Dear Dr. Juanzhen Sun,

The authors deeply appreciate you for your useful comments. Our sincere responses are shown as follows and will be presented in the final form of our manuscript.

Sincerely,
Takuya Kawabata on behalf of the authors

[Comment] 1. Page 2, line 9: "The objective of our study was thus to improve QPE and ..." How can you improve QPE by assimilating the dual-pol data with the developed operators?

[Response] As shown in Bauer et al. (2015), high resolution DA systems with rapid update cycle is able to produce rainfall distributions comparable to actual observations. We added the following words at P2L9.

",... which was discussed in Bauer et al. (2015) in the context of a data assimilation with high resolution and rapid update cycle,"

[Comment] 2. Page 2, line 16: "...our study is their first implementation in variational assimilation systems". Note that Li and Mecikalski (2010) implemented an dual-pol operator in WRF Var similar to your KD.

[Response] We removed the expression regarding KD and modify the sentence to “Although this emulator (Jung et al., 2008a, 2008b) has been used previously as observational operators in ensemble Kalman filter data assimilation systems, to our knowledge, our study is their first implementation in variational assimilation systems." (P2L15-16)

[Comment] 3. Page 4, line 18: Change "the fitting" to "a statistical fitting".

[Response] Thank you for your suggestion. We changed it like as it.

[Comment] 4. Page 8, section 4.2 and Figure 2: What DA system did you use to produce the results in Figure 2? If you developed the operators for the two systems, it should be natural to show the analyses from both systems, right?
Response] We used NHM-4DVAR for Fig. 2. Figures by WRF DA were added as Figure 3 and explanations were displayed in the 3rd paragraph of Section 4.2.

[Comment] Page 8, line 16: These errors are quite large. Have you tried to use smaller errors?

Response] The values of the observational errors were determined after statistical investigations (Kawabata et al. 2018) but they were adopted conservatively (larger than the statistics). The sensitivity of the errors will be examined over scientific explores.

[Comment] Page 9, line 8: "....reasonable results with both the FIT and KD operators". This statement is not accurate. The result from KD is reasonable and clearly better than that from FIT for Zh.

Response] Since this statement is from the comparison with the first guess field (FG), which catches no rain it is clear that the results are better than FG. However, since we agree with you that $Z_{DR}$ from both methods are poor, we will add “except for $Z_{DR}$” at the end of the sentence.

[Comment] The result of $Z_{DR}$ from FIT has some characteristics of the observed $Z_{DR}$ but not that from KD. The $K_{DP}$ from both FIT and KD differ quite significantly from the observation. Can you speculate why the $K_{DP}$ is so poorly represented?

Response] Since $Z_{DR}$ of our results are much poorer than $K_{DP}$, we assumed that you were confused between $Z_{DR}$ and $K_{DP}$. We are doubting some points to be modified in our method, for instance, axis ratio, quality control, and initial conditions, but no investigation has been done yet. This is one of future issues.

[Comment] From the Eq (19), $Q_{r}$ and $K_{DP}$ have a quite simple relationship but why Zh is rather reasonable but not $K_{DP}$?

Response] In KD, we can see quite small values of $K_{DP}$ (Fig. 2f), but good agreement with OBS in its horizontal distribution, while $Z_{b}$ looks better than $K_{DP}$ as Dr. Sun pointed out. However, since some of erroneous convections are seen in $Z_{b}$, it is difficult to determine which one is truly better. We added the following sentence to P9L3:

"In KD, we can see quite small values of $K_{DP}$ (Fig. 2f), but good agreement with OBS in its
horizontal distribution, while $Z_h$ looks better than $K_{DP}$
Dear Dr. Soichiro Sugimoto,

The authors deeply appreciate you for your useful comments. Our sincere responses are shown as follows and will be presented in the final form of our manuscript.

Sincerely,
Takuya Kawabata on behalf of the authors

[General comment] The results, however, show the low performance in a simple assimilation experiment. The authors have to revise this manuscript mainly the relevant section of this 4DVar assimilation experiment before acceptance for publication. I think the authors need to reconsider the experimental configurations with additional consideration of the results, especially on the performance of FIT and the errors found in differential reflectivity field. My comments are addressed below.

[Response] First of all, we do not propose new operators for dual polarimetric radar data assimilation in this manuscript, while we implemented forward operators, which were investigated their accuracy in Kawabata et al. (2018), and developed their TL and AD codes in the present study.

The first goal of our development of the operators was to obtain reasonable distribution of reflectivity (Zh) comparable to existing single polarimetric radar data assimilations. Since we thought to achieve the goal as illustrated in the examples (Fig. 2) and we believe that there is no vital problem existed in the operators, we finished the first stage of the development and moved on to the scientific stage for getting better results and further exploring scientific issues. Therefore, the purpose of this manuscript is providing technical information on the structure of the operators and their first examples, even though their performances were still poor.

To clarify abovementioned point, we modified the first sentence of the last paragraph of the introduction to “The scope of this paper is to provide the technical information on the observational operators and some evaluation results to help the users understand theoretical and practical aspects of the operators”.

[Comment] 1. The authors had better mention clearly the range of application in abstract and summary. For example, operators can be applicable to C-band radar data (possibly S-band radar using findings of previous works). Another work, however, is needed to perform a
statistical fitting of results simulated by a numerical radar simulator in applying to a radar system with a shorter wavelength (e.g., X-band radar). Besides, a dataset of beam-filling (effected by the ground) is required. In terms of a mesoscale model, the use of a two moment microphysical scheme is assumed.

