Dear Klaus Gierens,

please find below our responses to the reviews and short comments as well as the updated manuscript with tracked changes since our discussion paper and a new Supplement. The changes in the manuscript are mainly based on the suggestions we got. We decided to add a Supplement where we demonstrate some aspects in more detail than required for the average reader. Furthermore, we integrated a number of smaller improvements (e.g. correct space around ”=” or units) and mention that our irregular shape data set now covers size parameters up to 30.2 instead of 27.5.

We also uploaded our Fortran code and data set to https://zenodo.org but did not yet finalize/publish it so that we can do some last modifications if necessary. The code and data set will have a DOI as suggested by Lutz Gross.

Best regards,

Josef Gasteiger and Matthias Wiegner
Authors’ response to comments by reviewer 1:

We are grateful to reviewer 1 for his/her positive rating of our manuscript and useful suggestions. In the following, comments by the reviewer are in italic font, our answers in normal font.

Moreover we want to refer to the revised manuscript where all changes can easily be tracked. We decided to not repeat all changes below for reasons of clarity.

Page 13: it is described the core of the calculations needed to combine the optical properties of single particles to model particle ensembles, which is key for real-world applications. I found the description from lines 5 to 13 unclear. The authors state that each aerosol mode is decomposed into eight contributions, stemming from the combination of the interpolation weights on $m_r$, $m_i$ and $\epsilon'$. I suggest to include a simple example, perhaps with only two variables (e.g. $m_r$ and $m_i$), to allow a better understanding of this algorithm with summation over weights combinations.

We agree that this description needs to be improved and thus rephrased it in the revised manuscript. Furthermore, we added an example in Section S4 of the Supplement and refer to this example in the paper.

Page 13: in the last paragraph, it is mentioned that the combinations are generally $J$ and not eight, but the reason is unclear. Please clarify with an example.

This confusion is probably also a result of the previous paragraphs. The case mentioned (a fixed $m_r$, $m_i$ and $\epsilon_m$ given by the user) where eight contributions are necessary is just an example. For example, if an $\epsilon_m$ distribution is given by the user instead of one fixed $\epsilon_m$, a larger number of contributions is required for a mode. Furthermore, if the ensemble consists of more than one mode the contributions from the different modes need to be added, thus $J$ increases as well. We rearranged and improved this section such that this point should be more clear now. Moreover, Section S4 of the Supplement will certainly help to make clear how the interpolation is done (in an "easy to follow case").

Page 13: the final calculations of ensemble optical properties are a summation over the aerosol modes included by the user. I suggest to add the information, in this paragraph, that this is equivalent to the so-called "external mixing" assumptions, compared to possible "internal mixing" assumption on the mixing state.

The reviewer’s suggestion helps the reader to understand better the context, so we added a sentence 'This approach corresponds to the so-called external mixing of particles.' in this paragraph.

Page 18, line 12: the URL of the tools web interface is given here for the first time in the manuscript. I suggest to put the information also in the abstract,
for more immediacy.

We agree that the URL should be given in the abstract and changed it accordingly. Moreover, we mention the URL also at the end of the introduction and in the conclusions.
Authors’ response to comments by reviewer 2:

We appreciate the positive assessment and the useful comments of reviewer 2. In the following, comments by the reviewer are in italic font, our answers in normal font.

Moreover we want to refer to the revised manuscript where all changes can easily be tracked. We decided to not repeat all changes below for reasons of clarity.

1. Make the fonts in the Figures/plots large enough for easy visualization.

We agree that the fonts of several figures were too small. We went through the manuscript and improved the figures’ font sizes where required for better visualization.

2. Section 2.1: Provide the definition of the irregular-shape radius.

Eqs. 1, 2, and 4 are valid also for irregular particles. Maybe this was a bit unclear because the previous paragraph concerns mainly spheroids. We now mention explicitly that the size definitions are valid for ‘any kind of non-spherical particles’. This was included before the above mentioned equations.

3. Page 4, lines 27-28, page 5, line 1: The real part $mr$ determines the speed of light inside the particle and therefore the refraction of waves on the particle surface in the macroscopic sense: I think this is an over-simplification that may be misleading for a young scientist. It is better to omit it. Otherwise, please provide relevant reference.

As this part is not essential for the paper we removed it as suggested.

4. Page 7, line 21: The minimum size parameter was selected depending on the maximum size achieved with TMM.: An evaluation of the agreement between the 2 methods is missing here. Please provide an indicative plot, containing e.g. the scattering matrix elements $\alpha_1$ and $-b_1/a_1$ (two sub-plots) for indicative cases (e.g. see Fig.2 in Dubovik et al. (2006) -Application of spheroid models to account for aerosol particle nonsphericity in remote sensing of desert dust)

We have added several plots similar to Fig. 2 of Dubovik et al. (2006) as Section S3 to the supplement. They illustrate the transition from TMM to IGOM which be briefly discuss. In Section 2.3 of the paper we refer to this Supplement.

5. Page 10, lines 8-9: The transition size parameter between TMM and IGOM is in the range $5 < x < 125$, strongly depending on $m$ and particle shape.: Provide the corresponding ranges for different $m$ and particle shapes in an Appendix.

We have added an overview table as Section S2 of the Supplement and in addition have uploaded to https://zenodo.org a detailed list with maximum size parameters of TMM for all 22680 combinations of refractive indices and shapes
6. Page 13, lines 5-17: In case of fixed values of . . . for each mode.: Re-write this section in a more clear way, maybe using some examples. It is not clear what your methodology is here.

Reviewer 1 also had a similar concern. We have rewritten this part and also included an example as Section S4 of the Supplement.


We added relevant references (Petters and Kreidenweis, 2007; Markelj et al., 2017; Enroth et al., 2018; Psychoudaki et al., 2018) for the $\kappa$ values of different aerosol types.

8. Page 25, lines 12-13: In other words. . . radius definitions: Provide a visualization of this discussion in a plot with size distributions corresponding to the different radius definitions.

We added a new Figure 8 to clarify how the selected radius definition affects the results. The corresponding explanations in Sect. 5.4 were rephrased and extended.

9. Page 26, line 23-24: But it also needs . . . partial derivatives: It is not clear what you mean here, it is better to omit this.

We agree that this part is not essential for the understanding of this section. So we removed it as suggested.


Authors’ response to short comment by Maxim Yurkin:

We appreciate the short comment by Maxim Yurkin with his positive judgement and helpful suggestions. In the following, comments by Maxim Yurkin are in italic font, our answers in normal font.

Moreover we want to refer to the revised manuscript where all changes can easily be tracked. Sect. 2.2.4 of the manuscript and Sect. S1 of the new Supplement are attached to this reply.

1) The authors describe the discretization grid for the DDA in terms of the number of dipoles per wavelength. But this quantity is not relevant for particles smaller than the wavelength. I guess, the authors used some fixed number of dipoles for smaller particles, but that is not reflected in the text.

Yes, Maxim Yurkin is right, we overlooked this point when writing our discussion paper. It is now included in the ADDA section (2.2.4) of the revised version. We use the dipole set that has 11 dipoles per wavelength at $x = 10$ (with about 23000 dipoles) also for size parameters $x < 10$.

2) The orientation-averaging scheme (described in the Appendix A) seems fine, but it is a bit complicated. Thus, it would help if the authors test it for some simple problem (e.g. moderately-sized spheroid), where a reference solution is available. Or at least, mention the results of such tests in the text.

We agree that the description of the orientation-averaging scheme is a bit complicated in the discussion paper, mainly Eq. A2. In the revised paper we replaced Eq. A2 by a more simple equation describing the same method. We feel that it is not necessary to further simplify the scheme as the idea behind is straightforward. Note, that we moved the Appendix of the discussion paper to the Supplement (Section S1.1) of the revised version (to extend it with more details of the accuracy test, see below).

We decided to provide more details of the orientation averaging accuracy tests. However, in order to limit the size of the paper, details are swapped to Section S1.2 of the Supplement whereas our main results are still summarized in the paper. Furthermore, we considered a third irregular shape (F) for the orientation averaging accuracy tests.

Following the suggestion of testing our scheme for a simple problem like spheroids, we also added a test with spheroids at size parameter 2, 4, and 10 as Section S1.3 to the Supplement. We applied ADDA together with our orientation averaging scheme (for simplicity without considering the symmetry of spheroids) and compare the orientation-averaged properties of the spheroids to those calculated with TMM. A brief discussion of the agreement is also included in the Supplement.

3) Finally, I wonder if the DDA can be used for cases where neither TMM or IGOM is available (e.g., for 1:3 spheroids with $m_r < 1.04$ and size parameters
of about 30). The DDA is known to be particularly efficient for such regime (mr close to 1), due to the fast convergence of the (internal) iterative solver. So the authors may at least mention such possibility to extend their dataset.

As we are not very familiar with atmospheric applications in this refractive index range we did not put much effort in maximizing the size coverage for $m_r < 1.04$. However this suggestion points to a useful future extension of the data set, which is now mentioned in the outlook.
Authors’ response to short comment by Lutz Gross:

We thank Lutz Gross for his suggestion.

We uploaded the Fortran code including the dataset (about 30 GB compressed) for open access to https://zenodo.org. A DOI is assigned to the MOPSMAP model version v1.0 of the final GMD paper (in GMDD v0.9 was discussed) and a link is added to the code availability section of the paper. Furthermore, the code archive under https://zenodo.org includes a few examples of MOPSMAP applications corresponding to the examples presented in our paper.
MOPSMAP v0.9-1.0: A versatile tool for modeling of aerosol optical properties

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Abstract.

The spatiotemporal distribution and characterization of aerosol particles are usually determined by remote sensing and optical in-situ measurements. These measurements are indirect with respect to microphysical properties and thus inversion techniques are required to determine the aerosol microphysics. Scattering theory provides the link between microphysical and optical properties; it is not only needed for such inversions but also for radiative budget calculations and climate modeling. However, optical modeling can be very time consuming, in particular if non-spherical particles or complex ensembles are involved.

In this paper we present the MOPSMAP package (modeled optical properties of ensembles of aerosol particles) which is computationally fast for optical modeling even in case of complex aerosols. The package consists of a data set of pre-calculated optical properties of single aerosol particles, a Fortran program to calculate the properties of user-defined aerosol ensembles, and a user-friendly web interface for online calculations. Spheres, spheroids, and a small set of irregular particle shapes are considered over a wide range of sizes and refractive indices. MOPSMAP provides the fundamental optical properties assuming random particle orientation, including the scattering matrix for the selected wavelengths. Moreover, the output includes tables of frequently used properties such as the single scattering albedo, the asymmetry parameter or the lidar ratio. To demonstrate the wide range of possible MOPSMAP applications a selection of examples is presented, e.g., dealing with hygroscopic growth, mixtures of absorbing and non-absorbing particles, the relevance of the size equivalence in case of non-spherical particles, and the variability of volcanic ash microphysics.

The web interface is designed to be intuitive for expert and non-expert users. To support users a large set of default settings is available, e.g., several wavelength-dependent refractive indices, climatologically representative size distributions, and a parameterization of hygroscopic growth. Calculations are possible for single wavelengths or user-defined sets (e.g., of specific remote sensing application). For expert users more options for the microphysics are available. Plots for immediate visualization of the results are shown. The complete output can be downloaded for further applications. All input parameters and results are stored in the user’s personal folder so that calculations can easily be reproduced. The MOPSMAP package is available on
Aerosol particles in the Earth’s atmosphere are important in various ways, for example because of their interaction with electromagnetic radiation and their effect on cloud properties. Consequently aerosol particles are relevant for weather and climate. The temporal and spatial variability of their abundance as well as the variability of their properties is significant which poses huge challenges in quantifying their effects. This includes the need to establish extended networks of observations using instruments such as photometers (Holben et al., 1998), lidars (Pappalardo et al., 2014), or ceilometers (Wiegner et al., 2014), and the development of models to predict the influence of particles on the state of the atmosphere, see e.g. Baklanov et al. (2014).

Aerosol properties and distributions are often quantified by ground-based and space-borne optical remote sensing and by optical in-situ measurements. These measurements are indirect with respect to microphysical properties (e.g., particle size) because they measure optical quantities and require the application of inversion techniques to retrieve microphysical properties. Precise knowledge on the link between microphysical and optical properties is needed for the inversion. This link is provided by optical modeling, i.e. the optical properties of particles are calculated based on their microphysical properties. Optical modeling is required also for other applications, e.g., for radiative transfer, numerical weather prediction, and climate modeling. As optical modeling can be very time-consuming it is often inevitable to precalculate optical properties of particles and store them in a lookup table, which is then accessed by the inversion procedures or subsequent models.

In our contribution we describe the MOPSMAP (‘Modelled optical properties of ensembles of aerosol particles’) package which consists of a data set of pre-calculated optical properties of single aerosol particles, a Fortran program which calculates the properties of user-defined aerosol ensembles from this data set, and a user-friendly web interface for online calculations. Fig. 1 illustrates the overall scheme of the package, including the optical modeling codes (green box) needed once to prepare the underlying data set. MOPSMAP is either provided via an interactive web interface or is provided for offline calculations.
applications upon request at https://mopsmap.net or via download as offline application. The former is possible as MOPSMAP is computational very efficient. Compared to other data sets with predefined aerosol components, such as OPAC (Hess et al., 1998), compared to existing online Mie tools such as the one provided by Prahl (2018), and compared to GUI tools such as MiePlot Laven (2018), MOPSMAP is more flexible with respect to the characteristics of the aerosol ensembles. Moreover, our data set considers not only spherical particles but also spheroids and a small set of irregularly-shaped dust particles. The output includes ASCII tables for further evaluation, netcdf files for direct application in the radiative transfer model uvspec (Emde et al., 2016) and plots for e.g. educational purposes.

In Sect. 2, after defining aerosol properties, we describe how existing optical modeling codes were applied (green box in Fig. 1) to create the optical data set of single particles (yellow box). Subsequently, in Sect. 3, we describe the Fortran program (red box) that uses this data set to calculate optical properties of user-defined particle ensembles. The web interface for online application of the MOPSMAP package is introduced in Sect. 4. To demonstrate the potential of MOPSMAP, several applications are discussed in Sect. 5 before we sum up our paper and give an outlook.

2 Background and the MOPSMAP data set

The optical properties of a particle with known microphysical properties are calculated by optical modeling. For the creation of the basic data set of MOPSMAP, optical modeling of single particles has been performed. In this section we first define microphysical and optical properties of single particles and then describe how we created the data set using existing optical modeling codes.

We emphasize that the data set is, in principle, applicable to the complete electromagnetic spectrum, however we use, for simplicity, the term ’light’ and consequently ’optics’ instead of more general terms.

2.1 Definition of particle properties

The description of particle properties is well-established and can be found in textbooks in detail of variable depth. Thus, we can restrict ourselves to a brief summary of those properties that are of special relevance for MOPSMAP.

The microphysical properties of an aerosol particle are described by its shape, size, and chemical composition.

Atmospheric aerosols might be spherical in shape but many types consist of non-spherical particles, often with a large variety of different shapes. Mineral dust (e.g., Kandler et al., 2009) and volcanic ash aerosols (e.g., Schumann et al., 2011b) are important examples for the latter, but for example also pollen, dry sea salt, or soot particles are usually non-spherical. A quite common approach to consider the particle non-sphericity is to approximate the shape with spheroids (Kahn et al., 1997; Dubovik et al., 2006; Wiegner et al., 2009) is the approximation using spheroids or distributions of spheroids (Hill et al., 1984; Mishchenko et al., 1997; Kahn et al., 1997; Dubovik et al., 2006). Spheroids originate from rotation of ellipses about one of their axes. Only one parameter is required to describe the shape of a
Spheroids for the shape description. Mishchenko and Travis (1998) use the ‘axial ratio’ $\epsilon_m$, which is the ratio between the length of the axis perpendicular to the rotational axis and the length of the rotational axis. By contrast, Dubovik et al. (2006) use the ‘axis ratio’ $\epsilon_d$, defined as the inverse of $\epsilon_m$. Spheroids with $\epsilon_m < 1$, $\epsilon_d > 1$ are called prolate (elongated) whereas spheroids with $\epsilon_m > 1$, $\epsilon_d < 1$ are oblate (flat) spheroids. The aspect ratio $\epsilon'$ is the ratio between the longest and the shortest axis, i.e. $\epsilon' = \frac{1}{\epsilon_m} = \epsilon_d$ in case of prolate spheroids and $\epsilon' = \epsilon_m = \frac{1}{\epsilon_d}$ in case of oblate spheroids. Spheroids with $\epsilon' = 1$ are spheres.

