free}}$ leads to a smaller RMSE. This can be explained by the fact that the correction computed at the end of the 2\(^{nd}\) step, after $\text{Nb}_{\text{free}}$, is greater if the duration of the free-evolving simulation is longer. This means that the changes impose to the new basal drag coefficient is very similar to that obtained in $\text{Nb}_{\text{iter}}^{20}$, $\text{Nb}_{\text{year}}^{50}$, $\text{Nb}_{\text{cycle}}^{6}$ (Fig. ?c). This illustrates the robustness of the method and shows that it does not depend on the chosen initial distribution of the basal drag coefficient.

In Fig. 15 we show the evolution of the RMSE as a function of the number of cycles performed for a range of $\text{Nb}_{\text{inv}}$ values (20, 40, 80 and 160 yr). The RMSE difference for a given $\text{Nb}_{\text{cycle}}$ is generally less than 10 m, while this difference is sometimes larger than 20 m when $\text{Nb}_{\text{free}}$ varies from one value to the other. This suggests that $\text{Nb}_{\text{inv}}$ is of second importance relative to $\text{Nb}_{\text{free}}$. The RMSE appears to be slightly smaller for longer $\text{Nb}_{\text{inv}}$. For example, for $\text{Nb}_{\text{free}} = 200$ yr, increasing $\text{Nb}_{\text{inv}}$ from 20 yr to 40, 80 or 160 yr slightly reduces the minimum RMSE by 0.1 m, 1.7 m or 3.5 m respectively, and decreases the trend.
\[ \xi \] by 13.7\%, 7.2\% and 21.1\% for \( N_{\text{cycle}} \) equal to 12, 11 and 8 respectively. The minimum RMSE value (46.1 m) and trend \( \xi \) (12.3 cm yr\(^{-1}\)) are reached with \( N_{\text{cycle}} = 10 \) and with \( N_{\text{rev}} = 160 \) yr. Performing more cycles once the minimum RMSE is reached does not improve the results.

Overall, the combination of the highest \( N_{\text{rev}} \) (160 yr) with the highest \( N_{\text{free}} \) (400 yr) leads to the smallest RMSE (44.1 m) with a trend \( \xi \) of 9.9 cm yr\(^{-1}\) for \( N_{\text{cycle}} = 11 \). However, this minimum represents a considerable amount of computing time (6160 years) and does not represent the most efficient combination. As shown in Figures 5, 8 and 13, the minimum RMSE generally stabilises between \( N_{\text{cycle}} \) equal to 4 and 6. This means that similar RMSE and trend \( \xi \) could be obtained using fewer computing resources. For each set of combination, the mean value of the best RMSE values is equal to 51.1 m and is associated to a mean trend \( \xi \) of 15.5 cm yr\(^{-1}\). The experiment with \( N_{\text{rev}} = 20 \) yr, \( N_{\text{free}} = 200 \) yr and \( N_{\text{cycle}} = 6 \) produces a RMSE 0.6 m lower than the mean and is more than three times faster than best of the RMSE (1320 years to be compared to 6160 years).

6 Summary and discussion

In order to improve the reliability of Greenland ice sheet simulations in a future transient climate, an accurate evaluation of the present-day trend of ice flow dynamics is required. One the major difficulties in addressing this need lies in the poorly constrained observational data of the basal conditions that strongly control the ice motion in the entire ice sheet. Here, we present an inverse method to infer the spatial distribution of the basal drag coefficient in such a way that the mismatch between simulated and observed GrIS ice thickness is minimized. The best fit is minimised. As such, our target criteria are defined for the sets of minimisation procedure parameters providing minimum values of ice volume trend and difference between simulated and observed ice volume. This choice was motivated by the need to refine the projections of GrIS contribution to global sea level rise thickness RMSE (with respect to observations) and ice thickness trend, which are respectively as low as \( \sim 50 \) m and 15 cm yr\(^{-1}\) for our best fit. This remains in the range of PDC12 results. The great advantage of the method is its rapid convergence (a few hundred i.e. 1320 years) making it suitable for more computationally expensive models. Moreover, we have also shown that it only poorly depends on the initial guess of the spatial distribution of the basal drag coefficient and the initial temperature profile.