[Response] We added the following sentences to mention applicable frequencies in the manuscript. With respect to the beam filling data and the two-moment scheme, we already mentioned them in Section 2.11 and Section 3 of the original manuscript, respectively.

“FIT is also applicable for X- and S-bands by replacing the coefficients. Although we already prepared the coefficients for all bands in the source codes, the users should carefully investigate their validity.” (P5L9-10)

“Note that Eq. (19) is applicable for not only C-band but also X- and S-bands by putting their frequencies in $f$. “ (P5L15-16)

“Both of FIT and KD are applicable for not only C-band but also X- and S-bands.” (P9L19-20)

[Comment] 2. The descriptions in Section 2.1 are quite similar to the description found in the previous work of Kawabata et al. (2018). The authors should explain the essence of FIT concisely for the readers to understand that the forward operators have been already proposed in another work. Please revise Section 2.1 carefully avoid double posting. This revise may be reflected to title.

[Response] Since the scope of this manuscript is providing the technical information on the operators, we decided to describe equations again. For avoiding the confusion for double posting, we replace the word “developed” (P1L9) with “implemented” in the abstract and added the following sentence to the last of Section 2.1.2:

“Eqs. (4)-(19) follow Kawabata et al. (2018), and we put the equations with different order in this manuscript for the readers’ convenience to understand the flow of implementations of the forward, tangent linear and adjoint codes.”

[Comment] 3. The authors have to describe experimental configurations (Section 4.2) in detail, including the domain, the grid spacing of the mesoscale model used, 4DVar timeline, radar data configuration (e.g., resolutions, the number of elevation angles) at least. Which
mesoscale model is used, WRF or NHM? In terms of timeline, are several PPI data in one volume scan assimilated at the precise scanning timing during assimilation window? How about a method for preparing the background error?

[Response] We added the following sentences in Section 4.2:

“The horizontal resolution of NHM-4DVAR was 2 km and the length of assimilation window was 5 min, eleven PPI data from 0.5° to 4.8° elevations with the azimuth resolution at 0.7° and the range resolution of 150 m were assimilated at exact observation time as far as the time interval of NHM-4DVAR (10 s in this case) permits. The background errors were described in Kawabata et al. (2007) and (2011).”

“which displays the whole assimilation domain.” (P9L4)

“The second one was done using WRF 3DVAR with actual radar data from the DWD radar network (Helmert et al. 2014) for the same case with “Case 1” described in Kawabata et al. (2018). The horizontal resolution of WRF 3DVAR was 2 km, and polarimetric parameters and rain water content in single PPI data by Offenthal radar was assimilated (see Kawabata et al. 2018 for detailed information on the observation). The background errors were calculated with ensemble simulations by WRF initialized by ECMWF analysis using the “gen_be” tool compiled in WRFDA. Observational errors were the same with the first case. From the increments of polarimetric parameters (Fig. 3), although quite small impacts are seen, similar patterns are recognized in both methods and larger impact of $Z_h$ and $Z_{DR}$ were produced in FIT and KD, respectively.”

[Comment] 4. Observational errors are quite large, and, especially, the error for horizontal radar reflectivity seems to be unrealistic. A smaller error of radar reflectivity should be used. The authors may show the data to support the set-up of errors. The sensitivity of errors to the results should be discussed.

[Response] The values of the observational errors were determined after statistical investigations (Kawabata et al. 2018) but they were adopted conservatively (larger than the statistics). With respect to reflectivity, the value of 15 dBZ is the same with Kawabata et al. (2011). The sensitivity of the errors will be examined over the scientific explores. We added the following phrase at P8L21.
“which were determined after the statistical examination (Kawabata et al. 2018),”

How to estimate observational errors is one of the most important subjects in DA. The autocovariance method (see Appendix of Wulfmeyer et al. 2016) is more sophisticated than the try & error method and would be applied for radar data. We added the following phrase in Summary.

“it is necessary to estimate more appropriate observational errors (e.g., Wulfmeyer et al. 2016).” (P10L2)

[Comment] 5. Why not radial velocities (rv) assimilated in case of KD? Anyway, hail is associated with the event investigated. How does the authors consider the fall speed of hydrometeor in assimilating rv data?

[Response] We are sorry about the confusion on the rv assimilation. This was assimilated in KD as well. We modified the relevant sentence accordingly. Regarding the calculation method of rv, we used the same method with Sun and Crook (1997). We added the following sentence at the last of Section 4.

“In both cases, the radial velocity data were assimilated as the same method with Sun and Crook (1997).”

[Comment] 6. In Figure 2, non-zero differential reflectivity (Zdr) is calculated in non-stormy areas of retrieved and background fields, regardless of the type of operator. A possible reason regarding the axis ratio does not make sense to me. The authors should mention logically the reasons together with plotting smaller radar reflectivity Zh (< 15 dBZ). Assimilation of radar data (Zh, Zdr, Kdp) with quite weak echoes (e.g., clear air echo) is not appropriate in this framework. In operators, quite small Qr can lead to huge contribution in the perturbations of lambda, N0, and Zh (Zv). Therefore, the use of the minimum thresholds for Zh and Qr may remedy the low performance for the retrieval of Zdr, if the authors does not set the thresholds.