The size of a particle commonly is described by its radius or its diameter. While this is unambiguous in case of spheres, more detailed specifications are necessary in case for any kind of non-spherical particles. Often the size of an equivalent sphere is used for the description of the size of such particles non-spherical particle size: The volume-equivalent radius $r_v$ of a particle with known volume $V$ (containing the particle mass, i.e. without cavities) is

\[ r_v = \sqrt[3]{\frac{3V}{4\pi}}, \]  

(1)

whereas the cross-section-equivalent radius $r_c$ of a particle with the known orientation-averaged geometric cross sectional area section $C_{geo}$ is

\[ r_c = \sqrt{\frac{C_{geo}}{\pi}}. \]  

(2)

In case of spheroids, $r_c$ is equal to the radius of a sphere having the same surface area (as used by Mishchenko and Travis (1998)). For the conversion between $r_v$ and $r_c$, the radius conversion factor

\[ \xi_{vc} = \frac{r_v}{r_c} = \sqrt[3]{\frac{3\sqrt{\pi} \ V}{4 \ C_{geo}^{3/2}}} \]  

(3)

is used (Gasteiger et al., 2011b). $\xi_{vc}$ is equal to 1 in case of spheres and decreases with increasing deviation from spherical shape. Another definition of size is given by the radius of a sphere that has the same ratio between volume and geometric cross section as the particle

\[ r_{vcr} = \frac{3V}{4C_{geo}} = \xi_{vc}^3 r_c. \]  

(4)

This definition corresponds to the case ’VSEQU’ presented by Otto et al. (2011), to the ’effective radius’ in Eq. 5 of Schumann et al. (2011a), and is more sensitive to non-sphericity in terms of $\xi_{vc}$ than $r_v$ or $r_c$. For example, a particle with $r_c = 1 \mu m$ and $\xi_{vc} = 0.9$ implies $r_v = 0.9 \mu m$ and $r_{vcr} = 0.729 \mu m$, whereas $r_v = 0.9 \mu m$ and $r_{vcr} = 0.729 \mu m$.

For setting up a data set of optical properties for different wavelengths it is highly beneficial to make use of the size parameter

\[ x = \frac{2\pi r}{\lambda}. \]  

(5)

The size parameter $x$ describes the particle size relative to the wavelength $\lambda$. The advantage of using $x$ is that optical properties ($q_{ext}$, $\omega_0$, and $F$, as defined below) at a given wavelength are fully determined by its shape, refractive index $m$, and $x$. Equivalent size parameters $x_v$, $x_c$, and $x_{vcr}$ are calculated from the equivalent radii, analogously to Eq. 5.
The chemical composition of a particle determines its complex wavelength-dependent refractive index $m$. The real part $m_r$ determines the speed of light inside the particle and therefore the refraction of waves on the particle surface in the macroscopic sense. The imaginary part $m_i$ is relevant for the absorption of light inside the particle, whereby an imaginary part of zero corresponds to non-absorbing particles.

The optical properties of a non-spherical particle depend on the orientation of the particle relative to the incident light. In our data set we assume that particles are oriented randomly thus the optical properties are stored as orientation averages (Mishchenko and Yurkin, 2017).

The orientation-averaged optical properties at a given wavelength are fully described by the extinction cross section $C_{ext}$, the single scattering albedo $\omega_0$, and the scattering matrix $F(\theta)$ where $\theta$ is the angle by which the incoming light is deflected during the scattering process ('scattering angle'). The extinction cross section $C_{ext}$ can be normalized by the orientation-averaged geometric cross section $C_{geo}$ of the particle giving the extinction efficiency

$$q_{ext} = \frac{C_{ext}}{C_{geo}} = \frac{C_{ext}}{\pi r_c^2}$$

(6)

The single scattering albedo $\omega_0$ is given by

$$\omega_0 = \frac{C_{sca}}{C_{ext}}$$

(7)

where $C_{sca}$ is the scattering cross section.

For the scattering matrix $F$ of randomly oriented particles we use the notation of Mishchenko and Travis (1998), i.e.

$$F(\theta) = \begin{bmatrix} a_1(\theta) & b_1(\theta) & 0 & 0 \\ b_1(\theta) & a_2(\theta) & 0 & 0 \\ 0 & 0 & a_3(\theta) & b_2(\theta) \\ 0 & 0 & -b_2(\theta) & a_4(\theta) \end{bmatrix}$$

(8)

with six independent matrix elements. The scattering matrix describes the transformation of the incoming Stokes vector $I^{inc}$ to the scattered Stokes vector $I^{sca}$:

$$I^{sca}(\theta) = \frac{C_{sca}}{4\pi R^2} F(\theta) I^{inc}$$

(9)

where the Stokes vectors (van de Hulst, 1981) have the shape

$$I = \begin{bmatrix} I \\ Q \\ U \\ V \end{bmatrix}$$

(10)

and $R$ is the distance of the observer from the particle. The Stokes vectors $I$ describe the polarization state of light, with the first element $I$ describing its total intensity. Thus, $F$ is relevant for the polarization of the scattered light and its first element
\( a_1 \), which is known as the phase function, is important for the angular intensity distribution of the scattered light. The phase function is normalized such that
\[
\int_{0}^{180^\circ} a_1(\theta) \cdot \sin \theta \cdot d\theta = 2.
\] (11)

For many applications it is useful to expand the elements of the scattering matrix using generalized spherical functions (Hovenier and van der Mee, 1983; Mishchenko et al., 2016). The scattering matrix elements at any scattering angle \( \theta \) are then determined by a series of \( \theta \)-independent expansion coefficients \( \alpha_{l1}, \alpha_{l2}, \alpha_{l3}, \alpha_{l4}, \beta_{l1}, \) and \( \beta_{l2} \), with index \( l \) from 0 to \( l_{\text{max}} \), see Eqs. 11-16 in Mishchenko and Travis (1998). \( l_{\text{max}} \) depends on the required numerical accuracy as well as on the scattering matrix itself. E.g. in case of strong forward scattering peaks (typically occurring at large \( x \)), \( l_{\text{max}} \) needs to be larger than in case of more flat phase functions, to get the same accuracy.

The asymmetry parameter \( g \) is an integral property of the phase function:
\[
g = \frac{1}{2} \int_{0}^{180^\circ} \cos \theta \cdot a_1(\theta) \cdot \sin \theta \cdot d\theta.
\] (12)

\( g \) is the average cosine of the scattering angle of the scattered light and is calculated from the expansion coefficients by
\[
g = \alpha_{11}/3.
\] (13)

### 2.2 Optical modeling of single particles

Depending on the particle type, different approaches are available for calculating particle optical properties. For the creation of the MOPSMAP optical data set we use in case of spherical particles the well-known Mie theory (Mie, 1908; Horvath, 2009), which is a numerically exact approach over a very broad range of sizes. For spheroids we use the T-matrix method (TMM), which is a numerically exact method but limited with respect to maximum particle size. For larger spheroids not covered by TMM we apply the improved geometric optics method (IGOM). For irregularly-shaped particles the discrete dipole approximation (DDA) is applied.

#### 2.2.1 Mie theory

We use the Mie code developed by Mishchenko et al. (2002) for optical modeling of spherical particles. In contrast to the non-spherical particle types described below, we do not store the optical properties of single particles (in a strict sense) because the properties of spheres can be strongly size-dependent which would require a very high size resolution of our data set (e.g., Chýlek, 1990). Instead, we store data averaged over very narrow size bins, allowing us to use a lower size resolution resulting in a smaller storage footprint of the data set. Actually, for each size parameter grid point \( x \), we consider a size parameter bin covering the range from \( x/\sqrt{1.01} \) to \( x \cdot \sqrt{1.01} \), and apply the Mie code for 1000 logarithmically equidistant sizes within that bin before these results are averaged and stored.
2.2.2 T-matrix method (TMM)

We use the extended precision version of the code described by Mishchenko and Travis (1998) for modeling optical properties of spheroids. To improve the coverage of the particle spectrum \((x, \epsilon_m, \text{ and } m)\), internal parameter values of the TMM code, which primarily determine the limits of the convergence procedures, were increased \((\text{NPN1} = 290, \text{NPNG1} = -870, \text{NPN4} = -260)\) as discussed by Mishchenko and Travis (1998). Though, in general, the TMM provides exact solutions for scattering problems, non-physical results might be obtained due to numerical problems. To reduce the probability of non-physical results and to increase the accuracy of the results, the parameter DDELT, i.e. the absolute accuracy of computing the expansion coefficients, was set to \(10^{-6}\) (default \(10^{-3}\)). In non-converging cases, which happened near the upper limit of the covered size range, the requirements were relaxed to DDELT = \(10^{-3}\). Cases that did not converge even with the relaxed DDELT were not included in the data set. Nevertheless, some non-physical results were obtained by this approach, for example, \(\omega_0 > 1\), \(\omega_0 = 1\), or outliers of otherwise smooth \(\omega_0(x)\) or \(g(x)\) curves. Thus, for plausibility checks for each particle shape and refractive index, single scattering albedos \(\omega_0\) and asymmetry parameters \(g\) were plotted over size parameter \(x\) and outliers were recalculated with slightly modified size parameters. Recalculations with non-physical results were not included in the data set, which reduces the upper limit of the covered size range for that particular particle shape and refractive index.

2.2.3 Improved geometric optics method (IGOM)

Optical properties of large spheroids were calculated with the improved geometric optics method (IGOM) code provided by Bi et al. (2009); Yang et al. (2007) Yang et al. (2007); Bi et al. (2009). In general, this approximation is most accurate if the particle and its structures are large compared to the wavelength. In addition to reflection, refraction, and diffraction by the particle, which are considered by classical geometric optics codes, IGOM also considers the so-called edge effect contribution to the extinction efficiency \(q_{ext}\) (Bi et al., 2009). Classical geometric optics results in \(q_{ext} = 2\), whereas \(q_{ext}\) is variable in case of IGOM. The default settings of the code were used. The minimum size parameter was selected depending on the maximum size \(\text{achieved-calculated}\) with TMM.

2.2.4 Discrete dipole approximation code ADDA

Natural non-spherical aerosol particles, such as desert dust particles, comprise practically an infinite number of particle shapes, thus it is impossible to cover the full range of shapes in aerosol models. Moreover, the shape of each individual particle is never known under realistic atmospheric conditions. Consequently, typical irregularities such as flat surfaces, deformations or aggregation of particles, can be considered only in an approximating way. To enable the user of MOPS-MAP to investigate the effects of such irregularities the properties of six exemplary irregular particle shapes, as introduced by Gasteiger et al. (2011b), are provided. The geometric shapes were constructed using the object modeling language Hyperfun (Valery et al., 1999). The first three shapes are prolate spheroids with varying aspect ratios \((A: \epsilon' = 1.4, \epsilon'' = 1.4, B: \epsilon' = 1.8, \epsilon'' = 1.8, C: \epsilon' = 2.4, \epsilon'' = 2.4)\) and
surface deformations according to Gardner (1984). Shape D is an aggregate composed of ten overlapping oblate and prolate spheroids; surface deformations were applied as for shapes A-C. Shape E and F are edged particles with flat surfaces and varying aspect ratio.

The optical properties were calculated with the discrete dipole approximation code ADDA (Yurkin and Hoekstra, 2011). A large number of particle orientations needs to be considered for the determination of orientation-averaged properties. ADDA provides an optional built-in orientation averaging scheme in which the calculations for the required number of orientations is done within a single run. An individual ADDA run using this scheme requires approximately the time for one orientation multiplied with the number of orientations (typically a few hundreds), which can result in computation times of several weeks for large $x$. Because of the long computation times we split them up and performed independent ADDA runs for each orientation. The orientation-averaged properties are calculated in a subsequent step using the ADDA results for the individual orientations (see below).

The computational demand of DDA calculations increases strongly with size parameter $x$, typically with about $x^5$ to $x^6$. Thus, when aiming for large $x$, which is required for mineral dust in the visible wavelength range, it is necessary to find code parameters and an orientation averaging approach that provide a compromise between computation speed and accuracy.

The orientation sampling and averaging is described in Appendix ???. The ADDA code allows mainly the following code parameters to be optimized:

- DDA formulation
- Stopping criterion of the iterative solver
- Number of dipoles per wavelength

We estimate the accuracy of the ADDA results by comparing orientation-averaged $q_{ext}$, $q_{sca}$, $a_1(0^\circ)$, $a_1(180^\circ)$, and $a_2(180^\circ)/a_1(180^\circ)$ with results obtained using more strict calculation parameters. Accuracy tests are performed for shapes B and C (Fig. 1 of Gasteiger et al. (2011b)), for size parameters $x_p=10.0$, $x_p=12.0$, 14.4, 17.3, 19.0, and 20.8, and for refractive index $m=1.52+0.0043i$, i.e. 12 single particle cases are considered in total. By comparing the different DDA formulations available in ADDA, it was found that the filtered coupled-dipole technique (ADDA command line parameter 

$\text{-pol fcd -int fcd}$), as introduced by Piller and Martin (1998) and applied by Yurkin et al. (2010), offers the best compromise between computation speed and accuracy of modeled optical properties. Using a stopping criterion for the iterative solver of $10^{-4}$ instead of $10^{-3}$ has only negligible influence on orientation-averaged optical properties ($<0.1<0.1\%$) but requires approximately 30\% more computation time; thus, we used $10^{-3}$ for the ADDA calculations to create our data set. The extinction efficiency $q_{ext}$ and the scattering efficiency $q_{sca}$ change in all cases by less than 0.3\% if a grid density of 16 dipoles per wavelength is used instead of 11. The maximum relative changes due to the change in dipole density are 0.2\% for $a_1(0^\circ)$, 1.7\% for $a_1(180^\circ)$, and 1.9\% for $a_2(180^\circ)/a_1(180^\circ)$. Because of the large difference in computation time, which is about a factor 3-4, and the low loss in accuracy, about 11 dipoles per wavelength were selected for the MOPSMAP data set. For
we use the same dipole set as for \( x_v = 10 \) so that the number of dipoles per wavelength increases with decreasing \( x_v \), being about \( 110/x_v \).

The particle orientation is specified by three Euler angles \((\alpha, \beta, \gamma)\) as described by Yurkin and Hoekstra (2011) and basically a step size of \( 15^\circ \) is applied for \( \beta \) and \( \gamma \), resulting in 206 independent ADDA runs for each irregular particle. The orientation sampling and averaging is described in detail in Sect. S1.1 of the Supplement.

To test the accuracy of the selected orientation averaging scheme, orientation-averaged optical properties were calculated using a denser grid of orientation grid points \((\beta, \gamma)\), as defined in Appendix 2, but with a step size compared to results using a much smaller step of \( 5^\circ \). These calculations consider about 12 times more orientations than the calculations for our data set used for MOPSmap. Details are presented in Sect. S1.2 of the Supplement. Maximum deviations of \( 0.6\text{ less than }1\% \) are found for \( q_{ext} \) and \( q_{sca} \) and \( a_1(0^\circ) \). For backscatter properties, \( a_1(180^\circ) \) and \( a_2(180^\circ)/a_1(180^\circ) \) change by not more than \( 0.9\%, 6.5\%, \) and \( 12\% \), respectively. Typical deviations are of the order of a few percent (max. \( 14\% \)). Moreover, in Sect. S1.3 of the Supplement, the selected orientation averaging scheme is applied to spheroids and their optical properties are compared to reference TMM results. These deviations are comparable to those given in Sect. S1.2.

In summary, ADDA with the filtered coupled-dipole technique, at least 11 dipoles per wavelength, and a stopping criterion for the iterative solver of \( 10^{-3} \) was used for optical modeling of the irregularly-shaped particles in our data set together with the orientation averaging scheme described in Appendix 2, combining 206 ADDA runs. Tests demonstrate that the modeling accuracy is mainly determined by the applied orientation averaging scheme.