However, choosing the ice volume anomaly as the main criterion to assess the performance of the adjustment of the basal sliding, our method cannot be applied in regions of frozen bed and is only effective in thawed bed areas where basal sliding may occur. However, in case of a too large deformation rate in these regions, the basal drag coefficient is set to its maximum value to counteract the too fast ice flow. The limit of applicability of the method led us to investigate the impact of the enhancement factor which is expected to have a large influence on the deformation rate, and thus, on the ice flow and subsequently on the simulated ice thickness. We performed a series of simulations with a range of various values of the enhancement factor (from 0.5 to 5) and showed that the mismatch between the simulated and the observed GrIS topography is reduced with an appropriate tuning of the enhancement factor. This highlights that the overall performance of the method...
is critically dependent on the basal thermal state and points out that the finding of appropriate initial conditions with a simple adjustment procedure remains an undetermined issue. Actually, multiple combinations of the enhancement factor and the basal drag coefficient can produce a simulated ice thickness close the observed one, but this cannot discard the possibility of errors in modelled basal and vertical temperatures. A logical next step could lie in the adjustment of the spin-up method may lead to misinterpretations of the quality of the fitting procedure. As illustrated in Section 5, compensating effects may arise between regions of positive and negative ice thickness anomalies (w.r.t. observations). It is thus highly recommended to choose the best compromise between the minimisation of errors in ice volume on one hand and a low ice thickness root mean square error on the other hand. In this study we focused only on the results leading to RMSE values not greater than a few tens of meters. This remains in the range of PDC12 results who used the minimisation of ice thickness errors as the main target criterion. Because the basal sliding velocities are not computed in frozen bed areas in our hybrid model, reducing further the RMSE through inverse techniques of basal conditions, and thereby the compensating effects, is not an easy task. However, an appropriate tuning of the enhancement factor (Sect. 5.2) allows the adjustment of ice flow velocities in regions only governed by the shallow ice approximation and may improve the final GrIS topography. Basal drag coefficient combined with a similar approach for the adjustment of the enhancement factor in frozen bed areas. However, we have shown that the minimisation procedure presented in this paper is able to reduce the ice thickness mismatch regardless of the initial temperature profile. This offers the possibility to tune the thermal state to be as close as possible to the observations (inferred basal temperature as in ?, or vertical profiles at ice core locations) before running the iterative minimisation procedure. Increasing our confidence in the vertical temperature profile would therefore increase our confidence in the choice of Ef and β values.

Finally, we have shown in this paper that the iterative adjustment of β produces modelled surface velocities that compare well with the observed ones. This suggests that future work could include an additional metric related to surface ice velocities so as to further reduce the uncertainties associated with the choice of model parameters and variables.

Another limitation of the method may come from the model resolution. The succession of higher/lower ice thickness due to the succession of valleys/ridges in mountain areas may be poorly resolved. Owing to the insulation effect of the ice, this may lead to an erroneous representation of the basal temperature patterns, and SSA regions may be erroneously interpreted as frozen bed regions and vice versa (?). Higher-β. This drawback is clearly illustrated in our study in Fig. 6 (Ef = 1). Indeed, the simulated ice thickness obtained with the inversion procedure is generally less than 50 m in most GrIS areas, but can be greater than several hundred meters in coastal mountain ranges such the central eastern margin area where ice flow occurs in deep valleys. An alternative solution consists in correcting the basal temperature to account for bedrock roughness, similarly to what was done in PDC12 to improve their inversion procedure in the Transantarctics. On the other hand, higher resolution models can also better account for the dynamics of small-scale outlet glaciers and for their interactions with floating ice that strongly influence the ice sheet mass balance (e.g., ?). While this effect is less crucial for Greenland than for Antarctica, recent observations have highlighted increasing thinning rates in most coastal regions (??) causing grounding line retreat and significant destabilization of grounded glaciers. Ice sheet mass balance (e.g., ?). However, due to the elliptic character of the
SSA equation (e.g., ?), the local adjustment of the basal drag coefficient impact the ice velocity of neighbouring points. As a result increased resolution may increase the noise, unless introducing a smoothing function that filters the high frequency noise (?).