[Response] Thank you for your invaluable suggestion on improving the operators. Yes. It is necessary to polish up the quality control (QC) in the operators, although the same QCs are applied as described in Kawabata et al. (2018). We think there is no QC applicable for any radar site, any DA system, and (could be) any case. Thus, we have to improve continuously the QCs. In addition, one of the candidates for the low performance is the axis ratio model as
you mentioned. These are the future issues to be investigated. We mention these issues in the last paragraph of Summary.

“Furthermore, it is necessary to improve quality controls (QC) for polarimetric parameters, although the same QCs were applied as described in Kawabata et al. (2018), and the impact of axis ratio (Eq. (8)) and observational errors on assimilations will be investigated.”

[Comment] 7. Although FIT is theoretically more precise than KD, FIT shows the lower performance than KD. The authors jump to conclusions too quickly by regarding the nonlinearity in FIT as the low performance. In the background Zh and Kdp, the mesoscale model cannot resolve convections at all. One possible situation is that it is too dry in the background water vapor field. If so, I guess the adjustment of humidity is needed before assimilation to retrieve larger Zh and Kdp.

[Response] We agree with you. Actually, the initial conditions of these cases were quite poor. It is necessary to enhance the quality of the initial conditions, for instance, by using ensemble Kalman filter systems or downscaling the more appropriate analysis fields. We mentioned this matter in Summary.

“It would be possible to overcome the weaknesses of the Zh distributions in FIT and FG through assimilation–forecast cycles and/or by adding other types of observation data, such as conventional observations, Doppler (water vapour) lidar data, and water vapour data observed by GNSS.”

Specific and minor comments:
[Comment] 1. (Page 2) Why do the authors address on quantitative precipitation estimation (QPE)? I think QPE is out of the main topic of this manuscript.

[Response] As shown in Bauer et al. (2015), high resolution DA systems with rapid update cycle is able to produce rainfall distributions comparable to actual observations. We added the following words at P2L9.

“, which was discussed in Bauer et al. (2015) in the context of a data assimilation with high resolution and rapid update cycle,”

[Comment] 2. (Page 2 Line7) Change “is” to “are”.
Response] We got. Thank you.

[Comment] 3. (Pages 2 and 3) Operators proposed consider the relations between variables concerning rain water and radar observables. I feel something wrong with the mention of “cloud water”. Can C-band radar observe cloud water?

[Response] No. We removed all “cloud water” from the manuscript.

[Comment] 3. (Pages 2 and 3) Please check if the WRFDA “(WRF-Var” mentioned in the manuscript) deals with the “perturbation” of rainwater mixing ratio, not dealing with “the total part”.

[Response] The prognostic variable in WRF is rainwater itself, not the total water, if you mentioned this point.

[Comment] 4. (Page 4 Line 6) Change “proportional” to “polynomial”.


[Comment] 5. (Page 8 Line 17) Essentially, is Kdp assimilate in KD?

[Response] Yes. We added “derived from $K_{DP}$” after $Q_{\text{rain}}$.

[Comment] 6. (Page 9 Lines 8 & 9) The performance found in a simple assimilation test is far from a successful level.

[Response] Since these statements are from the comparison with the first guess field (FG), which catches no rain it is clear that the results are better than FG. However, since we agree with you that $Z_{\text{DR}}$ from both methods are poor, we will add “except for $Z_{\text{DR}}$” at the end of the sentence.
Observational operators for dual polarimetric radars in variational data assimilation systems (PolRad VAR v1.0)

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Abstract. We developed two observational operators for dual polarimetric radars and implemented them in two variational data assimilation systems: WRF Var, the Weather Research and Forecasting Model variational data assimilation system, and NHM-4DVAR, the nonhydrostatic variational data assimilation system for the Japan Meteorological Agency nonhydrostatic model. The operators consist of a space interpolator, two types of variable converters as well as their linearized and transposed (adjoint) operators. The space interpolator takes account of the effects of radar-beam broadening in both vertical and horizontal directions and climatological beam bending. The first variable converter emulates polarimetric parameters with model prognostic variables and includes attenuation effects, and the second one derives rainwater content from the observed polarimetric parameter (specific differential phase). We developed linearized and adjoint operators for the space interpolator and variable converters and then assessed whether the linearity of the linearized operators and the accuracy of the adjoint operators were good enough for implementation in variational systems. The results of a simple assimilation experiment showed good agreement between assimilation results and observations with respect to reflectivity and specific differential phase but not with respect to differential reflectivity.

1 Introduction

The Weather Research and Forecasting Model (WRF; Skamarock et al., 2008) is a widely used numerical weather model that was developed as a community model, and WRF Var, its data assimilation system (Barker et al., 2012), provides initial conditions for the model. NHM-4DVAR is a nonhydrostatic 4DVar system for the Japan Meteorological Agency nonhydrostatic model (JMANHM; Saito et al., 2012) that functions at storm scale (Kawabata et al., 2007, 2014a). Many remote sensing data are available for NHM-4DVAR, such as slant total delay, zenith total delay, and precipitable water vapour observed by global navigation satellite systems (GNSS; Kawabata et al., 2008); conventional radar data, including directly assimilated reflectivity data (Kawabata et al., 2011); and Doppler lidar data (Kawabata et al., 2014b). Because data assimilation associates observations with model fields, to make use of advanced observations, data assimilation methods need to be continuously developed and implemented into variational data assimilation systems.
Observations obtained by dual polarimetric radars are utilized by operational systems at many meteorological and hydrological operation centres (e.g., in the United States, France, Germany and Japan) to improve the accuracy of quantitative precipitation estimation (QPE). These radars provide polarimetric parameters, including the horizontally polarized reflectivity factor ($Z_H$), the vertically polarized reflectivity factor ($Z_V$), differential reflectivity ($Z_{DR}$), and the specific differential phase ($K_{DP}$). Many QPE methods that use these parameters have been proposed (e.g., Jameson, 1991; Jameson and Caylor, 1994; Ryzhkov and Zrnić, 1995; Anagnostou et al., 2008; Kim et al., 2010; Ryzhkov et al., 2014; Adachi et al., 2015). Because QPE methods using dual polarimetric radar parameters are expected to be better than methods using single polarization radar data, we developed assimilation methods for dual polarimetric radar observations for both WRF Var and NHM-4DVAR. The objective of our study was thus to improve QPE, which was discussed in Bauer et al. (2015) in the context of a data assimilation with high resolution and rapid update cycle, and quantitative rainfall forecasts (QPF) through the use of better analysis fields obtained by the assimilation of dual polarimetric radar observations.