### 2.3 Optical data set

Using the codes with the settings described above, a data set of modeled optical properties of single particles in random orientation was created. The complete data set requires about \( 42 \) gigabytes of storage capacity. For spheres we stored, instead of single particle properties, averages over narrow size bins as described above. An overview over the wide range of sizes, shapes, and refractive indices of the particles in the data set is given in Tables 1 and 2. For each combination of refractive index and shape a separate netCDF file was created, e.g., ’spheroid_0.500_1.5200_0.008600.nc’ for spheroids with \( \epsilon_m = 0.5 \) (prolate with \( \epsilon' = 2.0 \)) and \( m = 1.52 + 0.0086i \). Each file contains the optical properties on a grid of size parameters. The complete data set requires about \( 42 \) gigabytes of storage capacity.

For spheres and spheroids the minimum size parameter is set to \( 10^{-6} \), and the maximum size parameter is set to \( x \approx 1005 \) to cover \( r_e = 40 \mu m \) at \( \lambda = 250 \text{ nm} \) to \( r_e = 1005 \) to cover \( r_e = 80 \mu m \) at \( \lambda = 500 \text{ nm} \). The size increment is 1% (i.e. \( x_{i+1}/x_i = 1.01 \)) in case of spheres, 5% in case of TMM spheroids, and 10% for IGOM spheroids. In case of spheroids having refractive indices most relevant for atmospheric studies, the TMM is applied up to (or close to) the largest possible size parameter with the approach described in Sect. 2.2.2. The maximum size parameter for TMM of the TMM calculations is reduced for less relevant refractive indices. An overview is given in Sect. S2 of the Supplement and a detailed list of the maximum size parameters for
Table 1. Microphysics of spheres and spheroids considered in the MOPSMAP data set. *IGOM was not applied to $m \leq 1.0$

<table>
<thead>
<tr>
<th>method</th>
<th>Mie</th>
<th>TMM</th>
<th>IGOM</th>
</tr>
</thead>
<tbody>
<tr>
<td>particle shape</td>
<td>spheres</td>
<td>oblate and prolate spheroids</td>
<td>$\epsilon' = 1.2, 1.4, ..., 3.0, 3.4, 3.8, ..., 5.0$</td>
</tr>
<tr>
<td>size parameter</td>
<td>$10^{-6} &lt; x_c &lt; 1005$</td>
<td>$10^{-6} &lt; x_c &lt; (5 - 125)$</td>
<td>$(5 - 125) &lt; x_c &lt; 1005$</td>
</tr>
<tr>
<td>$\frac{x_{i+1}}{x_i} = 1.01$</td>
<td>$\frac{x_{i+1}}{x_i} = 1.05$</td>
<td>$\frac{x_{i+1}}{x_i} = 1.10$</td>
<td></td>
</tr>
<tr>
<td>size bins</td>
<td>single size</td>
<td>single size</td>
<td>single size</td>
</tr>
<tr>
<td>$m_r$</td>
<td>(0.1, 0.2, ..., 0.9)*, 1.04, 1.08, ..., 1.68, 1.76, ..., 2.0, 2.2, ..., 3.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$m_i$</td>
<td>0, 0.0005375, 0.001075, 0.0015203, 0.00215, 0.0030406, 0.0043, 0.0060811, 0.0086, 0.0121622, 0.0172, 0.0243245, 0.0344, 0.0486490, 0.0688, 0.0972979, 0.1376, 0.2752, 0.5504, 1.1008, 2.2016</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 2. Microphysics of irregularly-shaped particles considered in the MOPSMAP data set.

| particle shape | shapes A-F, Fig. 1 of Gasteiger et al. (2011b) |
| size parameter | $10^{-3} < x_r < 27.5; 10^{-3} < x_v < 30.2; x_r x_v \approx 1.10$; single size |
| $m_r$ | 1.48, 1.52, 1.56, 1.60 |
| $m_i$ | 0, 0.00215, 0.0043, 0.0086, 0.0172, 0.0344, 0.0688 |

All $m$ and $\epsilon_{im}$ combinations can be downloaded from Gasteiger and Wiegner (2018). The maximum size parameter for TMM is in the range $5 < x < 125$, strongly depending on $m$ and particle shape, and determines the lowest size parameter for which IGOM is at which IGOM may be applied. The first IGOM size parameter is between 0 and 10 % larger than the maximum TMM size parameter. The transition size parameter between TMM and IGOM is in the range $5 < x < 125$, strongly depending on $m$ and particle shape. The TMM and IGOM results for spheroids are merged into a single netcdf file covering the complete size range from $x = 10^{-6}$ to $x \approx 1005$, which is sufficient for most applications. For example, for prolate spheroids with $\epsilon' = 1.8$ and $m = 1.56 + 0i$, the size range from $x = 10^{-6}$ to $x = 88.22$ is covered by TMM, while IGOM starts at $x = 89.54$, $x = 89.54$. The transition from TMM to IGOM for several scattering angles is demonstrated in Sect. S3 of the Supplement. Since IGOM is an approximation unrealistic jumps of optical properties may occur at the transition size parameter. For typical mineral dust ensembles in the visible spectrum, particles in the TMM range contribute more than 90 % to the total extinction. IGOM was not applied to $m_r < 0.04$, thus the size parameter range is limited to the TMM range for these refractive indices. A step of 0.04 was selected for the $m_r$ grid in the most relevant range (from 1.00 to 1.68) and a wider $m_r$ step for the less common ranges elsewhere. Development of the data
set started with \( m_i = 0.0043 \) \( m_i = 0.0043 \) and beginning from this value, \( m_i \) was increased and decreased in steps of a factor \( \sqrt{2} \). Below \( m_i = 0.001 \) and above \( m_i = 0.1 \) \( m_i = 0.001 \) and above \( m_i = 0.1 \) the step width is a factor of 2.

The optical data for the irregularly-shaped particles (Tab. Table 2) is limited to \( x_v \leq 27.5 \) \( x_v \leq 30.2 \) because of the huge computation requirements for optical modeling of large particles. Nonetheless, this range already covers the most important range for many applications - Eis covered, e.g., at \( \lambda = 1064 \) \( \lambda = 1064 \) nm particles up to \( x_v = 4.6 \mu m \) are covered \( x_v = 5.1 \mu m \) can be modeled. The \( m \) grid for the irregularly-shaped particles is limited to the most relevant range for desert dust in the visible spectrum and the \( m_i \) step is set to a factor of 2. The quantification of the conversion factor \( \xi_{ove} \) of the six irregular shapes requires the determination of their orientation averaged geometric cross sections which is done numerically.

The optical properties stored for each particle are the extinction efficiency \( q_{ext} \), the scattering efficiency \( q_{sca} \), and the expansion coefficients \( \alpha_1^i, \alpha_2^i, \alpha_3^i, \alpha_4^i, \beta_1^i \), and \( \beta_2^i \) of the scattering matrix. The ADDA and the IGOM code provide the angular-resolved scattering matrix elements which we converted to the expansion coefficients stored in the data set following the method described by Hovenier and van der Mee (1981); Mishchenko et al. (2016). We optimized the expansion coefficients for accurate scattering matrices at \( 180^\circ \), which probably is the most error sensitive angle. As a by-product lidar applications will certainly benefit from this optimization.

In case of asymmetric shapes in random orientation, the scattering matrix has 10 independent elements as discussed by van de Hulst (1981). By using only six elements of \( F \) (Eq. 8) in our data set we implicitly assume that each irregular model particle (shapes A-F) occurs as often as its mirror particle, which is formed by mirroring at a plane (van de Hulst, 1981).

Fig. 2 shows an example of the MOPSMAP optical data set. The refractive index is set to \( m = 1.56 + 0.00215 i \), which is representative for desert dust particles at visible wavelengths. The properties of spherical particles are shown in blue, while whereas the properties of prolate spheroids with \( c' = 1.4 \) \( c' = 1.4 \) and 3.0 are shown in orange and green, respectively. Red and violet lines denote irregularly-shaped particles D and F, respectively. The upper panel shows the extinction efficiency \( q_{ext} \) as function of cross-section-equivalent size parameter \( x_c \). The general shape of the \( q_{ext}(x_c) \)-curve is similar for the different shapes; nonetheless, with increasing deviation from spherical shape, the amplitudes of the oscillations of \( q_{ext}(x_c) \) get smaller and a shift of the maximum \( q_{ext} \) towards larger \( x_c \) is found. The middle panel shows the single scattering albedo \( \omega_0 \) for the same particles as the upper panel. For particle sizes comparable to the wavelength, \( \omega_0 \) reaches maxima with values of about 0.991, almost independent of particle shape. \( \omega_0 \) approaches a value of about 0.551 at \( x_c \approx 1000 \) for spheres and spheroids. The lower panel shows the asymmetry parameter \( g \). When the particle size becomes comparable to the wavelength, \( g \) increases and oscillates as function of \( x_c \) with the strongest oscillations occurring in case of spheres. There is some shape dependence of \( g \) for \( x_c \approx 5 \), in particular the aggregate shape results in systematically smaller \( g \) than the other shapes for \( \approx 10 \).

The transition from the numerically exact TMM to the IGOM approximation occurs at \( x_v \approx 125 \) for \( c' = 1.4 \) \( x_c \approx 125 \) for \( c' = 1.4 \) (orange line) and at \( x_v \approx 27 \) for \( c' = 3.0 \) \( x_c \approx 27 \) for \( c' = 3.0 \) (green line) and is quite smooth.
Figure 2. Optical properties of single particles (or narrow size bins in case of spheres) with fixed refractive index $m = 1.56 + 0.00215i$ as function of size parameter. The different colors denote different particle shapes. The upper panel shows the extinction efficiency $q_{\text{ext}}$, the middle panel the single scattering albedo $\omega_0$, and the lower panel the asymmetry parameter $g$.

3 MOPSMAP Fortran program

In this section the basic characteristics of the MOPSMAP Fortran program to calculate optical properties of particle ensembles is described. Besides a modern Fortran compiler, e.g., gfortran 6 or above, the netCDF Fortran development source code is required to build the executable. The computation time and memory requirements depend on the ensemble complexity and the number of wavelengths but in general are low for state-of-the-art personal computers. The Fortran code and the data set are available upon request. A for download from Gasteiger and Wiegner (2018) and a web interface (see Sect. 4) provides online access to most of the functionality of the Fortran program without the requirement to download the code and the data set.
Within each MOPSMAP run the optical properties of a specific user-defined ensemble are calculated at a user-defined wavelength grid. The ensemble microphysics and the wavelength grid are defined in an input file. The details about the options available for the input file are described in a user manual which is provided together with the code.

Fig. 3 shows a flow chart of the MOPSMAP Fortran program. The program is initialized by reading the input file and a data set index. The latter contains information on the refractive index and shape grid and the size parameter ranges covered by the data set. Then, all information required for the optical modeling is initialized, for example the set of wavelengths, the refractive indices as function of wavelength, shape distributions, and the effect of the hygroscopic growth, before the optical calculations are performed for each wavelength, as described in the following.

### 3.1 Calculation of optical properties of particle ensembles

Usually aerosol particles occur as ensembles of particles of different size, refractive index, and/or shape. The different particles contribute to the optical properties of the ensemble. Assuming that the distance between the particles is large enough so that
interaction of light with each particle occurs without influence of any other particle (‘independent scattering’; van de Hulst, 1981), the contribution of each particle can be added as described below.

In MOPSMAP particle ensembles are composed of one or more independent modes (the terms ’mode’ and ’component’ are often used synonymously in the literature). Each mode in MOPSMAP is characterized by particle size, shape, and refractive index, whereby each property can be described as a fixed value or as a distribution (see below). As these parameters do not necessarily correspond to the grid points of the MOPSMAP data set, interpolation and decomposition into different contributions is performed. In for each mode (and each wavelength) decomposition into contributions from the different available m and shapes of the data set needs to be performed.

For a mode containing spheroids, in the most simple but probably most frequently used case of fixed values of m, m', and for a mode, the mode is decomposed into eight contributions. This is done as follows: If the next grid points of the data set that are smaller and larger are and respective for the real part of the refractive index, and for the imaginary part, and and for the aspect ratio with:

The weight of the grid points and grid points are

Finally the weights for each of the eight contributions are calculated as the products of the three corresponding weights; for example, the contribution from the and grid point is weighted with . This approach basically results in a linear interpolation of extensive properties between the m, m', and grid points of the data set. The error of the interpolation of the user-specified values between the grid points of the data set is discussed in Sect. 3.3 of the Supplement. The error of the interpolation
Under other conditions more or less than eight contributions have to be considered. In case a $\epsilon'$ distribution is given for a mode of spheres or a single irregular shape, an interpolation in the shape dimension is not necessary, so that four contributions are sufficient. In case of a spheroid aspect ratio distribution, contributions from the required range of $\epsilon'$ all required $\epsilon_{in}$ grid points are considered and weighted according to the $\epsilon'$ distribution. The decomposition into contributions is done independently for each mode.

Because given distribution. In case a mode contains a non-absorbing fraction (see below), an additional $m_i$ grid point, $m_i = 0$, may be required. Furthermore, because of the limited size range of irregularly-shaped particles (see above) they require a special treatment. For cases when the size range of a user-defined mode exceeds size parameter $x_v = 27.5$, which is the upper limit for these particles in the data set, a special treatment can be applied: A MOPSMAP option is available which substitutes irregularly-shaped particles above this $x_v$ limit a selected size parameter with other particle shapes, spherical or non-spherical, as selected by the user. As a consequence, the particle shape of that mode becomes size- and wavelength-dependent and the number of different contributions increases. The total number of contributions for an ensemble, denoted as $J$ in the following, varies because the number of modes is not fixed and, as just discussed, the number of contributions from each mode depends on the characteristics of each mode. This underlines the flexibility of MOPSMAP.

The optical properties of the particle ensemble are calculated for each wavelength by summation over extensive properties of all particles. Assume that the user-defined ensemble is decomposed into described by the $J$ contributions as just described. This approach corresponds to the so-called external mixing of particles. Each contribution has a size distribution $n_j(r)$, i.e. a particle number concentration per particle radius interval from $r$ to $r + dr$, in the range from $r_{min,j}$ to $r_{max,j}$, which is obtained by multiplying the user-defined size distribution of the mode with the weights obtained during the decomposition.

The extinction coefficient $\alpha_{ext}$ and the scattering coefficient $\alpha_{sca}$ are calculated by

$$\alpha_{ext} = \sum_{j=1}^{J} \left( \int_{r_{min,j}}^{r_{max,j}} C_{ext,j}(r) \cdot n_j(r) \cdot dr \right) \quad \text{and} \quad \alpha_{sca} = \sum_{j=1}^{J} \int_{r_{min,j}}^{r_{max,j}} C_{sca,j}(r) \cdot n_j(r) \cdot dr$$

(17)

$$\alpha_{sca} = \sum_{j=1}^{J} \left( \int_{r_{min,j}}^{r_{max,j}} C_{sca,j}(r) \cdot n_j(r) \cdot dr \right)$$

(18)

The expansion coefficients need to be weighted with $C_{sca,j}(r)$, for example $\alpha'_1$ of a particle ensemble is calculated by

$$\alpha'_1 = \frac{1}{\alpha_{sca}} \sum_{j=1}^{J} \left( \int_{r_{min,j}}^{r_{max,j}} \alpha'_1,j(r) \cdot C_{sca,j}(r) \cdot n_j(r) \cdot dr \right).$$

(19)

For the integration of extensive properties over the size distribution, we apply the trapezoidal rule, which assumes linearity between the $r$ grid points.
The size distribution \( n(r) = \frac{dN}{dr} \) for each mode can be specified in various ways. The MOPSMAP user can either specify a single size, apply size distribution tables in ASCII format, or apply a size distribution parameterization. The following size distribution parameterizations are available:

1. \( n(r) = \frac{1}{\sqrt{2\pi}\sigma_{ar}} \exp \left[ \frac{1}{2} \left( \frac{\ln r - \ln r_{\text{mod}}}{\sigma_{ar}} \right)^2 \right] \) — log-normal distribution

2. \( n(r) = a r^\alpha \exp(-b r^\gamma) n(r) = Ar^\alpha \exp(-Br^\gamma) \) — modified gamma distribution, \(^{10}\) Deirmendjian (1964), Deirmendjian (1964)

3. \( n(r) = a \exp(-br) n(r) = A \exp(-Br) \) — exponential distribution, \( \alpha = 0, \gamma = 1 \)

4. \( n(r) = a r^\alpha n(r) = Ar^\alpha \) — power law distribution, Junge distribution, \( b \cdot B = 0 \), Deirmendjian (1964), Deirmendjian (1964)

5. \( n(r) = a r^\alpha \exp(-br) n(r) = Ar^\alpha \exp(-Br) \) — gamma distribution, \( \gamma = 1 \), Twomey (1977), Twomey (1977)

with \( r_{\text{mod}} \) the mode radius, \( \sigma \) a dimensionless parameter for the relative width of the distribution, and \( \sum_{i} N_0 \) the total number density (in the range from \( r_{\text{min}} = 0 \) to \( r_{\text{max}} = \infty \), \( r_{\text{min}} = 0 \) to \( r_{\text{max}} = \infty \)) of the log-normal distribution. For the subsequent size distributions, parameters \( a A, \alpha, b B, \) and \( \gamma \) are positive and \( a A \) controls the scaling of total number density while whereas \( \alpha, b B, \) and \( \gamma \) are relevant for the shape of the size distributions. The exponential distribution, power law distribution, and the gamma distribution, are a subset of the modified gamma distribution with the specific parameter values as given above (see also Petty and Huang, 2011).