The reliability of the method also depends on the quality of observations data and of climate forcing. Errors in observed surface or bedrock topography or in SMB patterns different from those associated to the observed ice thickness would give rise to errors in the present-day estimated ice thickness and thus to an erroneous choice of the best spin-up parameters. In the same way, large uncertainties remain in the reconstruction of the geothermal heat flux that strongly impacts the basal temperature.

Finally, we would like to stress that in our simulations, the spatial distribution of the basal drag coefficient does not change through time. However, changes in basal hydrologic conditions along with changes in ice surface elevation and ice extent are likely to occur in a changing climate. While a constant spatial distribution of the $\partial/\partial t$ coefficient may seem reasonable for short-term projections, it is more questionable at the century time scale, and future modelling efforts should therefore be undertaken to compute interactively the basal drag coefficient as a function of changes in basal conditions.

7 Code and data availability

The developments of...

Code and data availability

The developments on the GRISLI source code are hosted at https://forge.ipsl.jussieu.fr/mailman/listinfo.cgi/grisli-grisli (last access: 23 March 2019 IPSL, 2019). At present, it is in a transitional phase with the aim of being released publicly in the future, but it is currently not publicly available. Access to those who conduct research in collaboration with the GRISLI users group can be granted upon request to C. Ritz (catherine.ritz@univ-grenoble-alpes.fr). The model outputs from the simulations described in this paper are freely available from the authors upon request.

Author contributions. The implementation of the iterative process in the GRSILI model was initially done by C. Ritz and further optimized by S. Le clec’h, A. Quiquet and C. Dumas. Analyses of the experiments were performed by S. Le clec’h and discussed with the co-authors.

The paper was written by S. Le clec’h, A. Quiquet and S. Charbit with contributions from M. Kageyama.

Competing interests. The authors declare that they have no conflict of interest.

Acknowledgements. We are very grateful to David Pollard and Stephen Price for their fruitful comments that helped us to refine our approach and improve the manuscript. The authors would like to thank X. Fettweis for providing outputs from the MAR model used as climate forcing.
S. Le clec'h, M. Kageyama, S. Charbit and C. Dumas acknowledge the financial support from the French-Swedish GIWA project French projects OSCAR (LEFE/INSU) and the ANR AC-AHC2 – as well as the CEA for the PhD funding from the French-Swedish GIWA project.

S. Le clec'h has been funded by the CEA. He also acknowledges the iceMOD project funded by the Research Foundation – Flanders (FWO – Vlaanderen). A. Quiquet is funded by the European Research Council grant ACCLIMATE no 339108 and by the Louis Bachelier Institute.
Table 1. Integrated metrics computed from the last 5 years of the 200 yr free-evolving simulations of the 2nd step (green box in Fig. 3) for $N_{b_{PTE}} = 20$ and $N_{b_{ECE}} = 200$ with varying enhancement factors ($Ef$) ranging from 0.5 to 5.