We chose an emulator (Zhang et al., 2001) and an estimator (Bringi and Chandrasekar, 2001) to use as forward operators after evaluating their accuracy (Kawabata et al., 2018). In addition, because both WRF Var and NHM-4DVAR consider only perturbations to cloud water and rainwater in their tangent and adjoint models, our operators also deal only with cloud water and rainwater and exclude ice particles. Although both this emulator (Jung et al., 2008a, 2008b) and this estimator (Yokota et al., 2016) have been used previously as observational operators in ensemble Kalman filter data assimilation systems, to our knowledge, our study is their first implementation in variational assimilation systems. We call the current version of the operators as PolRad VAR v1.0.

The first author has mainly contributed to the WRF Var version of these operators was developed over the rapid update WRF 3DVAR system at the University of Hohenheim, Germany (see, e.g., Schwitalla et al., 2011; Schwitalla and Wulfmeyer, 2014; Bauer et al., 2015), and to the version for NHM-4DVAR was developed at Meteorological Research Institute, Japan Meteorological Agency. mainly by the first author.

In the scope of this paper, we describe theoretical and practical aspects of the is to provide the technical information on the observational operators and some evaluation results to help the users understand theoretical and practical aspects of the operators. The forward operators (space interpolator and variable converters) and their linearized (tangent linear) and transposed (adjoint) operators are described in Sect. 2. Section 3 describes setup options of the observational operators, Sect. 4 presents verification and assimilation test results, and Sect. 5 is a summary.

2 Observational operators

In variational data assimilation systems, a cost function is defined and then iteratively minimized until its gradient becomes zero. The cost function and its gradient are defined as

$$J(x) = \frac{1}{2}(x - x^b)^T B^{-1} (x - x^b) + \frac{1}{2} (H(x) - y)^T R^{-1} (H(x) - y),$$

(1)
\[ \nabla J(x) = B^{-1}(x - x^b) + H^T R^{-1}(H(x) - y), \]

where \( T \) denotes the transpose of a matrix; \( x, x^b, \) and \( y \) are model fields, first-guess fields, and observations, respectively; and \( H(x), H, \) and \( H^T \) represent the observational operators, their linearized operators (tangent linear), and their transposed (adjoint) operators, respectively. The observational operators work as variable converters \( (H_s) \) from model fields \( x \) to observational values related to observations \( y, \) and as space interpolators \( (H_s) \) from model to observational space as follows:

\[ H(x) = H_s H_s(x). \]

We developed two types of variable converters, a single space interpolator, and their tangent linear and adjoint operators. Both WRF Var and NHM-4DVAR consider only perturbations to the mixing ratios of the hydrometer variables (cloud water and rainwater) and not those to their number densities in the tangent linear and adjoint models. However, in the tangent and adjoint operators described here (Sect. 2.2), non-perturbed number densities of cloud water and rainwater are included. These variables are initialized to zero at the beginning or end of the operators, and their effects are directly considered in the cost functions of WRF Var and NHM-4DVAR, whereas their gradients are indirectly considered through perturbations of the mixing ratios of cloud water, rainwater, water vapour and other variables like temperature and pressure.

It is recommended that users of WRF Var run the system with CLOUD_CV (required) and the CV7 (optional) switches. The former adds mixing ratios of cloud water and rainwater to the default control variable set (Wang et al., 2013), and the latter replaces the control variables of stream function and velocity potential with momentum control variables to improve the performance of WRF simulations at high horizontal resolution (Sun et al., 2016). With these selections, the control variables in WRF Var are almost the same as those in NHM-4DVAR (Kawabata et al., 2011).

### 2.1 Variable converters

#### 2.1.1 Model variables to polarimetric parameters (FIT)

Among the many numerical precipitation scheme options (e.g., single-moment scheme, large-scale condensation scheme) for WRF and JMANHM, we chose two double-moment schemes (WRF, Morrison et al., 2009; JMANHM, Hashimoto, 2008) for our observational operators because such schemes predict both the number density \( (N; \text{m}^{-3}) \) and the mixing ratio \( (Q; \text{kg kg}^{-1}) \) of rainwater, whereas single-moment schemes predict only \( Q \). Therefore, two of three unknown parameters in the drop size distribution (DSD) function are detected by the schemes. Following Morrison et al. (2009), the DSD function is given by

\[ N(D) = N_0 D^\mu \exp(-\Lambda D), \]