The particle shape can be specified independently for each mode and is, within each mode, independent of size and refractive index. In case of spheroids, either a fixed aspect ratio \( \epsilon' \) or an aspect ratio distribution is used. The latter can be given as a table in an ASCII file or it can be parameterized by a modified log-normal distribution (Kandler et al., 2007)

\[
n(\epsilon') = \frac{dN}{n_0 \cdot d\epsilon'} = \frac{1}{\sqrt{2\pi}\sigma_{ar}(\epsilon'-1)} \exp \left[ -\frac{1}{2} \left( \frac{\ln(\epsilon'-1) - \ln(\epsilon'_0 - 1)}{\sigma_{ar}} \right)^2 \right]
\]

with parameters \( \epsilon'_0 \) for the location of the maximum of \( n(\epsilon') \) and \( \sigma_{ar} \) for the width of the distribution.

The refractive index of each mode can either be wavelength-independent or specified as function of wavelength in an ASCII file. In addition, it is possible to specify for each mode a non-absorbing fraction \( \chi \). If \( \chi > 0 \), the mode is divided, for all sizes and shapes, into a non-absorbing \( (m_{i,1} = 0, m_{i,1} = 0, \text{relative abundance } \chi) \) and an absorbing fraction \( (m_{i,2} = m_i/(1-\chi), m_{i,2} = m_i/(1-\chi), \text{relative abundance } 1-\chi) \). As a consequence, the average \( m_i \) over all particles of the mode remains equal to the \( m_i \) as specified.
by the user. This non-absorbing fraction approach can be used as a parameterization of the refractive index variability within desert dust ensembles as described by Gasteiger et al. (2011b) and below in Sect. 5.6.

For the hygroscopic particle growth the following parameterization (Petters and Kreidenweis, 2007; Zieger et al., 2013)

\[
\frac{r_{\text{wet}}(RH)}{r_{\text{dry}}} = \left(1 + \kappa \cdot \frac{RH}{1 - RH} \right)^{\frac{1}{3}}
\]

is implemented in MOPSMAP where $RH$ is the relative humidity and $\kappa$ the hygroscopic growth parameter of the particles of each mode. This equation describes the ratio between the size of the particle at a given $RH$ and the size of the particle in a dry environment ($RH=0\%$). The parameterization implies that this ratio is independent of size, thus for example in case of a log-normal size distribution, $r_{\text{min}}$, $r_{\text{max}}$, and $r_{\text{mod}}$ are multiplied with this ratio, while the relative width $\sigma$ of the distribution is not modified. This is the usual approach though modal representations of aerosol size distributions may also predict higher moments (Binkowski and Shankar, 1995; Zhang et al., 2002), and thus $\sigma$ can be a prognostic variable as well. The refractive index is modified by the taken up water following the volume weighting rule. Both $RH$ and $\kappa$ can be chosen by the user. This parameterization is valid for particles with $r > 40\, \text{nm}$ where the Kelvin effect can be neglected (Zieger et al., 2013). It is worth noting that this parameterization differs from the relative humidity dependence implemented in OPAC which was adapted from Hänel and Zankl (1979).

### 3.2 Output of Fortran program

As output of MOPSMAP the following properties of aerosol ensemble are available. Redundant properties, such as the lidar-related properties, are available to facilitate the usage of the results.

- Extinction coefficient $\alpha_{\text{ext}} \left[ \text{m}^{-1} \text{m}^{-1} \right]$
- Single scattering albedo $\omega_0$
- Asymmetry parameter $g$
- Effective radius $r_{\text{eff}} = \int r^3 n(r) dr / \int r^2 n(r) dr \left[ \mu m \right]$ (referring to $r_c$, $r_w$, or $r_{3\sigma}$ as selected by the user)
- Number density $n \left[ \text{m}^{-3} \right]$ (number of particles per atmospheric volume)
- Cross section density $a \left[ \text{m}^{-1} \text{m}^{-1} \right]$ (particle cross section per atmospheric volume)
- Volume density $v$ (particle volume per atmospheric volume)
- Mass concentration $M \left[ \text{gm}^{-3} \text{gm}^{-3} \right]$ (particle mass per atmospheric volume)
- Expansion coefficients ($\alpha_1$ to $\beta_2$) for elements of scattering matrix
Scattering matrix elements \((a_1 \text{ to } b_2)\) at user defined angle grid

Volume scattering function \(\tilde{a}_1 = \frac{\alpha_{\text{ext}} \cdot \omega_0}{4\pi} \cdot a_1 \left[ m^{-1} \cdot \text{sr}^{-1} \right]\) at user defined angle grid

Backscatter coefficient \(\beta = \frac{\alpha_{\text{ext}} \cdot \omega_0}{4\pi} \cdot a_1 (180^\circ) \left[ m^{-1} \cdot \text{sr}^{-1} \right]\)

Lidar ratio \(S = \frac{4\pi \omega_0 a_1 (180^\circ)}{\text{sr}} \left[ \text{sr} \right]\)

Linear depolarization ratio \(\delta_l = \frac{a_1 (180^\circ) - a_2 (180^\circ)}{a_1 (180^\circ) + a_2 (180^\circ)}\)

Ångström exponents \(\text{AE}_\zeta = -\frac{\log(\zeta(\lambda_1))}{\log(\lambda_1/\lambda_2)}\) for \(\zeta \in \{\alpha_{\text{ext}}, \alpha_{\text{sca}}, \alpha_{\text{abs}}, \beta\}\)

Extinction to mass conversion factor \(\eta = \frac{M}{\alpha_{\text{ext}}} \left[ g m^{-2} \right]\)

Mass to backscatter conversion factor \(Z = \frac{\beta}{M} \left[ m^2 \cdot \text{sr}^{-1} \cdot g^{-1} \right]\)

Scattering matrix elements and the quantities derived from them are calculated from the expansion coefficients. Wavelength-independent properties \(r_{\text{eff}}, N_a, a, v,\) and \(M,\) are calculated for each wavelength to demonstrate the numerical accuracy of the integration.

The results are available in ASCII and in netcdf format. The format of the program output is described in the user manual. The netcdf output files can be read by the radiative transfer model uvspec, which is included in libRadtran (Mayer and Kylling, 2005; Emde et al., 2016).

### 3.3 Interpolation and sampling error

Due to the limited size resolution in the data set and required interpolations between refractive index and aspect ratio grid points, deviations from exact model calculations for specific microphysical properties occur. As examples, Fig. 4 illustrates deviations introduced for single particle properties, while Table \ref{tab:deviations} shows deviations for particle ensembles.

On the left-hand side of Fig. 4 effects of the limited size resolution on the extinction efficiency \(q_{\text{ext}}\) and the asymmetry parameter \(g\) are shown for non-absorbing spheres and spheroids with \(m=1.52 + 0i\). In particular for spheres with \(x > 10\) deviations for single particles can be considerable because of small-scale features that are not resolved in the data set. In case of spheres these features are implicitly considered in the data set by storing the average over 1000 sizes within each size bin as described above. In case of spheroids the data set contains properties calculated for single sizes which may not be fully representative for close-by sizes. However, since the small-scale features are much weaker for spheroids than for spheres, the average deviation for spheroids is much smaller than for spheres.

On the right-hand side of Fig. 4 effects due to the required interpolation between the refractive index grid points are illustrated for spheres with \(m=1.54 + 0.005i\). While the red lines show the properties calculated from the data set, the black lines show Mie calculations done explicitly for \(m=1.54 + 0.005i\) with the same size grid as used in
Figure 4. Examples illustrating the effect of the limited size resolution of the MOPSMAP data set (left-hand side) and the effect of the interpolation between the refractive index grid points of the data set (right-hand side); extinction efficiencies $q_{ext}$ (upper panels) and asymmetry parameters $g$ (lower panels) as functions of the size parameter from $x=0$ to $x=40$ are compared; in a) and c) the high size resolution calculations (grey lines) were performed with linear $x$ steps of 0.002 in case of spheres and 0.01 in case of spheroids; in b) and d) the red lines show properties calculated for $m=1.54+0.005i$ with MOPSMAP for $m = 1.54 + 0.005i$ by interpolation between refractive indices included in the data set (their data i.e., between $m = 1.52 + 0.0043i$, $m = 1.52 + 0.0060811i$, $m = 1.56 + 0.0043i$, and $m = 1.56 + 0.0060811i$, for which the properties are shown as four thin grey lines), and the black lines show for comparison the properties calculated by Mie theory explicitly for $m=1.54+0.005i$ ($m = 1.54 + 0.005i$ using the same $x$ grid as used by the data set).

The comparison illustrates that MOPSMAP calculates optical properties on average correctly but some smaller scale features are lost: for example, the extinction efficiency $q_{ext}(x)$ in the size parameter range from 20 to 40 is dampened compared to the Mie calculation for $m=1.54+0.005i$ because of the interference of the $q_{ext}(x)$ curves for $m_r=1.52$ and $m_r=1.56$ (see grey lines in Fig. 4b; note that curves for different $m_i$ lie almost on top of each other).

For other size ranges, refractive indices, and optical quantities, the effects on the single particle properties are in principle similar but they may vary in magnitude.

Table 3 investigates the sampling and interpolation errors for a mono-modal log-normal size distribution with a typical width of $\sigma=2.0\sigma=2.0$. The effective radius is $r_{eff}=1.44\mu m$.
Table 3. Optical properties calculated for a log-normal mode with $r_{\text{mod}}=0.5\mu m$, $\sigma=2.0\sigma=2.0$, $r_{\text{min}}=0.01\mu m$, $r_{\text{max}}=4\mu m$ at $\lambda=0.62832\mu m\lambda=628.32\text{nm}$. Two cases of particle shapes are considered: spheres and prolate spheroids with $\epsilon=2.0\epsilon=2.0$. Columns 'data set' contain values calculated using MOPSMAP with the data set described in Sect. 2.3. For comparison, the same properties are calculated in columns 'highres' using high size resolution and in columns 'explicit' using Mie theory or TMM explicitly at $m=1.54+0.005i$.

<table>
<thead>
<tr>
<th>size sampling example</th>
<th>$m$-interpolation example</th>
</tr>
</thead>
<tbody>
<tr>
<td>for $m=1.52+0i$</td>
<td>for $m=1.54+0.005i$</td>
</tr>
<tr>
<td>spheres</td>
<td>spheroids</td>
</tr>
<tr>
<td>data set</td>
<td>highres</td>
</tr>
<tr>
<td>$\alpha_{\text{ext}}$ [km$^{-1}$]</td>
<td>4.808</td>
</tr>
<tr>
<td>$\omega_0$</td>
<td>1.0000</td>
</tr>
<tr>
<td>$g$</td>
<td>0.7045</td>
</tr>
<tr>
<td>$S$ [sr]</td>
<td>10.52</td>
</tr>
<tr>
<td>$\delta_l$</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

Sizes up to $r_{\text{max}}=4\mu m$, which corresponds to size parameter $x_c=40$ at $\lambda=0.62832\mu m$, $x_c=40$ at $\lambda=628.32\text{nm}$, are considered. The left half of Tab. 3 compares optical properties calculated from the MOPSMAP data set (columns 'data set') with properties calculated using a high size resolution (columns 'highres'), the same resolutions as displayed in Fig. 4a. For spheres the results are equal up to at least the fourth digit. In case of prolate spheroids with $\epsilon=2.0\epsilon=2.0$, deviations are found for the forth digit of $\alpha_{\text{ext}}$ and $g$. For the lidar-related quantities $S$ and $\delta_l$ the differences are larger with the relative deviation of $\delta_l$ being 2.6%. These differences are caused by the high sensitivity of lidar-related quantities and it is expected that deviations become smaller when shape distributions or wider size distributions are applied.

The right half of Tab. 3 demonstrates the effect of the $m$-interpolation for an exemplary $m=1.54+0.005im=1.54+0.005i$. MOPSMAP calculations (columns 'data set') are compared to results obtained using explicitly this refractive index in the Mie and TMM calculations. While the effect of the $m$-interpolation is very small for $\alpha_{\text{ext}}$, $g$, and $\delta_l$, it is slightly larger for $\omega_0$ and $S$. The maximum relative effect is found for the lidar ratio $S$ of spheres with a deviation of 1.7%.

These comparisons demonstrate that deviations found for single particles are largely smoothed out in case of particle ensembles due to the averaging over a large number of single particles, different particles. Only for a few special atmospheric applications, for example modeling of a rainbow, the limited resolution of the data set may still lead to a considerable error.
4 MOPSMAP web interface

A web-interface is provided as part of MOPSMAP at https://mopsmap.net. It was designed to be intuitive for expert and non-expert users, e.g., for the demonstration of sensitivities of optical properties on microphysical properties in the framework of lectures, but also for a lot of scientific problems as outlined in the following section. The web interface is written in PHP and uses the SQLite library. After the registration as a user, online calculations of optical properties of a large range of particle ensembles can be performed. Input and output can be defined by the user; for non-expert users a lot of default ensembles representative for specific climatological conditions are already available. The input parameters primarily include the microphysical properties of the particles. The particles’ microphysics are described by up to four components (each described by an individual log-normal size distribution), the wavelength dependence of the refractive index and the shape. Any log-normal size distribution can be used; to facilitate the usage (e.g., for non-expert users) the aerosol components from the OPAC data set (Hess et al., 1998), e.g., "mineral coarse mode", "water-soluble", or "soot", are already included. The same is true for the ten "aerosol types" defined in OPAC, e.g., "continental clean", "urban" or "maritime polluted", consisting of a combination of components. Calculations can be made for a single wavelength, for wavelength ranges or a prescribed wavelengths-set (e.g., for a typical aerosol lidar or a AERONET sun photometer). Moreover, the user can define own wavelengths-sets, e.g., for a specific radiometer. The relative humidity is selected by the user and it is effective for all hygroscopic components according to Eq. 21. The hygroscopic growth of the OPAC components in MOPSMAP differs from the original OPAC version (Hess et al., 1998); it follows the $\kappa$-parameterization with the values proposed by Zieger et al. (2013). In the 'expert user mode' the flexibility is further increased: the number of components can be larger than four, and the size distribution can be given as discrete values on a user-defined size grid.

The output comprises the complete set of optical properties as described in Section 3.2. It can be downloaded for further applications and includes ASCII tables as well as a netcdf-file as required that can be used for radiative transfer calculations using with uvspec of the widely used Libradtran-lbradtran package (Emde et al., 2016). To provide an immediate overview over the results, the most important parameters, such as extinction coefficient ($\alpha_{ext}$), single scattering albedo ($\omega_0$), asymmetry parameter ($g$), Ångström exponent (AE), or lidar ratio ($S$), are displayed as tables when the calculations have been completed. In addition plots of the results as function of wavelength and scattering angle are shown as selected by the user.