<table>
<thead>
<tr>
<th>Enhancement factor value</th>
<th>$Ef=0.5$</th>
<th>$Ef=1$</th>
<th>$Ef=1.5$</th>
<th>$Ef=2$</th>
<th>$Ef=2.5$</th>
<th>$Ef=3$</th>
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<th>$Ef=4.5$</th>
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<tr>
<td>$N_{b_{cycle}} = 6$</td>
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<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>RMSE in m</td>
<td>53.0</td>
<td>50.3</td>
<td>50.8</td>
<td>52.3</td>
<td>55.4</td>
<td>59.3</td>
<td>64.6</td>
<td>70.4</td>
<td>74.8</td>
<td>78.8</td>
</tr>
<tr>
<td>Volume difference in Gt</td>
<td>33089</td>
<td>20671</td>
<td>7579</td>
<td>-7224</td>
<td>-22290</td>
<td>-36570</td>
<td>-49727</td>
<td>-64113</td>
<td>-76951</td>
<td>-90385</td>
</tr>
<tr>
<td>Trend in ice thickness $\xi$ in cm yr$^{-1}$</td>
<td>18.3</td>
<td>16.3</td>
<td>14.8</td>
<td>15.1</td>
<td>15.7</td>
<td>16.5</td>
<td>18.5</td>
<td>18.9</td>
<td>19.2</td>
<td>20.1</td>
</tr>
<tr>
<td>$N_{b_{cycle}}$ for lowest RMSE</td>
<td>15</td>
<td>13</td>
<td>9</td>
<td>13</td>
<td>15</td>
<td>11</td>
<td>15</td>
<td>11</td>
<td>13</td>
<td>13</td>
</tr>
<tr>
<td>Minimal RMSE in m</td>
<td>52.1</td>
<td>49.9</td>
<td>49.8</td>
<td>51.9</td>
<td>54.2</td>
<td>57.9</td>
<td>63.6</td>
<td>68.9</td>
<td>74.0</td>
<td>78.2</td>
</tr>
<tr>
<td>Volume difference in Gt for the cycle with lowest RMSE</td>
<td>30738</td>
<td>18072</td>
<td>4922</td>
<td>-10254</td>
<td>-27240</td>
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<td>-67400</td>
<td>-79316</td>
<td>-93313</td>
</tr>
<tr>
<td>Trend in ice thickness $\xi$ in cm yr$^{-1}$ for the cycle with lowest RMSE</td>
<td>18.3</td>
<td>15.0</td>
<td>13.4</td>
<td>16.3</td>
<td>16.3</td>
<td>16.5</td>
<td>17.3</td>
<td>24.6</td>
<td>26.9</td>
<td>21.8</td>
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</table>
Figure 1. Climate forcing averaged over the 1979-2005 period simulated by the atmospheric regional model MAR (?) and interpolated on the GRISLI ice sheet model grid (5 km x 5 km): a/ mean surface mass balance (in m yr$^{-1}$, i.e. $10^3$ kg m$^{-2}$ yr$^{-1}$) with the black line representing the equilibrium line indicating the frontier between accumulation and ablation areas; b/ mean annual surface temperature (in °C) with the white dashed lines representing the 5 °C iso-contours. In addition, c/ basal temperature difference with respect to the pressure melting point (in °C) after the 30 000 years equilibrium temperature computation for a fixed topography.
Figure 2. Spatial distribution of the basal drag coefficient (in $\log_{10}$ Pa yr $m^{-1}$) in: a/ the initial condition, used in the GRISLI Ice2Sea simulations; b/ the iterative cycle that produces the minimal RMSE ($Nb_{cycle} = 9$) when using $Ef = 1$ for $Nb_{inv} = 20$ yr and $Nb_{free} = 200$ yr (Sec. 4.2); c/ the iterative cycle that produces the minimal RMSE ($Nb_{cycle} = 9$) when using $Ef = 1$ for $Nb_{inv} = 20$ yr and $Nb_{free} = 200$ yr but starting from a uniform basal drag coefficient (Sec. 5.2).
Figure 3. Schematic representation of the iterative minimisation procedure method. The iterative process itself (step 1 and step 2) is shown in the red box. The assessment of the performance of the method for a given cycle (e.g., RMSE and trend discussed in Sec. 4 and 5) is performed at 200 years of the 2nd step (green box), independently from the value of Nb\textsubscript{free}. The initial conditions for the iterations are the results of the 30 000 year temperature computation using a fixed topography (black box) followed by a relaxation of the surface topography (blue box).