where \( D \) (mm) is the raindrop diameter, \( N_0 \) (mm\(^{-1}\) m\(^{-3}\)) is the intercept parameter, \( \mu \) is the shape parameter, and \( \Lambda \) (mm\(^{-1}\)) is the slope parameter. \( \Lambda \) is given by

\[ \Lambda = \left( \frac{\pi \rho_w N_r}{10^3 \rho_d Q_r} \right)^{\frac{1}{3}}, \]
where $\rho_w$ is the density of water (997 kg m$^{-3}$ in this study) and $\rho_a$ is air density (kg m$^{-3}$), a model diagnostic variable. $\rho_a$ and $N_0$ are given by

$$\rho_a = \frac{p}{RT(1 + 0.61q_v)},$$  \hspace{1cm} (6)$$

$$N_0 = N_r \Lambda,$$  \hspace{1cm} (7)

where $p$ is atmospheric pressure (Pa), $R$ is the gas constant, $T$ is temperature (K), and $q_v$ is the mixing ratio of water vapour (kg kg$^{-1}$).

In our study, the remaining unknown parameter $\mu$ is fixed at zero, and $N(D)$ is based on bulk sampling, the minimum and maximum values of $D$ are set to 0.05 mm and 5 mm, respectively.

Because in the rainwater prognostic variables, raindrops are assumed to be spherical in both WRF and JMANHM, we introduce the axis ratio of a raindrop, which is proportional polynomial to $D$ (Brandes et al., 2002, 2005), as follows:

$$r = 0.9951 + 2.51 \times 10^{-2}D - 3.644 \times 10^{-2}D^2$$
$$+ 5.303 \times 10^{-3}D^3 - 2.492 \times 10^{-4}D^4.$$  \hspace{1cm} (8)

Radar observations are derived from measurements of the scattering of electromagnetic waves by raindrops. The first converter is based on fitting functions that relate equivolume diameters $D$ to scattering amplitude (Zhang et al., 2001). The backscattering amplitudes are represented by a power law function as follows:

$$|S_{h,v}(D)| = \alpha_{h,v}D^{\beta_{h,v}},$$  \hspace{1cm} (9)

where the coefficients $\alpha_{h,v}$ and $\beta_{h,v}$ are determined by fitting $D$ to the backscattering amplitudes $|S_{h,v}|$ calculated by the T-matrix method (Mishchenko et al., 1996). The difference between the horizontal and vertical forward scattering amplitudes is defined as

$$\text{Re}(f_h(D) - f_v(D)) = \alpha_k D^{\beta_k},$$  \hspace{1cm} (10)

where $f_h(D)$ and $f_v(D)$ represent the horizontal and vertical forward scattering amplitudes, and $\alpha_k$ and $\beta_k$ are determined by the fitting. Zhang et al. (2001) proposed fitting functions for S-band radars, and Kawabata et al. (2018) derived new fitting parameters for C-band radars. Following Zhang et al. (2001), horizontal (H) and vertical (V) reflectivity factors are

$$Z_{H,V} = \frac{4\lambda^4}{\pi^4|K_w|^2}\left(\alpha_{h,v}^2 N_0 \Lambda^{-2(2\beta_{h,v}+1)} \Gamma(2\beta_{h,v}+1)\right),$$  \hspace{1cm} (11)

where $\lambda$ (m) is the radar wavelength; $K_w$ is a constant, defined as $K_w = (\varepsilon - 1)/(\varepsilon + 2)$, where $\varepsilon$ is the complex dielectric constant of water estimated as a function of wavelength and temperature (Sadiku, 1985); and $\Gamma$ represents the Gamma function. The horizontal reflectivity $Z_H$ is converted to conventional reflectivity $Z_h$ (dBZ) by

$$Z_h = 10\log_{10}(Z_H),$$  \hspace{1cm} (12)

and $Z_{DR}$ (dB) is defined as

$$Z_{DR} = 10\log_{10}\left(\frac{Z_H}{Z_v}\right) = Z_h - Z_v.$$  \hspace{1cm} (13)

$K_{DP}$ ($^\circ$ km$^{-1}$) is defined as
\[ K_{DP} = \frac{180\lambda}{\pi} N_0 \alpha_k \Lambda^{-(\beta_k+1)} \Gamma(\beta_k + 1). \]  

(14)

The attenuation effects are calculated as follows:

\[ Z_h^{att}(x) = Z_h(x) - 2 \int_0^x A_H(s) ds, \]  

(15)

\[ Z_{DR}^{att}(x) = Z_{DR}(x) - 2 \int_0^x A_{DP}(s) ds, \]  

(16)

where \( Z_h^{att} \) and \( Z_{DR}^{att} \) represent attenuated \( Z_h \) and \( Z_{DR} \), respectively. \( A_H \) and \( A_{DP} \) are the specific attenuation (dB km\(^{-1}\)) and the specific differential attenuation (dB km\(^{-1}\)), respectively, defined as

\[ A_H = \alpha_H K_{DP} \beta_H, \]  

(17)

\[ A_{DP} = \alpha_d K_{DP} \beta_d, \]  

(18)

The values of the coefficients \( \alpha_h, \alpha_v, \alpha_k, \alpha_H, \beta_h, \beta_v, \beta_k, \beta_H \) and \( \beta_d \) for C-band in these equations are listed in Table 1.

Hereafter, this converter is called FIT.

FIT is also applicable for X- and S-bands by replacing the coefficients. Although we already prepared the coefficients for all bands in the source codes, the users should carefully investigate their validity.