All results are stored in the user’s personal folder so that all calculations can be reproduced. Furthermore, all calculations can also easily be rerun with a slightly modified input parameter set.

5 Applications

In this section a selection of examples is presented to demonstrate the wide range of applications of MOPSMAP. Many of them can be performed by using the web interface. Some examples need a local version of MOPSMAP alongside with scripts that
repeatedly call the Fortran program. These scripts are written in Python and can be requested together with downloaded from Gasteiger and Wiegner (2018) as part of the MOPSMAP package.

It is worth mentioning that numerous studies demonstrate the need for optical modeling of aerosol ensembles, thus illustrating the range of possible applications of MOPSMAP. Moreover, optical modeling is essential for many different related modeling activities. It is required, for example, for closure experiments (consistency checks between different measurement methods involving an aerosol model, e.g., Wiegner et al., 2009; Gasteiger et al., 2011b; Müller et al., 2012; Bell et al., 2013; Ma et al., 2014; Zieger et al., 2014; Düsing et al., 2018), radiative transfer studies (e.g., Otto et al., 2009; Emde et al., 2010), inversion of remote sensing measurements (e.g., Dubovik et al., 2006; Gasteiger et al., 2011a; Müller et al., 2016), inversion of in-situ data (e.g., Weinzierl et al., 2009; Szymanski et al., 2009; Kassianov et al., 2014), aerosol layer visibility simulations (e.g., Weinzierl et al., 2012), dynamic aerosol transport models (e.g., Heinold et al., 2007; Balzarini et al., 2015), aerosol characterization (e.g., Gasteiger et al., 2017; Che et al., 2018; Zhuang et al., 2018), and solar energy (e.g., Polo et al., 2016; Kosmopoulos et al., 2017).

5.1 Effect of hygroscopicity

The first example of applications deals with hygroscopic growth. If aerosol particles are hygroscopic their microphysical and optical properties change with relative humidity RH. Fig. 5 shows how optical properties of the 10 OPAC aerosol types (Hess et al., 1998), which contain up to four components, some of which being hygroscopic, change with RH. These calculations were performed using the MOPSMAP web interface, where the OPAC aerosol types are available as pre-defined ensembles and the relative humidity can be chosen by the user. MOPSMAP considers the hygroscopic effect by application of the κ-parameterization (Eq. 21) which differs from the RH dependency implemented in OPAC.

The upper row of Fig. 5 shows the normalized extinction coefficient of the different types (indicated by color) at three wavelengths λ (each in a subplot) calculated for RH values of 0 %, 50 %, 70 %, 80 %, and 90 %. The extinction at all λ is normalized to the extinction at RH=0% and λ = 532 nm. As a consequence, the differences between the columns illustrate the wavelength dependency of the extinction, while changes with RH illustrate the hygroscopic effects. For example, for the desert aerosol type (orange color), the wavelength dependency is low, which is related to the large size of the dominant mineral particles, and the hygroscopic effect is relatively weak because mineral particles are hygrophobic. By contrast, for maritime (bluish colors) and antarctic types (purple color), the wavelength dependence is stronger and the hygroscopic effect is strong because of the domination by highly hygroscopic sulfate and sea salt particles. For the continental as well as the urban and arctic types, the wavelength dependence is even stronger while the hygroscopic effect is weaker, which may be explained by strong contributions from the soot and watersoluble components which contain quite small particles with κ values significantly smaller than the κ values of sea salt particles (e.g., Petters and Kreidenweis, 2007; Markelj et al., 2017; Enroth et al., 2018; Psichoudaki et al., 2018).
The single scattering albedo $\omega_0$ is shown in the second row of Fig. 5. $\omega_0$ varies strongly with aerosol type, with the highest values of almost 1.0 for the antarctic, maritime clean, and maritime tropical aerosol types. Since water is almost non-absorbing at the considered wavelengths, the water uptake does hardly change $\omega_0$ if $\omega_0$ is already close to 1.0. The single scattering
albedo of the desert type is much lower but it is also virtually independent on the RH as this aerosol type does not take up much water. For the other types, an increase of RH results in an increase of ω0.

The extinction to mass conversion factor η, which is plotted in the third row of Fig. 5, is necessary to calculate mass concentrations from extinction coefficient measurements or mass loadings from AOD measurements. An important parameter for η is the particle size (e.g., Gasteiger et al., 2011a) with the consequence that the desert aerosol type, which contains the highest fraction of coarse particles of the considered types, shows the highest η values. Again, the wavelength dependency is significant for the other aerosol types so that the η values at λ = 1064 nm (right column) are significantly larger than at λ = 532 nm (middle column). The dependence of η on RH is significantly weaker than the dependence of the extinction on RH (upper row), which may be explained with the compensation of the increase of extinction by the increase of mass with increasing RH.

The bottom row of Fig. 5 illustrates the mass to backscatter conversion factor Z as function of the relative humidity RH. Z is useful for example for comparisons of vertical profiles simulated with aerosol transport models to profiles measured with lidar or ceilometer. Multiplication of simulated aerosol mass concentrations M with Z provides simulated β profiles which can be compared with the measurements. The figure shows that there is considerable spread between the different aerosol types in particular at short wavelengths. RH has strong effects only on the maritime and arctic aerosol types.

Currently the hygroscopic growth of different aerosol components is not ultimately understood, and different κ-values are discussed. With MOPSMAP their influence on the optical properties can easily be determined and used in validation studies.

5.2 Optical properties for sectional aerosol models

Aerosol transport models in combination with the optical properties of the aerosol allow one to model the radiative effect of the aerosol. The aerosol is typically modeled in terms of mass concentrations for a limited number of aerosol types divided over a few size bins (sectional aerosol model) or a few modes (modal aerosol models). Thus, realistic optical properties for each size bin of each aerosol type are required for modeling the radiative effects (e.g., Curci et al., 2015).

In this example, we calculated the optical properties of dust at λ = 500 nm for the five size bins of the COSMO-MUSCAT model (Heinold et al., 2007). The size bins are determined by the radius limits 0.1 μm, 0.3 μm, 0.9 μm, 2.6 μm, 8 μm, and 24 μm. We assumed constant dv/dlnr within each bin. Each bin was modeled through the expert mode of the MOPSMAP web interface. The refractive index is m = 1.53 + 0.0078i which is equal to the value given for the mineral components in OPAC. We considered two cases for the particle shape, on the one hand spherical particles and on the other hand prolate spheroids with the aspect ratio distribution given by Kandler et al. (2009). For the latter case we assumed volume-equivalent sizes to keep the particle mass constant.

The calculated phase functions are presented in Fig. 6, where each size bin is represented by an individual color. The difference between both lines of same color represents the shape effect. For size bin 1 (0.1 μm < r < 0.3 μm, black lines)
**Figure 6.** Phase functions at $\lambda = 500$ nm of the five COSMO-MUSCAT dust size bins (different colors) assuming spherical particles (solid lines) and prolate spheroids (dashed lines). For details see text.

**Table 4.** Optical properties at $\lambda = 500$ nm of the five COSMO-MUSCAT dust size bins. Two cases for the particle shape are considered: Spheres / prolate spheroids. For details see text.

<table>
<thead>
<tr>
<th></th>
<th>bin 1</th>
<th>bin 2</th>
<th>bin 3</th>
<th>bin 4</th>
<th>bin 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\omega_0$</td>
<td>0.9632 / 0.9628</td>
<td>0.9216 / 0.9264</td>
<td>0.7903 / 0.7934</td>
<td>0.6450 / 0.6485</td>
<td>0.5561 / 0.5601</td>
</tr>
<tr>
<td>$g$</td>
<td>0.6567 / 0.6585</td>
<td>0.6866 / 0.7111</td>
<td>0.8088 / 0.8109</td>
<td>0.8998 / 0.9017</td>
<td>0.9442 / 0.9419</td>
</tr>
<tr>
<td>$\eta \ [gm^{-2}gm^{-2}]$</td>
<td>0.2905 / 0.3000</td>
<td>0.5594 / 0.5236</td>
<td>2.230 / 2.071</td>
<td>6.989 / 6.633</td>
<td>22.09 / 20.90</td>
</tr>
<tr>
<td>$Z \ [m^2sr^{-1}g^{-1}m^2sr^{-1}g^{-1}]$</td>
<td>4.234·10^{-2} /</td>
<td>1.185·10^{-1} /</td>
<td>1.403·10^{-2} /</td>
<td>1.204·10^{-3} /</td>
<td>8.225·10^{-5} /</td>
</tr>
<tr>
<td></td>
<td>3.981·10^{-2}</td>
<td>5.421·10^{-2}</td>
<td>8.901·10^{-3}</td>
<td>7.457·10^{-4}</td>
<td>8.651·10^{-5}</td>
</tr>
</tbody>
</table>

The difference is small, while whereas for all other bins the shape effect is larger for all other bins. The strongest effects are found for $\theta > 100^\circ$ with differences of up to a factor of 4 between the particle shapes. These angular ranges can be important for example for the backscattering of sun light into the space and thus for the aerosol radiative effect. The very strong effect at $\theta=180^\circ$ is relevant for any lidar application, e.g, the intercomparison of modeled and measured attenuated backscatter profiles (Chan et al., 2017)(Chan et al., 2018).

Calculated parameters relevant for radiative transfer and remote sensing are given in Table 4. The shape effect on the single scattering albedo $\omega_0$ and the asymmetry parameter $g$ is small except for size bin 2 where $g$ is significantly larger for the spheroids than for the spheres. The extinction to mass conversion factor $\eta$ is systematically smaller for spheroids than for spheres in bins 2-5 because the geometric cross section of the spheroids is $\approx 5.5\%$ larger than the cross section of the volume-equivalent spheres. The mass to backscatter conversion factor $Z$ of the spheroids is for most size bins lower than $Z$ of spheres with maximum differences being larger than a factor of 2.
5.3 Effect of cut-off at maximum size

Many in-situ measurement setups are limited with respect to the maximum particle size they are able to sample, e.g., because of losses at the inlet or the tubing. In this example, we illustrate the effect of the cut-off for the desert aerosol type from OPAC at $\text{RH}=0\%$ (Koepke et al., 2015).

![Graph 1](image1)

**Figure 7.** Optical and microphysical properties of the OPAC desert aerosol type as function of cutoff radius $r_{\text{max}}$. The upper panel shows properties that are normalized by the values found at $r_{\text{max}}=60 \mu m$ (where 99.988 % of the total particle cross section is covered, referring to $r_{\text{max}} = \infty$). The PM10 mass, i.e. the mass in the particles with diameter smaller than 10 $\mu m$ ($r_{\text{max}}=5 \mu m$), and the PM2.5 mass ($r_{\text{max}}=1.25 \mu m$) are standard parameters to quantify pollution (e.g., Querol et al., 2004). In our example, PM10 and PM2.5 contains only 59.5 % and 21.6 % of the total particle mass, respectively. However, PM10 and PM2.5 measurement setups cover 94.4 % and 69.0 % of the total cross section area or extinction geometric cross section, respectively. The single scattering albedo in this case of PM2.5 is about 0.035-0.071 higher than for the total...
aerosol, whereas the asymmetry parameter is lowered-reduced by about 0.02-0.04. As further example, if the cutoff is $r_{\text{max}}=10\mu m$, 97.8% of the total cross section and 75.6% of the mass are covered; the single scattering albedo and the asymmetry parameter deviate from the total aerosol by less than 0.008.

This example shows that consideration of maximum size is essential when derived optical properties or mass concentrations are interpreted and results can be severely misleading if the cut-off radius is not considered. These effects can be easily quantified with MOPSMAP and its web interface.

5.4 Effect of selection of size equivalence of non-spherical particles

In case of a

![Figure 8. Log-normal size distributions (SD) with same $r_{\text{mod}}, \sigma$, $N_0$, and $r_{\text{max}}$ assuming different size equivalences for aggregate particles (shape D, $\xi_{\text{ve}} = 0.8708$) as applied in Table 5. The size distributions are plotted in terms of cross-section-equivalent sizes (i.e., $dN/dr_{c}(r_{c})$ referring to black axes and grid). For comparison also axes valid for the other size interpretations are plotted in red and green, which allows each size distribution to be interpreted in terms of each size equivalence.

This example demonstrates how the selection of the size equivalence in case of non-spherical particle, the particle size is usually described by the size of an equivalent sphere. As introduced in particles affects various ensemble properties. In MOPSMAP the size-related parameters are either interpreted as $r_{c}$ (default), or as $r_{v}$ or $r_{\text{ve}}$ (see Sect. 2.1, this can be...) according to the choice of the user. Each size equivalence can be transformed into another by Eqs. 3 and 4, E.g., if 'volume cross section ratio equivalent' has been chosen in the web interface, and '0.5' for $r_{\text{mod}}$, this would be equivalent to setting $0.5 \cdot \xi_{\text{ve}}^{-3}$ for example, a sphere with the same cross section area $r_{\text{mod}}$ when the default 'cross section equivalent' is kept ($\xi_{\text{ve}}$ depending on shape).
Table 5. Properties of one-modal size distribution at $\lambda = 532\text{nm}$, $\xi_{\text{vc}} = 0.8708$, consisting of spheres or aggregate particles (shape D, $\xi_{\text{vc}} = 0.8708$, Fig. 1 of Gasteiger et al. (2011b)) assuming different size equivalences. For details see text.

<table>
<thead>
<tr>
<th>properties</th>
<th>spheres</th>
<th>aggregate particles</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>using $r$</td>
<td>using $r_c$</td>
</tr>
<tr>
<td>$\alpha_{\text{ext}} \text{ [km}^{-1}\text{km}^{-1}]$</td>
<td>0.350</td>
<td>0.347</td>
</tr>
<tr>
<td>$\omega_0$</td>
<td>0.897</td>
<td>0.922</td>
</tr>
<tr>
<td>$g$</td>
<td>0.722</td>
<td>0.679</td>
</tr>
<tr>
<td>$a_1(0^\circ-90^\circ)$</td>
<td>100</td>
<td>97.4</td>
</tr>
<tr>
<td>$\bar{a}_1(0^\circ-90^\circ) \text{ [km}^{-1}\text{sr}^{-1}\text{km}^{-1}\text{sr}^{-1}]$</td>
<td>2.51</td>
<td>2.48</td>
</tr>
<tr>
<td>$a_1(180^\circ-180^\circ)$</td>
<td>1.21</td>
<td>0.405</td>
</tr>
<tr>
<td>$S \text{ [sr]}$</td>
<td>11.6</td>
<td>33.6</td>
</tr>
<tr>
<td>$\delta_l$</td>
<td>0.000</td>
<td>0.450</td>
</tr>
<tr>
<td>cross section density $a \text{ [km}^{-1}\text{km}^{-1}]$</td>
<td>0.141</td>
<td>0.141</td>
</tr>
<tr>
<td>mass concentration $M \text{ [µg} \cdot \text{m}^{-3}]$</td>
<td>482</td>
<td>318</td>
</tr>
</tbody>
</table>

To further elucidate the role of the different representations of radii, the same volume, or the same ratio between volume and cross section as the non-spherical particle. These size equivalences can be selected in MOPSMAP and its web interface parameters of a log-normal size distribution are applied to the different size interpretations. For this purpose, the parameters are set to $r_{\text{mod}} = 0.5 \mu m$, $\sigma = 2$ with $r_{\text{min}} = 0.001 \mu m$, $r_{\text{max}} = 1.75 \mu m$ ($r_{\text{eff}} = 0.98 \mu m$), and $N_0 = 103.66 \text{ cm}^{-3}$, which results in a concentration of $N = 100 \text{ cm}^{-3}$ in the range from $r_{\text{min}}$ to $r_{\text{max}}$. The effect of the three alternative interpretations on particle size is demonstrated in Fig. 8 for irregular shape D having $\xi_{\text{vc}} = 0.8708$. All three size distributions (curves of different color) are plotted in terms of $dN/dr_c (r_c)$ (black axes). For comparison, also axes for $dN/dr_v (r_v)$ (red axes) and $dN/dr_{\text{vc}} (r_{\text{vc}})$ (green axes) are shown. Using these axes, the size distribution curves can be interpreted in terms of the various size equivalences. The comparison between the size distributions clearly shows a shift towards larger sizes when $r_{\text{vc}}$ or $r_v$ instead of $r_c$ is assumed. For example, assuming $r_{\text{vc}}$, for the log-normal size distribution (green curve) describes the same ensemble as using $r_{\text{mod}} = 0.5 \mu m$, $\sigma = 2$, $r_{\text{min}} = 0.001 \mu m$, and $r_{\text{max}} = 1.75 \mu m$ ($r_{\text{eff}} = 0.98 \mu m$) is assumed. $N_0$ is set to 103.66 cm$^{-3}$, which results in a concentration of 100 cm$^{-3}$ in the range from $r_{\text{min}}$ to $r_{\text{max}}$. Furthermore, the since the size distributions depend on the selected size equivalence various (optical) properties of the ensemble are also different; a quantification has been provided by MOPSMAP (Table 5). The particle mass density is set to 2600 kg m$^{-3}$ kg m$^{-3}$, the refractive index is $m = 1.54 + 0.005i$, and the wavelength is $\lambda = 532 \mu m$. In this example particles larger than the wavelength are optically dominant. Results calculated with MOPSMAP are shown in Tab. 5 $\lambda = 532 \text{ nm}$. The first column of Table 5 shows the optical properties of spherical particles.
In the subsequent columns, all particles are assumed to be aggregate particles (shape D) with the same \( r_c \) (second column, red curve), the same \( r_v \) (third column, green curve), and the same \( r_{vcr} \) (last column, black curve) as the spheres in the first column. In other words, this means that \( r_{mod}, r_{min}, \) and \( r_{max} \) alternately refer to these different radius definitions.