Figure 4. (a) Ice thickness difference (in m) simulated at the end of a 200-year-long simulation without any specific initialization procedure with respect to the observed ice thickness from ?. (b) Difference (in m yr$^{-1}$) between the surface ice velocity in the same simulation and the observed surface velocity from ?. The dashed lines correspond to the 1000 m surface elevation iso-contours for the simulated topography. Grey areas represent non-ice-covered areas. Note that a logarithmic scale is used for the ice velocity difference.
Figure 5. Ice thickness root mean square error w.r.t. observations from ?, in meters for $\text{Nb}_{\text{inj}} = 20$, $\text{Nb}_{\text{free}} = 200$ and with enhancement factors (Ef) ranging from 0.5 to 5 as a function of the number of iterations ($\text{Nb}_{\text{cycle}}$).
Figure 6. Difference between the simulated and the observed (?) ice thickness (in meters) for Ef ranging from 0.5 to 5 for the iterative cycle Nb cycle that produces the lowest RMSE (Table 1). Here, Nb free and Nb ice are set to 20 and 200 years respectively.
Figure 7. Spatial distribution of the basal drag coefficient (in $\log_{10}$ Pa yr $m^{-1}$) for $Ef$ ranging from 0.5 to 5 for the iterative cycle $Nb_{\text{cycle}}$ that produces the lowest RMSE (Table 1). Here, $Nb_{\text{inv}}$ and $Nb_{\text{rece}}$ are set to 20 and 200 years respectively.
Figure 8. GrIS volume difference w.r.t. observations from ?, in Gt, for $Nb_{UPR} = 20$ yr and $Nb_{FREE} = 200$ yr and with enhancement factors (Ef) ranging from 0.5 to 5 as a function of the number of iterations ($Nb_{CYCLE}$).
Figure 9. Temporal evolution of GrIS total volume difference in Gt (solid lines) and RMSE in m (dashed lines) for $\text{Nb}_{\text{int}} = 20$ yr and $\text{Nb}_{\text{free}} = 200$ yr, with varying enhancement factors (Ef) ranging from 0.5 to 5. The $\text{Nb}_{\text{cycle}}$ chosen here corresponds to the one producing the minimum ice thickness RMSE (see Table 1).
Figure 10. a/ MEaSUREs Greenland ice sheet velocity (in m yr$^{-1}$) map from InSAR for the 2016-2018 mean period (º). b/ Simulated surface ice velocity using Ef = 1 for Nb$_{inv}$ = 20 yr, Nb$_{free}$ = 200 yr and Nb$_{cycle}$ = 13 (the one producing the minimal ice thickness RMSE, Table 1).
Figure 11. Simulated ice surface velocity difference (in m yr$^{-1}$) with respect to observations (?) using $E_f$ ranging from 0.5 to 5 for $Nb_{UL} = 20$ yr, $Nb_{free} = 200$ yr and $Nb_{cycle}$ that corresponds to the one producing the lowest ice thickness RMSE (see Table 1).
Figure 12. Vertical temperature profiles (in °C) from the ice sheet surface to the bedrock over Greenland central region (73-74.5 °N, 40-43 °W). The black dashed line is the non-equilibrated temperature profile used in ?. The coloured lines are the profiles in the course of the long 30-kyr experiment for the temperature calculation. The red profile is the one used as initial condition for the experiments shown in Sec. 4.2.
Figure 13. Ice thickness root mean square error w.r.t. observations from $\ldots$ in meters for $N_{\text{INV}} = 20$ yr, $N_{\text{FREE}} = 200$ yr as a function of the number of iterations ($N_{\text{ITER}}$). Dark blue dots are for the experiment that uses the non-equilibrated temperature profile as initial condition and $E_f = 3$. Cyan and orange dots are for the experiments using the equilibrated temperature and $E_f = 3$ and $E_f = 1$, respectively.
Figure 14. Ice thickness root mean square error w.r.t. observations from $\cdot$, in meters for a fixed $\text{Nb}_{\text{INV}} = 20$ and four $\text{Nb}_{\text{FREE}}$ values (50, 100, 200, 400) as a function of the number of iterations ($\text{Nb}_{\text{CYCLE}}$). The experiments use an enhancement factor of 1.
Figure 15. Ice thickness root mean square error w.r.t. observations from ?, in meters for a fixed $N_{\text{free}} = 200$ and four $N_{\text{inv}}$ values (20, 40, 80, 160) as a function of the number of iterations ($N_{\text{cycle}}$). The experiments use an enhancement factor of 1.