### 2.1.2 Observations of polarimetric parameters to model variables (KD)

The second converter (hereafter KD) converts observed \( K_{DP} \) to rainwater content \( (Q_{\text{rain}}) \) according to the following relation:

\[ Q_{\text{rain}} = 3.565 \left( \frac{K_{DP}}{f} \right)^{0.77}, \]  

(19)

where \( f \) (GHz) is the radar frequency and the power law coefficients are from Bringi and Chandrasekar (2001). \( Q_{\text{rain}} \) in the model is defined as \( Q_{\text{rain}} = Q_r \rho_a \) (kg m\(^{-3}\)). Note that Eq. (19) is applicable for not only C-band but also X- and S-bands by putting their frequencies in \( f \).

Eqs. (4)-(19) follow Kawabata et al. (2018), and we put the equations with different order in this manuscript for the readers’ convenience to understand the flow of implementations of the forward, tangent linear and adjoint codes.

### 2.2 Tangent linear and adjoint operators

#### 2.2.1 Tangent linear and adjoint operators of FIT

Because only \( p, T, \) and \( q_v \) are perturbed in WRF Var and NHM-4DVAR, the linearized form of Eq. (6) is

\[ \Delta \rho_a = \frac{\Delta p}{RT(1 + 0.61 q_v)} - \frac{p \Delta T}{RT^2(1 + 0.61 q_v)} - \frac{0.61 p \Delta q_v}{RT(1 + 0.61 q_v)^2}, \]  

(20)

and the perturbations of \( \Lambda \) and \( N_0 \) are given as

\[ \Delta \Lambda = \frac{1}{3} \Lambda (-\Delta Q_r Q_r^{-1} - \Delta \rho_a \rho_a^{-1}), \]  

(21)

\[ \Delta N_0 = N_r \Delta \Lambda, \]  

(22)
where $\Delta Q_r$ and $\Delta N_0$ are perturbations of the mixing ratio and number density of rainwater, respectively. Note that the perturbation of $N_r$ is not considered in the adjoint model (see Sect. 2). Thus, the perturbations of $Z_{HV}$, $Z_{DR}$, and $K_{DP}$ are represented as

$$\Delta Z_{HV} = \frac{4\lambda^4}{\pi^4 |K|^2} (\alpha_{hv}^2 \Gamma(2\beta_{hv} + 1)(\Delta N_0 \Lambda^{-2(\beta_{hv} + 1)} - (2\beta_{hv} + 1)\Delta N_0 \Lambda^{-2(\beta_{hv} + 2)})$$

(23)

$$\Delta Z_{DR} = \Delta Z_h - \Delta Z_v$$

(24)

$$\Delta K_{DP} = \frac{180\lambda}{\pi} \alpha_k \Gamma(\beta_k + 1)(\Delta N_0 \Lambda^{-(\beta_k + 1)} - (\beta_k + 1)\Delta N_0 \Lambda^{-(\beta_k + 2)})$$

(25)

Finally, the perturbations of $A_H$ and $A_{DP}$ are

$$\Delta A_H = \alpha_h \beta_H \Delta K_{DP} K_{DP}^{\beta_H-1}$$

(26)

$$\Delta A_{DP} = \alpha_d \beta_d \Delta K_{DP} K_{DP}^{\beta_d-1}$$

(27)

The adjoint operators are represented by the transposed form of Eqs. (20)–(27), that is, $(\text{tangent linear})^T$. As an example, the adjoint of Eq. (27) is

$$\Delta K_{DP} = \Delta K_{DP} + \alpha_d \beta_d \Delta A_{DP}$$

(28)

### 2.2.2 Tangent linear and adjoint operators of KD

Because $K_{DP}$ in Eq. (19) is an observed value, it is not necessary to linearize the equation. However, the equation that relates $Q_{\text{rain}}$ to $Q_r$ (Sect. 2.1.2) needs to be linearized as follows:

$$\Delta Q_{\text{rain}} = \Delta Q_r \rho_a + Q_r \Delta \rho_a$$

(29)

The transposed form of this equation is used for the adjoint model (see Sect. 2.2.1).

### 2.3 Space interpolator

Space interpolators in data assimilation systems map the model space to the observational space according to the representativeness of the observations. In the case of radar data, the effect of beam broadening stands for the representativeness, typically for a beam width of approximately 1.0°. The broadening is characterized by a Gaussian distribution orthogonal to the direction to the radar beam. Most previous studies (e.g., Seko et al., 2004; Wattrelot et al., 2014), except Zeng et al. (2016), consider only vertical beam broadening, because numerical models have horizontal grid spacings of several kilometres, whereas they have vertical grid spacings in the lower troposphere of less than one kilometre. However, data assimilation systems must have sub-kilometre horizontal grid spacings as well (e.g., Kawabata et al. 2014a, Miyoshi et al., 2016) so that the space interpolators can take account of horizontal beam broadening. In addition, several phased array radars recently
deployed in Japan have different beam widths in the vertical and horizontal directions. Our operator thus considers beam broadening in both the vertical and horizontal directions.

In addition, it is important for the space interpolator to include beam-bending effects, which depend on atmospheric conditions. In this study, the bending is determined by considering the climatological vertical gradient of the refractive index of the atmosphere in accordance with the effective earth radius model (Doviak and Zrnić, 1993), following Haase and Crewell (2000), who showed statistically that the climatological refractive index is close to the actual refractive index at elevation angles higher than 1°, instead of by considering the actual atmospheric conditions, although Zeng et al. (2014) developed an excellent radar simulator that considers the actual refractivity of the atmosphere.