The results show that the size of non-spherical particles increases are consistent with the increase of particle size from assuming \( r_c \) over \( r_v \) to \( r_{vcr} \) (see cross section density \( \sigma \) and mass concentration \( M \)) from assuming \( r_c \) over \( r_v \) to \( r_{vcr} \), and also Fig. 8). The extinction coefficient \( \alpha_{ext} \) and the forward volume scattering \( \tilde{a}_1 (0^\circ\theta) \) of the non-spherical particles best agree with the spherical counterparts if cross section equivalence cross section equivalence is assumed. These properties are known to be sensitive to the particle cross section for particles larger than the wavelength. The absorption is in first approximation proportional to the particle volume if absorption is weak. As a consequence, for the single scattering albedo \( \omega_0 \) both cross section and volume are relevant and dependencies are more complicated than for \( \alpha_{ext} \). The single scattering albedo \( \omega_0 \) of shape D decreases in Fig. 8) from left to right due to the strong increase in particle volume. The selection of the size equivalence has a small effect on the asymmetry parameter \( g \), the backward phase function \( a_1 (180^\circ\theta) \), the lidar ratio \( S \), and the linear depolarization ratio \( \delta_1 \).

These results highlight the importance of a thoughtful selection of the size equivalence. Considering that the most appropriate size equivalence certainly depends on the concept how the size distribution is measured. For example, if scattering by coarse dust particles is measured and the size is inverted assuming spherical particles, assuming cross-section equivalence in subsequent applications with non-spherical particles seems natural as scattering mainly depends on the particle cross section. MOPSMAP and its web interface provides the flexibility to investigate this topic theoretically.

### 5.5 Uncertainty estimation of calculated optical properties

In general, the knowledge on microphysical properties is limited, thus they are subject to uncertainties. If these uncertainties can be quantified, it is consistent to also quantify the corresponding uncertainties of the optical properties.

In this regard, the sensitivity of a calculated optical property \( \zeta \) to changes of a microphysical property \( \chi, \psi \) is an important aspect that can be expressed by the first partial derivative \( \partial \zeta / \partial \chi \partial \zeta / \partial \psi \). The Jacobian matrix \( \mathbf{J} \) is the \( M \times N \) matrix containing all first partial derivatives for \( M \) optical properties and \( N \) microphysical properties. The elements of \( \mathbf{J} \) of an aerosol ensemble can be numerically calculated by perturbing the microphysical properties of the ensemble. For demonstration in the following example we perturb \( \chi, \psi \) with a factor of 0.99 and 1.01 to numerically calculate the first partial derivatives. A sample script for the calculation of \( \mathbf{J} \) is provided together with MOPSMAP.

Table 6 shows an example of \( \mathbf{J} \) for the optical properties \( \zeta \in \{ \omega_0, g, S \} \) and the microphysical properties \( \chi \in \{ \epsilon_i, m_i, \epsilon_i, m_i, \epsilon_i, m_i, \epsilon_i, m_i \} \phi \in \{ m_\ell, \ell, m_\ell, \ell, m_\ell, \ell \}

\( \mathbf{J} \) was calculated for a simplified dust ensemble described by one log-normal size mode with \( r_{mod}, r_{min}, r_{max} \) and \( m_\ell, \ell, m_\ell, \ell, m_\ell, \ell, m_\ell, \ell \)

and prolate spheroids with \( \epsilon_i, \ell = 2.0 \). The wavelength is set to \( \lambda = 532 \text{ nm}, \lambda = 532 \text{ nm} \). This results in \( \omega_0 = 0.9020, \nu = 0.7319 \).
Table 6. Elements of the Jacobian matrix, i.e. first partial derivatives, of a dust-like ensemble (see text for details).

<table>
<thead>
<tr>
<th></th>
<th>$\partial \omega_0$</th>
<th>$\partial g$</th>
<th>$\partial S$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\partial \epsilon'$</td>
<td>+0.010</td>
<td>+0.058</td>
<td>+48.3 sr</td>
</tr>
<tr>
<td>$\partial m_r$</td>
<td>-0.037</td>
<td>-0.428</td>
<td>-360 sr</td>
</tr>
<tr>
<td>$\partial m_i$</td>
<td>-11.0</td>
<td>+3.69</td>
<td>+2839 sr</td>
</tr>
</tbody>
</table>

and $S=69.95\omega_0=0.9020$, $g=0.7319$, and $S=69.95$ sr. These properties are most sensitive to $m_i$ which can be clearly seen from Table 6. For example, a change of $m_i$ by 0.001 would result in a change of $\omega_0$ of 0.011. But it also needs to be considered that the absolute values of $m_i$ are usually low, which contributes to the high values for the partial derivatives. Nonetheless, by comparing the derivatives of $\omega_0$ it is clear that $m_i$ is by far most important for $\omega_0$. An increase in $\epsilon'$ or $m_i$ increases $g$ and $S$, while whereas an increase in $m_r$ reduces their values. The sensitivity to perturbations of the microphysical properties is particularly strong for the lidar ratio $S$ which can be seen by comparing $S=69.95$ sr of the ensemble with the partial derivatives. We emphasize that the accuracy of $J$ is limited by the sampling in the MOPSMAP data set (see also Sect. 3.3), for example partial derivatives $\partial \zeta/\partial m_r$ are constant between the $m_r$ grid points of the data set.

The Jacobian matrix $J$ is valid for a certain set of microphysical properties values and, as mentioned, $J$ can be used to quantify the uncertainty of the calculated properties for a given microphysical uncertainty. However, when uncertainties of the microphysical properties get larger, $J$ may change significantly within the uncertainty range of $\chi$ and other approaches may be required to estimate the uncertainty of the calculated optical properties. A simple approach applicable to this problem is the Monte Carlo method (e.g., JCGM, 2008). Repeated calculations with microphysical properties randomly chosen within the uncertainty range are performed. The uncertainty of the calculated quantities is determined by the statistics over the different sampled ensembles. In general, the computation time is longer than using $J$ and is proportional to the number of calculated ensembles. Due to the statistical nature of the Monte Carlo method, the final results get more precise with increasing number of sampled ensembles. A Monte Carlo script for the Monte Carlo uncertainty propagation is provided together with MOPSMAP. For example, based on the ensemble described above, sampling within the uncertainty ranges $r_{mod}=0.1\pm0.01\mu m$, $m_{mod}=0.1\pm0.01\mu m$, $\sigma=2.6\pm0.1$, $m_r=1.53\pm0.03$, $m_r=0.0063\pm0.002$, and $\epsilon'=2.0\pm0.5$ results in the ranges $0.85<\omega_0<0.94$, $0.68<g<0.78$, and $29\text{ sr}<S<103\text{ sr}$.

5.6 Effect of refractive index variability

Mineral dust aerosols are ensembles of different minerals having different refractive indices. Usually the variability of the refractive index of the particles within a dust aerosol ensemble is neglected when modeling its optical properties. In this example, we compare optical properties calculated using the full measured variability of the imaginary part of the refractive index $m_i$ to properties calculated with the common assumption of all particles in an ensemble having an average $m_i$. Furthermore, a parameterization of the variability is considered.
We use the desert aerosol type of OPAC (Koepke et al., 2015). Prolate spheroids with the aspect ratio distribution of Kandler et al. (2009) are assumed for the mineral components and spherical particles for the WASO component (RH=0 RH = 0 %). The real part of the refractive index is $m_r=1.53$, $m_r=1.53$ for all particles. The wavelength in this example is set to $\lambda = 355 \text{ nm}$, which is a wavelength where absorption by iron oxide is strong. Because of the variable iron oxide content of individual particles, the variability of $m_i$ is large at this wavelength. Consequently, a significant influence on optical properties can be expected. In this example we consider three cases of imaginary part variability: First, we apply the size-resolved distribution of the imaginary part of the refractive index for Saharan dust as derived from mineralogical analysis (Kandler et al., 2011). Second, we assume the average imaginary part for all particles (it is $0.0175$ which is close to $0.0166$ given for the mineral components in OPAC at $\lambda = 355 \text{ nm}$). Finally, we parameterize the $m_i$ distribution with the non-absorbing fraction approach as introduced in Sect. 3.1. In this case we set $X=0.5$, $X=0.5$, resulting in 50% of the mineral particles having $m_i=0$, whereas the other 50% of the particles having $m_i=0.0349$.

![Figure 9](image_url)

**Figure 9.** Volume scattering function of dust at $\lambda = 355 \text{ nm}$ ($\lambda = 355 \text{ nm}$) (arbitrary scale) for different using either the $m_i$ distributions distribution (see text) measured by Kandler et al. (2011), the average $m_i$ of these measurements (black), or applying the non-absorbing fraction parameterization with different $X$ (blue).

Fig. 9 shows the volume scattering function for the three cases. This figure shows that the sensitivity of the forward scattering to the $m_i$ distribution is negligible whereas the sensitivity increases with increasing scattering angle $\theta$. For backward scattering, the difference between the measured $m_i$ distribution (red line) and using the average $m_i$ (black line) is more than a factor of two. The parameterization assuming $X=50\%, X=0.5$ (thick blue line) is in much better agreement with the measured case. The root-mean-square relative deviation between the volume scattering function for the measured distribution and for the average $m_i$ is 30%, whereas it is only 4% for the parameterization. For comparison also two additional $X$ values, i.e. $X=25\%, X=0.25$ (thin dashed blue line) as well as $X=75\%, X=0.75$ (thin solid blue line), are shown in Fig. 9, but their deviation is larger than for the parameterization with $X=50\%, X=0.5$. The extinction coefficient $\alpha_{\text{ext}}$ changes only by less than 0.03% between the
three representations of \( m_i \). For \( \omega_0 \) we find 0.852 using the measured \( m_i \)-distribution, whereas \( \omega_0 = 0.741 \) when using the average \( m_i \) and \( \omega_0 = 0.834 \) using the parameterization with \( \chi = 50\% \), \( \chi = 0.5 \). For the asymmetry parameter \( g \) we find 0.744, 0.789, and 0.749 for the measured, averaged, and parameterized cases, respectively. For the lidar ratio \( S \) values of 41 sr, 78 sr, and 42 sr are calculated for the three cases, whereas for the linear depolarization ratio \( \delta_l \) values of 0.241, 0.212, and 0.220 are obtained.

These results emphasize that it is important to consider the non-uniform distribution of the absorptive components in the desert dust ensembles for optical modeling of such aerosols at short wavelengths. We have shown in this example that optical properties of Saharan dust can be well simulated with \( \chi = 0.5 \). Whether this conclusion holds for other cases of desert dust can easily be investigated by means of MOPSMAP when measurements of \( m_i \) distributions of further dust types are available.

### 5.7 Effect of particle shape on the nephelometer truncation error

Integrating nephelometers aim to measure in situ the total scattering coefficient \( \alpha_{sca}^{true} \) of aerosol particles by detecting all scattered light. The angular sensitivities of real nephelometers however deviate from the ideal sensitivity which is the sine of scattering angle \( \theta \). For example nearly-forward or nearly-backward scattered light does not reach the detectors because of the instrument geometry (Müller et al., 2011). This has to be considered during the evaluation of measurements and can be done by applying a truncation correction factor \( C_{ts} = \alpha_{sca}^{true} / \alpha_{sca}^{meas} \) to the measured scattering coefficients \( \alpha_{sca}^{meas} \). \( C_{ts} \) can be calculated theoretically using optical modeling if aerosol microphysical properties and the angular sensitivity of the instrument are known. Some nephelometers not only measure the total scattering coefficient but also the hemispheric backscattering coefficient which is the scattering integrated from \( \theta = 90^\circ \) to \( 180^\circ \). Also for the hemispheric backscattering coefficient a correction factor needs to be applied to correct the measured hemispheric backscattering coefficient affected by the non-ideal instrument sensitivity. This correction factor \( C_{bs} \) is defined analogously to \( C_{ts} \) as the ratio between the true coefficient and the measured one. Note that this hemispheric backscattering coefficient is defined different than \( \beta \) which is measured by lidars and used elsewhere in this paper.

Fig. 10 shows modeled correction factors for the total (upper panel) and the backscatter (lower panel) channel of an Aurora 3000 nephelometer. The angular sensitivity of the instrument is taken from Müller et al. (2011). For the following sensitivity study the mineral dust refractive index from OPAC (Hess et al., 1998), the parameterized \( m_i \) distribution with \( \chi = 50\% \), \( \chi = 0.5 \) (as shown in Sect. 5.6), a log-normal size mode with \( \sigma = 1.6 \), \( \sigma = 1.6 \) and a maximum radius of \( r_{max} = 5 \mu m \) (corresponding to a PM10 inlet) is assumed. The mode radius \( r_{mod} \) is varied from 0.01 to 1 \( \mu m \) (horizontal axis) and two cases for the particle shape, i.e. spherical particles (solid lines) and cross-section-equivalent prolate spheroids with the \( \epsilon' \) distribution from Kandler et al. (2009) (dashed lines) are considered. The colors denote the three operating wavelengths of the instrument (450, 525, and 635 nm). The figure shows that the total scattering correction factor \( C_{ts} \) mainly depends on particle size. In case of large particles (\( r_{mod} = 1 \mu m \)) the nephelometer underestimates total scattering by a factor of \( \approx 2 \) if...
the truncation error is not corrected. Shape has only a small effect on forward scattering, thus its influence on the correction of the truncation error is less than 3\% (compare dashed and solid lines of same color). The maximum shape effect on $C_{bs}$ is 7\%, i.e. indicating that assuming spherical particles for the truncation correction may result in an overestimation of the hemispheric backscattering coefficient.

The correction factors might be recalculated for example when new data on the refractive index or particle shape become available. This example highlights the potential of MOPSMAP as a useful tool for the characterization of optical in-situ instruments. In addition, it could be used for the interpretation of angular measurements, for example as performed with a polar photometer by Horvath et al. (2006).

### 5.8 Optical properties of ash from different volcanoes close to the source

Vogel et al. (2017) present a data set comprising shape-size distributions of ashes from nine different volcanoes as well as wavelength-dependent refractive indices for five different ash types. The particles were collected between 5 and 265 km from the volcanoes. While refractive indices can be expected to be valid also at larger distances from the volcanoes, the effective radii in the range from 9.5 $\mu$m to 21 $\mu$m are probably not realistic for long-range transported ash. Based on this data set, which is available in the supporting information of Vogel et al. (2017), we calculate optical properties of these volcanic ashes with MOPSMAP. Each single particle is modeled as a prolate spheroid with the given size and aspect ratio, as well as with the refractive index given for the type of ash the volcano emits. In addition, we assume a non-absorbing fraction of $\chi = 50\% \rightarrow \chi = 0.5$ (as used in Sect. 5.6). The application of this non-absorbing fraction approach seems reasonable when taking into account the variability of the transparency of the particles shown in Fig. 5 of Vogel et al. (2017). Due to the data set limits
of MOPSMAP, particles with \( r > 47.5 \mu m \) are modeled as \( r = 47.5 \mu m \) and aspect ratios \( > 5 \) are set to 5. For each volcano, less than 0.5 % of the particles were affected by these modifications.

**Figure 11.** Modeled wavelength-dependent optical properties for ashes from different volcanoes. More details on the ash samples is given in Tab. 1 of Vogel et al. (2017). The colors indicate ash type: basalt in dark blue, basaltic andesite in light blue, andesite in green, dacite in orange, and rhyolite in red (see Fig. 7 of Vogel et al. (2017) for reference).

Fig. 11 shows the single scattering albedo \( \omega_0 \) and the asymmetry parameter \( g \) for the nine ashes as function of wavelength between 300 nm and 1500 nm. Differences of \( \omega_0 \) are up to about 0.12 with ash from Chaiten (Chile) and Mt. Kelud (Indonesia) being the least and most absorbing species, respectively. \( \omega_0 \) is correlated with the ash type, which is mainly a result of the significant variability of \( m_i \) (see Fig. 16b of Vogel et al. (2017)). For all ashes, \( \omega_0 \) increases slightly with wavelength, typically by about 0.05 over the wavelength range shown. The variability of \( g \) is less than 0.05 and for all ashes the changes with wavelength are weak with values of less than 0.02. The mass to backscatter conversion factor \( Z \) varies between 1.16 to 3.38 \( \cdot 10^{-2} \) \( m^2 \cdot sr^{-1} \cdot g^{-1} \) for the nine ashes. The extinction to mass conversion factor \( \eta \) at \( \lambda = 550 \text{ nm} \) ranges from 14.8 to 33.0 \( \text{gm}^{-2} \cdot \text{gm}^{-2} \) which is considerably higher than known for typical aerosols (e.g., Fig. 5) or volcanic ash transported over continental scales (e.g., \( \eta \) between 1.10 and 1.88 \( \text{gm}^{-2} \cdot \text{gm}^{-2} \) found by Wiegner et al. (2012)). In particular the different values of \( \eta \) clearly demonstrate that optical properties of volcanic ash layers drastically change with the distance from the eruption due to changing microphysics.

This example suggests that it is worthwhile to consider the specific microphysical properties of each volcano. However, for realistic MOPSMAP calculations valid in the long-range regime certainly size distributions different from the ones used in this example must be applied whereas the refractive indices are more likely representative.
6 Conclusions

Radiative properties of atmospheric aerosols are relevant for a wide range of meteorological applications, in particular for radiative transfer calculations and remote sensing and in-situ techniques. Optical properties strongly depend on the microphysical properties of the particles – size, refractive index and shape, properties that are highly variable under ambient conditions. As a consequence, the application of mean properties could be questionable. However, the determination of optical properties of specific aerosol ensembles can be quite time consuming, in particular when non-spherical particles shall be considered.

For this purpose we have developed the MOPSMAP package that provides the full set of optical properties of arbitrary randomly-oriented aerosol ensembles: single particles of the ensemble can be spherical or spheroidal with size parameters up to \( a \approx 1000, \ x \approx 1000 \). Moreover, a small set of irregular particles is considered. The refractive index can be \( 0.1 \leq m_r \leq 3.0 \) and \( 0 \leq m_i \leq 2.2 \). The size distribution of the ensemble can either be parametrized as log-normal distribution, (modified) gamma distribution or freely chosen according to individual data. MOPSMAP includes a web interface for online calculations \( \text{at} \) https://mopsmap.net, offering the most frequently used options; for advanced applications or large sets of computations the full package is \( \text{available on request} \text{freely available for download} \). Key applications of MOPSMAP are expected to be the evaluation of radiometer measurements in the UV, VIS and N-IR spectral range, or aerosol lidar measurements. They can help to improve the inversion of such measurements for aerosol characterization. Furthermore, MOPSMAP can be used to refine optical properties of aerosols in radiative transfer models, or in numerical weather prediction and chemistry transport models.

The details of the concept underlying MOPSMAP are discussed in this paper. Several examples are presented to illustrate the potential of the package, including an example to calculate optical properties for sectional aerosol models and an example illustrating the effect of maximum size cut-off that occurs in the inlet system of in-situ instruments. In another example, conversion factors between the backscatter coefficient (available from lidar/ceilometer measurements or from numerical forecast models) and the mass concentration of volcanic ashes have been calculated. These conversion factors are relevant to estimate flight safety after volcanic eruptions and vary by about a factor of three between the nine ashes under investigation.

The concept of MOPSMAP allows continuous upgrades to further extend the range of applications. E.g., the resolution of the refractive index grid \( \text{can} \text{could} \) be increased, new versions of underlying scattering codes \( \text{could} \text{be applied when available} \), larger size parameters \( \text{can} \text{could} \) be considered, e.g., using DDA for \( m \) close to 1 (Yurkin and Hoekstra, 2011), and new sets of irregular particles \( \text{can} \text{could} \) be implemented, e.g., those presented by Mehri et al. (2018). However such extensions can be quite time consuming, so that extensions are expected to be limited. Moreover, conceptional upgrades will be investigated without knowing yet whether they can be included in the web interface. Here, a trade-off between scientific complexity and user-friendliness must be found. In this context \( \text{Whereas internal mixing in case of homogeneous particles is already covered in the present version} \), the implementation of a core-shell particle model can be discussed. Finally, we want to emphasize that the feedback of the users will help us to set up a priority list of further actions.
Orientation averaging applied to irregular-shaped particles

The particle orientation is specified by three Euler angles \((\alpha_e, \beta_e, \gamma_e)\) as described by Yurkin and Hoekstra (2011). Averaging over \(\beta_e\) is done with a step width of 15°, and for each \(\beta_e\) up to 24 \(\gamma_e\) are used for averaging (see details in Tab. ??). The averaging over \(\alpha_e\) is done within a single ADDA computation because rotation over \(\alpha_e\) is equivalent to the rotation of the scattering plane and is computationally cheap. The optical properties are averaged over 32 \(\alpha_e\). In total, for a single particle 206 individual ADDA computations are performed and, if the averaging over \(\alpha_e\) is considered, 6592 orientations are evaluated. For the numerical calculation of orientation averages of extensive optical properties \(\zeta \in \{C_{\text{ext}}, C_{\text{sca}}, C_{\text{sca}} \cdot F\}\), the following steps were applied.

\[\beta_e\] and \(\gamma_e\) grid points for orientation averaging of ADDA calculations: \(\beta_e, \gamma_e = 0, 180015, 1650, 60, 120, \ldots, 30030, 1500, 30, 60, \ldots, 33045, 60, 75, 90, 105, 120, 1350, 15, 30, \ldots, 345\)

Averaging over \(\alpha_e\) is done by ADDA for each \(\beta_e - \gamma_e\) pair using the option ‘phi_integr 1’. The \(\alpha_e\) averaged quantity is denoted as \(\zeta(\beta_e, \gamma_e)\).

As the next step, averaging over \(\gamma_e\) is done (outside ADDA) for each \(\beta_e\) using:

\[
\zeta_{\alpha, \gamma}(\beta_e) = \frac{1}{N_{\gamma}} \sum_{i_{\gamma} = 1}^{N_{\gamma}} \zeta_{\alpha}(\beta_e, \gamma_e[i_{\gamma}]).
\]

where \(N_{\gamma}\) is the number of equidistant \(\gamma_e\) grid points for the given \(\beta_e\) (Tab. ??).

Finally, averaging over \(\beta_e\) is done numerically using linear interpolation of \(\zeta_{\alpha, \gamma}\) between the \(\beta_e\) grid points and subgrid sampling with \(N_s = 100\) subgrid points:

\[
\zeta = \sum_{i_{\beta} = 1}^{N_{\beta} - 1} \sum_{i_s = 1}^{N_s} \left[ \left( \zeta_{\alpha, \gamma}(\beta_e[i_{\beta} + 1]) - \zeta_{\alpha, \gamma}(\beta_e[i_{\beta}]) \right) \frac{i_s}{N_s} + \zeta_{\alpha, \gamma}(\beta_e[i_{\beta}]) \right] \cdot \\
\left( \cos((\beta_e[i_{\beta} + 1] - \beta_e[i_{\beta}]) \frac{i_s}{N_s} + \beta_e[i_{\beta}]) - \cos((\beta_e[i_{\beta} + 1] - \beta_e[i_{\beta}]) \frac{i_s + 0.5}{N_s} + \beta_e[i_{\beta}]) \right)
\]

Code and data availability. The MOPSMAP data set and the Fortran code including scripts related to examples presented in this paper are available at https://doi.org/10.5281/zenodo.1284217 (Gasteiger and Wiegner, 2018).
Competing interests. The authors declare that they have no conflict of interest.

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gmd-2018-56

Contents

S1 Orientation averaging of irregularly-shaped particles and accuracy assessment 2
  S1.1 Orientation averaging scheme ........................................ 2
  S1.2 Accuracy assessment using dense Euler angle grid .................... 3
  S1.3 Accuracy assessment using spheroids ................................ 6

S2 Maximum TMM size parameters ............................................. 7

S3 Transition from TMM to IGOM ............................................. 8

S4 Example for decomposition of a mode into contributions from data set grid points 17
S1 Orientation averaging of irregularly-shaped particles and accuracy assessment

S1.1 Orientation averaging scheme

The particle orientation is specified by three Euler angles ($\alpha_e$, $\beta_e$, $\gamma_e$) as described by Yurkin and Hoekstra (2011). Averaging over $\beta_e$ is done with a step width of $15^\circ$, and for each $\beta_e$ up to 24 $\gamma_e$ are used for averaging (see details in Table S1). The averaging over $\alpha_e$ is done within a single ADDA computation because rotation over $\alpha_e$ is equivalent to the rotation of the scattering plane and is computationally cheap. The optical properties are averaged over 32 $\alpha_e$. In total, for a single particle 206 individual ADDA runs are performed and, if the averaging over $\alpha_e$ is considered, 6592 orientations are evaluated. For the numerical calculation of orientation averages of extensive optical properties $\zeta \in \{C_{\text{ext}}, C_{\text{sca}}, C_{\text{sca}} \cdot F\}$, the following steps were applied.

Table S1: $\beta_e$ and $\gamma_e$ grid points for irregularly-shaped particles in MOPSMAP data set.

<table>
<thead>
<tr>
<th>$\beta_e$</th>
<th>$\gamma_e$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0°, 180°</td>
<td>0°</td>
</tr>
<tr>
<td>15°, 165°</td>
<td>0°, 60°, 120°, ..., 300°</td>
</tr>
<tr>
<td>30°, 150°</td>
<td>0°, 30°, 60°, ..., 330°</td>
</tr>
<tr>
<td>45°, 60°, 75°, 90°, 105°, 120°, 135°</td>
<td>0°, 15°, 30°, ..., 345°</td>
</tr>
</tbody>
</table>

Averaging over $\alpha_e$ is done by ADDA for each $\beta_e$-$\gamma_e$-pair using the option `phi_integr 1`. The $\alpha_e$-averaged quantity is denoted as $\zeta_{\alpha}(\beta_e, \gamma_e)$.

As the next step, averaging over $\gamma_e$ is done (outside ADDA) for each $\beta_e$ using

$$
\zeta_{\alpha,\gamma}(\beta_e) = \frac{1}{N_{\gamma}} \sum_{i=1}^{N_{\gamma}} \zeta_{\alpha}(\beta_e, \gamma_e[i]).
$$

(S1)

where $N_{\gamma}$ is the number of equidistant $\gamma_e$ grid points for the given $\beta_e$ (Table S1).

Finally, the orientation-averaged $\zeta$ is obtained by averaging over $\beta_e$ using

$$
\zeta = \frac{1}{2} \int_{0^\circ}^{180^\circ} \tilde{\zeta}_{\alpha,\gamma}(\beta_e) \cdot \sin \beta_e \cdot d\beta_e
$$

(S2)

where $\tilde{\zeta}_{\alpha,\gamma}(\beta_e)$ is linearly interpolated between the available $\zeta_{\alpha,\gamma}(\beta_e)$ grid points (Eq. S1, Table S1). Numerical integration of Eq. S2 is performed using a step width of $\Delta \beta_e = 0.15^\circ$. 

2
S1.2 Accuracy assessment using dense Euler angle grid

Here we present a comparison of single particle properties calculated either with the orientation averaging scheme used in the MOPSMAP data set (Sect. S1.1) or using a much denser grid of orientation angles ($5^\circ$ step size for $\beta_e$ and $\gamma_e$). This comparison provides an estimation of the accuracy of the former using the latter as reference.

Irregular shapes B, C, and F with $m = 1.52 + 0.0043i$ at six size parameters $x_v$ from 10.0 to 20.8 are considered, i.e., 18 different single particles. In the following, tables are shown for the extinction efficiency $q_{\text{ext}}$ (Table S2), scattering efficiency $q_{\text{sca}}$ (Table S3), forward scattering phase function $a_1(0^\circ)$ (Table S4), backscattering phase function $a_1(180^\circ)$ (Table S5), and the normalized 2,2-element of the scattering matrix at backward direction $a_2(180^\circ)/a_1(180^\circ)$ (Table S6), where each line corresponds to a size parameter and the three values separated by slashes correspond to the different irregular shapes B/C/F.

In case of $q_{\text{ext}}$ and $q_{\text{sca}}$, i.e. properties integrated over all scattering angles, the deviation between MOPSMAP and the reference is virtually zero (Tables S2, S3). The same is true for forward scattering (Table S4). Larger deviations typically in the order of a few percent (max. 14%) occur in case of backscattering. It is well known that scattering under $180^\circ$ is very sensitive to various parameters of a scattering problem, here the particle orientation. The effect on atmospheric aerosols is however reduced as under realistic conditions over- and underestimates partly compensate according to the ensemble of different particles.

Table S2: Extinction efficiency $q_{\text{ext}}$ from the MOPSMAP data set compared to results obtained using a dense grid of step size $5^\circ$ for $\beta_e$ and $\gamma_e$. The three values separated by slashes correspond to shapes B, C, and F. The relative deviation of $q_{\text{ext}}$ of MOPSMAP from the reference is rounded to full percent values.

<table>
<thead>
<tr>
<th>size parameter</th>
<th>MOPSMAP data set</th>
<th>dense $\beta_e$ and $\gamma_e$ grid</th>
<th>rel. deviation in %</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_v$=10.0</td>
<td>2.269/2.064/2.069</td>
<td>2.267/2.053/2.076</td>
<td>0/+1/0</td>
</tr>
<tr>
<td>$x_v$=12.0</td>
<td>2.521/2.442/2.279</td>
<td>2.530/2.430/2.075</td>
<td>0/+1/0</td>
</tr>
<tr>
<td>$x_v$=14.4</td>
<td>2.180/2.367/2.272</td>
<td>2.184/2.371/2.280</td>
<td>0/0/0</td>
</tr>
<tr>
<td>$x_v$=17.3</td>
<td>2.203/2.097/2.090</td>
<td>2.205/2.100/2.101</td>
<td>0/0/-1</td>
</tr>
<tr>
<td>$x_v$=19.0</td>
<td>2.303/2.200/2.106</td>
<td>2.306/2.200/2.114</td>
<td>0/0/0</td>
</tr>
<tr>
<td>$x_v$=20.8</td>
<td>2.229/2.289/2.195</td>
<td>2.232/2.297/2.181</td>
<td>0/0/+1</td>
</tr>
</tbody>
</table>
Table S3: Scattering efficiency $q_{sca}$ from the MOPSMAP data set compared to results obtained using a dense grid of step size 5° for $\beta_e$ and $\gamma_e$. The three values separated by slashes correspond to shapes B, C, and F. The relative deviation of $q_{sca}$ of MOPSMAP from the reference is rounded to full percent values.