Remote sensing observations usually have higher spatial resolutions than the model grid spacings. To avoid correlations of the observational errors in such high-resolution data, it is necessary either to thin the data or to use “super observations”. In this study, we chose the super observation method, in which observations are averaged over each model grid cell. Super observation methods also have the advantage that they remove undesirable fluctuations associated with sub-grid-scale phenomena, the assimilation of which makes the numerical model unnecessarily noisy (e.g., Seko et al., 2004; Zhang et al., 2009).

First, we calculated the path of the centre of the radar beam in the model domain, including its elevation, azimuth, and bending angles (Fig. 1a). Once sufficient data are included within a model grid cell, they are averaged and mapped onto an interpolation point along the radar beam (IP in Fig. 1). This value at this point is a “super observation”, and it is compared with the modelled value, which was interpolated by using Gaussian weights (Fig. 1b). Moreover, we also developed the tangent and adjoint codes of the space interpolator.

3 Setup options

The operators are controlled by the namelist (“namelist.polradar”) as follows:

```
&name_obs
  o_dir='/home/usr/datadir',
  o_stn(1)='OFT',
  o_stn(2)='TUR',
  icnv=0/
```

Here, ‘o_dir’ is the directory for the input observational data; ‘o_stn’ indicates the station names of radar sites, where ‘max_stn’, the number of names, is set in ‘da_setup_obs_structures_polradar.inc’ in WRF Var and in ‘obs_dual_pol.f90’ in NHM-4DVAR; and ‘icnv’ is a switch for the selection of the observational operator, where “0” and “5” mean FIT and KD, respectively.

In addition, a file that defines for each radar areas where the beam is blocked by topography, named ‘beam_block_rate_${radar_site}.dat’, must be supplied by the user. This file is made by another program and should be prepared before the assimilation.
4 Results

4.1 Verification of the tangent linear and adjoint operators of FIT

In this section, we examine the linearity of only the FIT variable converter; it is not necessary to examine the linearity of the KD converter because of the intrinsic linearity of Eq. (19). We evaluated the linearity of FIT by performing a Taylor expansion.

If the original equation is given as
\[ y = H(x), \]
then the linearized equation is defined as
\[ \delta y = H \delta x. \]
If the linear equation is derived with no errors, the following Taylor expansion of Eq. (31)
\[ \frac{|H(x + \alpha \delta x) - H(x)|}{|\alpha||H\delta x|} = 1 + O(\alpha) \]
should be accurate within the rounding error of the computer. The results for \( Z_H, Z_V, \) and \( K_{DP} \) in Eqs. (11) and (14) are 1.00 when \( \alpha \) is \( 10^{-7} \) to \( 10^{-15} \).

Regarding the adjoint operator, we evaluated the following equation:
\[ (H\delta x)^T(H\delta x) = \delta x^T[H^T(H\delta x)], \]
where the left-hand side of Eq. (33) is calculated using the tangent linear operator, and on the right-hand side, the output variables of the tangent linear operator are input into the adjoint operator. This equation must be accurate within the rounding error. In FIT, the difference between the left- and right-hand sides was \(-8.215650382 \times 10^{-15} \), which we consider accurate enough.

4.2 Actual data assimilation test

We conducted a simple data assimilation test two simple data assimilation tests. Observational errors of \( Z_H, Z_{DR}, K_{DP}, \) and \( Q_{rain} \), which were determined after the statistical examination (Kawabata et al. 2018), were 15.0 dBZ, 2.0 dB, 4.0° km\(^{-1} \), and 4 g m\(^{-3} \), respectively.

The first one was done using NHM-4DVAR with actual radar data from the C-band dual polarimetric radar at the Meteorological Research Institute in Tsukuba, Japan (Yamauchi et al., 2012; Adachi et al., 2013). Observational errors of \( Z_H, Z_{DR}, K_{DP}, \) and \( Q_{rain} \) were 15.0 dBZ, 2.0 dB, 4.0° km\(^{-1} \), and 4 g m\(^{-3} \), respectively. In this experiment, only both of radial velocity data in addition to polarimetric parameters of \( Z_H, Z_{DR}, \) and \( K_{DP} \) were assimilated in FIT, and only radial velocity and \( Q_{rain} \) derived from \( K_{DP} \) was assimilated in KD. The assimilation window was from 2100 to 2105 UTC 23 June 2014, a day on which intense hail fell in Tokyo, Japan.

The horizontal resolution of NHM-4DVAR was 2 km and the length of assimilation window was 5 min, eleven PPI data from 0.5° to 4.8° elevations with the azimuth resolution at 0.7° and the range resolution of 150 m were assimilated. Each
PPI data was assimilated at exact observation time as far as the time interval of NHM-4DVAR (10 s in this case) permits. The background errors were described in Kawabata et al. (2007) and (2011). Analysis (KD and FIT) and observational (OBS) fields of $Z_h$, $Z_{DR}$, and $K_{DP}$ are shown in Fig. 22, which displays the whole assimilation domain. Although there was no rain region in the first-guess field (FG; Fig. 2d), $Z_h$ in KD was comparable to that in OBS from the standpoint of rainfall distribution and intensity, but $Z_h$ in FIT covered a much smaller area than it did in OBS. This smaller coverage may be due to nonlinearity in FIT. In KD, we can see quite small values of $K_{DP}$ (Fig. 2f), but good agreement with OBS in its horizontal distribution, while $Z_h$ looks better than $K_{DP}$. $K_{DP}$ values were smaller in both KD and FIT than in OBS. This result is similar to that of a statistical analysis performed by Kawabata et al. (2018). In contrast, $Z_{DR}$ values in KD and FIT were larger than OBS over large areas. This result implies that the calculation of the axis ratio of raindrops (Eq. 8) may need modification, because in the FG field, $Z_{DR}$ values and coverage were already too large, in comparison with those of OBS. The second one was done using WRF 3DVAR with actual radar data from the DWD radar network (Helmert et al. 2014) for the same case with “Case 1” described in Kawabata et al. (2018). The horizontal resolution of WRF 3DVAR was 2 km, and polarimetric parameters and rain water content in single PPI data by Offenthal radar was assimilated (see Kawabata et al. 2018 for detailed information on the observation). The background errors were calculated with ensemble simulations by WRF initialized by ECMWF analysis using the “gen_be” tool compiled in WRFDA. Observational errors were the same with the first case. From the increments of polarimetric parameters (Fig. 3), although quite small impacts are seen, similar patterns are recognized in both methods and larger impact of $Z_h$ and $Z_{DR}$ were produced in FIT and KD, respectively. In both cases, the radial velocity data were assimilated as the same method with Sun and Crook (1997).

5 Summary

We implemented two variable converters for polarimetric radars in the WRF variational data assimilation system (WRF Var) and the JMANHM data assimilation system (NHM-4DVAR). FIT simulates polarimetric parameters using a double moment cloud microphysics scheme, and KD estimates rainwater contents with the observed specific differential phase. Both of FIT and KD are applicable for not only C-band but also X- and S-bands. The advantage of FIT over KD is that it includes theoretically precise formulations for both the mixing ratio and number density of rainwater, as well as attenuation effects, whereas KD has advantages due to its linear formulation and small computational cost.

These operators work in conjunction with an advanced space interpolator, which considers 1) beam broadening in three dimensions, 2) different beam widths in vertical and horizontal directions, 3) the climatological beam-bending effect. The interpolator also simulated attenuation effects.

Tangent and adjoint operators of the two variable converters and the space interpolator were developed and implemented along with the forward operators. In a simple data assimilation experiment, we succeeded in assimilating actual polarimetric observations and obtained reasonable results with both the FIT and KD operators, except for $Z_{DR}$. However, our results show a need for further improvements of the $K_{DP}$ and $Z_{DR}$ estimates. It would be possible to overcome the weaknesses of the $Z_h$
distributions in FIT and FG through assimilation–forecast cycles and/or by adding other types of observation data, such as conventional observations, Doppler (water vapour) lidar data, and water vapour data observed by GNSS. Furthermore, it is necessary to improve quality controls (QC) for polarimetric parameters, although the same QCs were applied as described in Kawabata et al. (2018), the impact of axis ratio (Eq. (8)) and observational errors on assimilations will be investigated, and it is necessary to estimate more appropriate observational errors (e.g., Wulfmeyer et al. 2016). These challenges would improve QPE and QPF with the current forms of the operators.

**Code data availability**

The source code of PolRad VAR v1.0 for NHM-4DVAR belongs to the Meteorological Research Institute of the Japan Meteorological Agency and is not publicly available. Any researcher interested in the code is encouraged to contact the corresponding author. The observational operators and sign a contract for license to get the code. PolRad VAR v1.0 for WRF Var is currently being implemented within the community version of the system. WRF Var and researchers will be able to access its code via the WRF repository (http://www2.mmm.ucar.edu/wrf/users/downloads.html) in the near future. Any researchers interested in the current form of the code can get it from the corresponding author via e-mail.

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References


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Table 1. Values of coefficients $\alpha$ and $\beta$, $\alpha_{h,v,k}$ and $\beta_{h,v,k}$ in Eqs. (9) and (10) and $\alpha_{H,d}$ in Eqs. (17) and (18) are from Kawabata et al. (2018), and $\beta_{H,d}$ in Eqs. (17) and (18) are from Bringi and Chandrasekar (2001).

<table>
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<th>(subscript)</th>
<th>$h$</th>
<th>$v$</th>
<th>$k$</th>
<th>$H$</th>
<th>$d$</th>
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<td>0.0017</td>
<td>$2.36\times10^{-5}$</td>
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<td>0.013</td>
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<tr>
<td>$\beta$</td>
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<td>2.77</td>
<td>5.36</td>
<td>0.99</td>
<td>1.23</td>
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Figure 1. Schematic diagrams of (a) a super observation and (b) the space interpolator. Boxes represent model grid cells, and the red box indicates the grid cell in which the super observation is defined. The cross marks represent the interpolation point (IP). In (b), the grey curve indicates the Gaussian weights at various grid points (black circles); the solid black line shows the beam propagation; and the dashed lines illustrate beam broadening.
Figure 2. Horizontal distributions of polarimetric parameters of, from left to right, observations (OBS), assimilation results by KD and FIT with NHM-4DVAR, and the first-guess field (FG) at 2104 UTC on 23 June 2014. (a)–(d) $Z_h$; (e)–(h) $K_{DP}$; (i)–(l) $Z_{DR}$. 
**Figure 3.** Horizontal distributions of differences of polarimetric parameters between assimilation results by KD and FIT with WRF 3DVAR and observations at 1100 UTC on 14 August 2014.