<table>
<thead>
<tr>
<th>size parameter $x_v$</th>
<th>MOPSMAP data set</th>
<th>dense $\beta_e$ and $\gamma_e$ grid</th>
<th>rel. deviation in %</th>
</tr>
</thead>
<tbody>
<tr>
<td>10.0</td>
<td>2.076/1.881/1.904</td>
<td>2.074/1.870/1.910</td>
<td>0/+1/0</td>
</tr>
<tr>
<td>12.0</td>
<td>2.295/2.226/2.082</td>
<td>2.303/2.214/2.075</td>
<td>0/+1/0</td>
</tr>
<tr>
<td>14.4</td>
<td>1.916/2.114/2.042</td>
<td>1.920/2.118/2.049</td>
<td>0/0/0</td>
</tr>
<tr>
<td>17.3</td>
<td>1.898/1.803/1.822</td>
<td>1.899/1.807/1.832</td>
<td>0/0/-1</td>
</tr>
<tr>
<td>19.0</td>
<td>1.975/1.885/1.816</td>
<td>1.977/1.886/1.824</td>
<td>0/0/0</td>
</tr>
<tr>
<td>20.8</td>
<td>1.879/1.954/1.883</td>
<td>1.881/1.961/1.869</td>
<td>0/0/+1</td>
</tr>
</tbody>
</table>

Table S4: Forward scattering phase function $a_1(0^\circ)$ from the MOPSMAP data set compared to results obtained using a dense grid of step size 5° for $\beta_e$ and $\gamma_e$. The three values separated by slashes correspond to shapes B, C, and F. The relative deviation of $a_1(0^\circ)$ of MOPSMAP from the reference is rounded to full percent values.

<table>
<thead>
<tr>
<th>size parameter $x_v$</th>
<th>MOPSMAP data set</th>
<th>dense $\beta_e$ and $\gamma_e$ grid</th>
<th>rel. deviation in %</th>
</tr>
</thead>
<tbody>
<tr>
<td>10.0</td>
<td>74.00/70.20/78.66</td>
<td>73.94/69.62/78.62</td>
<td>0/+1/0</td>
</tr>
<tr>
<td>12.0</td>
<td>119.8/121.9/126.7</td>
<td>120.3/121.6/126.8</td>
<td>0/0/0</td>
</tr>
<tr>
<td>14.4</td>
<td>154.7/176.6/188.7</td>
<td>154.8/176.8/188.8</td>
<td>0/0/0</td>
</tr>
<tr>
<td>17.3</td>
<td>224.8/225.4/253.3</td>
<td>224.8/225.5/253.2</td>
<td>0/0/0</td>
</tr>
<tr>
<td>19.0</td>
<td>288.1/288.4/309.4</td>
<td>288.0/288.2/309.6</td>
<td>0/0/0</td>
</tr>
<tr>
<td>20.8</td>
<td>340.4/366.8/391.0</td>
<td>340.7/367.2/389.5</td>
<td>0/0/0</td>
</tr>
</tbody>
</table>

Table S5: Backward scattering phase function $a_1(180^\circ)$ from the MOPSMAP data set compared to results obtained using a dense grid of step size 5° for $\beta_e$ and $\gamma_e$. The three values separated by slashes correspond to shapes B, C, and F. The relative deviation of $a_1(180^\circ)$ of MOPSMAP from the reference is rounded to full percent values.

<table>
<thead>
<tr>
<th>size parameter $x_v$</th>
<th>MOPSMAP data set</th>
<th>dense $\beta_e$ and $\gamma_e$ grid</th>
<th>rel. deviation in %</th>
</tr>
</thead>
<tbody>
<tr>
<td>10.0</td>
<td>0.3855/0.3277/0.3184</td>
<td>0.3758/0.3262/0.3084</td>
<td>+3/0/+3</td>
</tr>
<tr>
<td>12.0</td>
<td>0.3445/0.2643/0.3270</td>
<td>0.3238/0.2557/0.3254</td>
<td>+6/+3/+1</td>
</tr>
<tr>
<td>14.4</td>
<td>0.3488/0.2409/0.4018</td>
<td>0.3423/0.2492/0.3844</td>
<td>+2/-3/+5</td>
</tr>
<tr>
<td>17.3</td>
<td>0.3244/0.2423/0.4634</td>
<td>0.3091/0.2581/0.4306</td>
<td>+5/-6/+8</td>
</tr>
<tr>
<td>19.0</td>
<td>0.2866/0.2200/0.4625</td>
<td>0.2702/0.2174/0.4115</td>
<td>+6/+1/+12</td>
</tr>
<tr>
<td>20.8</td>
<td>0.2882/0.2185/0.4352</td>
<td>0.2800/0.2170/0.3822</td>
<td>+3/+1/+14</td>
</tr>
</tbody>
</table>
Table S6: Normalized \((2,2)\)-element of the scattering matrix at backward direction
\(a_2(180°)/a_1(180°)\) from the MOPSMAP data set compared to results obtained using a dense grid of step size 5° for \(\beta_e\) and \(\gamma_e\). The three values separated by slashes correspond to shapes B, C, and F. The relative deviation of \(a_2(180°)/a_1(180°)\) of MOPSMAP from the reference is rounded to full percent values.

<table>
<thead>
<tr>
<th>size parameter</th>
<th>MOPSMAP data set</th>
<th>dense (\beta_e) and (\gamma_e) grid</th>
<th>rel. deviation in %</th>
</tr>
</thead>
<tbody>
<tr>
<td>(x_v=10.0)</td>
<td>0.4894/0.5856/0.3342</td>
<td>0.5081/0.5166/0.3347</td>
<td>-4/+13/0</td>
</tr>
<tr>
<td>(x_v=12.0)</td>
<td>0.5398/0.5713/0.3195</td>
<td>0.5135/0.5407/0.3417</td>
<td>+5/+6/-6</td>
</tr>
<tr>
<td>(x_v=14.4)</td>
<td>0.5356/0.5489/0.3229</td>
<td>0.5200/0.5385/0.3396</td>
<td>+3/+2/-5</td>
</tr>
<tr>
<td>(x_v=17.3)</td>
<td>0.4884/0.5684/0.3274</td>
<td>0.4890/0.5454/0.3574</td>
<td>0/+4/-8</td>
</tr>
<tr>
<td>(x_v=19.0)</td>
<td>0.4799/0.5362/0.3343</td>
<td>0.4800/0.5159/0.3606</td>
<td>0/+4/-7</td>
</tr>
<tr>
<td>(x_v=20.8)</td>
<td>0.5188/0.5382/0.3294</td>
<td>0.4973/0.5387/0.3590</td>
<td>+4/0/-8</td>
</tr>
</tbody>
</table>
S1.3 Accuracy assessment using spheroids

Here ADDA together with the orientation averaging scheme used for the irregular shapes in the MOPSMA data set (as described in Sect. S1.1) is applied to prolate spheroids with $m = 1.52 + 0.0043i$ and $\epsilon' = 2.0$ (ADDA option 'shape ellipsoid 1.0 2.0'). Volume-equivalent size parameters $x_v = 2, 4, \text{and } 10$ are considered. As reference the optical properties of the same randomly-oriented particles are calculated with the TMM code of Mishchenko and Travis (1998).

The comparison again shows that the integrated parameters and the forward scattering almost perfectly agree whereas for backscattering few cases with larger deviations up to 17% are obtained. In general, the relative deviations are of similar magnitude as those found in our tests of Sect. S1.2 though the number of independent ADDA calculations is lower for spheroids than for irregular particles because of the rotation symmetry of spheroids.

Table S7: Properties of prolate spheroids with $\epsilon' = 2.0$ and $m = 1.52 + 0.0043i$. The three values separated by slashes correspond to size parameters $x_v = 2, 4, \text{and } 10$. The relative deviation of ADDA from TMM is rounded to full percent values.

<table>
<thead>
<tr>
<th>optical parameter</th>
<th>ADDA + orient. avg.</th>
<th>TMM</th>
<th>rel. deviation in %</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q_{ext}$</td>
<td>1.656/3.861/2.274</td>
<td>1.650/3.860/2.256</td>
<td>0/0/+1</td>
</tr>
<tr>
<td>$q_{sca}$</td>
<td>1.621/3.780/2.068</td>
<td>1.615/3.778/2.048</td>
<td>0/0/+1</td>
</tr>
<tr>
<td>$a_1(0^\circ)$</td>
<td>5.741/18.86/69.30</td>
<td>5.748/18.84/69.20</td>
<td>0/0/0</td>
</tr>
<tr>
<td>$a_1(180^\circ)$</td>
<td>0.1069/0.1700/0.3597</td>
<td>0.0948/0.1686/0.3568</td>
<td>+13/+1/+1</td>
</tr>
<tr>
<td>$a_2(180^\circ)/a_1(180^\circ)$</td>
<td>0.9350/0.5175/0.4721</td>
<td>0.9230/0.4927/0.5678</td>
<td>+1/+5/-17</td>
</tr>
</tbody>
</table>
S2 Maximum TMM size parameters

A detailed list of maximum TMM size parameters for all 22680 combinations of refractive index and shape from the MOPSMAP spheroid data set is provided for download together with the data set at https://doi.org/10.5281/zenodo.1284217. A summary is given in Table S8 for different aspect ratios $\epsilon'$ and refractive index ranges. The TMM calculations for a given refractive index and shape (always iterating from small to large $x_c$ with steps of 5%) are terminated either when a size parameter equal or larger than the minimum upper size parameter given in Table S8 is successfully modeled or if TMM does not converge. Furthermore, the results were checked for plausibility as discussed in our paper. As a consequence, the maximum TMM size parameter in the MOPSMAP data set is often 0% - 5% larger than given in Table S8 but can be smaller in case of numerical problems for specific shape and refractive index combinations.

Table S8: Minimum upper size parameters $x_c$ for the TMM calculations of oblate and prolate spheroids as function of aspect ratio $\epsilon'$ for different refractive index ranges. For details see text.

<table>
<thead>
<tr>
<th>$\epsilon'$</th>
<th>1.28 $\leq m_r \leq$ 1.6 and $m_i \leq 0.1376$</th>
<th>1.64 $\leq m_r \leq$ 2 and $m_i \leq 0.1376$</th>
<th>all other $m$ with $m_r \geq 1.0$</th>
<th>$m_r &lt; 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.2</td>
<td>120</td>
<td>118</td>
<td>60</td>
<td>25</td>
</tr>
<tr>
<td>1.4</td>
<td>120</td>
<td>84</td>
<td>60</td>
<td>25</td>
</tr>
<tr>
<td>1.6</td>
<td>110</td>
<td>76.2</td>
<td>55</td>
<td>25</td>
</tr>
<tr>
<td>1.8</td>
<td>88</td>
<td>65.8</td>
<td>44</td>
<td>25</td>
</tr>
<tr>
<td>2.0</td>
<td>63</td>
<td>56.8</td>
<td>31.5</td>
<td>25</td>
</tr>
<tr>
<td>2.2</td>
<td>51</td>
<td>44.5</td>
<td>25.5</td>
<td>10</td>
</tr>
<tr>
<td>2.4</td>
<td>40</td>
<td>31.6</td>
<td>20.0</td>
<td>10</td>
</tr>
<tr>
<td>2.6</td>
<td>33</td>
<td>24.8</td>
<td>16.5</td>
<td>10</td>
</tr>
<tr>
<td>2.8</td>
<td>29</td>
<td>21.4</td>
<td>14.5</td>
<td>10</td>
</tr>
<tr>
<td>3.0</td>
<td>25</td>
<td>20.4</td>
<td>12.5</td>
<td>10</td>
</tr>
<tr>
<td>3.4</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3.8</td>
<td>19.4</td>
<td>16.7</td>
<td>9.7</td>
<td>5</td>
</tr>
<tr>
<td>4.2</td>
<td>16.7</td>
<td>14.5</td>
<td>8.4</td>
<td>5</td>
</tr>
<tr>
<td>4.6</td>
<td>14.5</td>
<td>13.1</td>
<td>7.3</td>
<td>5</td>
</tr>
<tr>
<td>5.0</td>
<td>12.5</td>
<td>11.9</td>
<td>6.3</td>
<td>5</td>
</tr>
<tr>
<td>5.0</td>
<td>11.3</td>
<td>10.0</td>
<td>5.7</td>
<td>5</td>
</tr>
</tbody>
</table>
S3 Transition from TMM to IGOM

Figs. S1 to S8 illustrate the angle-specific scattering as function of size parameter calculated by the TMM (solid lines) and the IGOM approximation (dashed lines). The transitions between TMM and IGOM are shown as dotted lines. The upper plots are similar to those presented in Fig. 2a of Dubovik et al. (2006) (shifted by some factor) but also include results for backscattering (180°). The lower plots show the ratio between the scattering matrix elements $a_2$ and $a_1$. All plots are for $m=1.52+0.0030406i$, close to the value used by Dubovik et al. (2006), and the different plots show different particle shapes.

The figures show a very smooth transition between TMM and IGOM for the scattering intensity in the forward direction. For the backward direction some cases with jumps at the transition size parameters are found, indicating uncertainties of the IGOM approximation for the corresponding scattering angle. It is well known that backscattering is particularly difficult to model. Note that the contribution of IGOM results to the optical properties of typical dust ensembles at visible wavelengths is low so that uncertainties of ensemble backscattering properties arising from IGOM uncertainties are normally limited.
Figure S1: Comparison of IGOM (dotted lines) with TMM (solid lines) for prolate spheroids with $\epsilon'=5.0$ and $m=1.52+0.0030406i$ at different scattering angles $\theta$ (indicated by color).
Figure S2: Comparison of IGOM (dotted lines) with TMM (solid lines) for prolate spheroids with $\epsilon' = 2.8$ and $m = 1.52 + 0.0030406i$ at different scattering angles $\theta$ (indicated by color).
Figure S3: Comparison of IGOM (dotted lines) with TMM (solid lines) for prolate spheroids with $\epsilon'=2.0$ and $m=1.52+0.0030406i$ at different scattering angles $\theta$ (indicated by color).
Figure S4: Comparison of IGOM (dotted lines) with TMM (solid lines) for prolate spheroids with $\epsilon'=1.4$ and $m=1.52+0.0030406i$ at different scattering angles $\theta$ (indicated by color).
Figure S5: Comparison of IGOM (dotted lines) with TMM (solid lines) for oblate spheroids with $\epsilon'=1.4$ and $m=1.52+0.0030406i$ at different scattering angles $\theta$ (indicated by color).
Figure S6: Comparison of IGOM (dotted lines) with TMM (solid lines) for oblate spheroids with $\epsilon'=2.0$ and $m=1.52+0.0030406i$ at different scattering angles $\theta$ (indicated by color).
Figure S7: Comparison of IGOM (dotted lines) with TMM (solid lines) for oblate spheroids with $\epsilon' = 2.8$ and $m=1.52+0.0030406i$ at different scattering angles $\theta$ (indicated by color).
Figure S8: Comparison of IGOM (dotted lines) with TMM (solid lines) for oblate spheroids with $\epsilon' = 5.0$ and $m = 1.52 + 0.0030406i$ at different scattering angles $\theta$ (indicated by color).
### S4 Example for decomposition of a mode into contributions from data set grid points

Suppose, a mode has spheroids with $m_r = 1.53$, $m_i = 0$, and $\epsilon_m = 1.92$. Then the MOPSMAP grid points of each dimension and their weights are

- $m_{r,i} = 1.52$ with $w_{m_{r,i}} = 0.75$
- $m_{r,i+1} = 1.56$ with $w_{m_{r,i+1}} = 0.25$
- $m_{i,j} = 0$ with $w_{m_{i,j}} = 1$
- $m_{i,j+1} = 0.0005375$ with $w_{m_{i,j+1}} = 0$
- $\epsilon_{m,k} = 1.8$ with $w_{\epsilon_{m,k}} = 0.4$
- $\epsilon_{m,k+1} = 2.0$ with $w_{\epsilon_{m,k+1}} = 0.6$

The contributions from the different $(m_r, m_i, \epsilon_m)$-grid points (i.e., netcdf files) included in the data set are therefore

1. $m_{r,i} = 1.52$, $m_{i,j} = 0$, $\epsilon_{m,k} = 1.8$ with $w = 0.75 \cdot 1 \cdot 0.4 = 0.30$
2. $m_{r,i} = 1.52$, $m_{i,j} = 0$, $\epsilon_{m,k+1} = 2.0$ with $w = 0.75 \cdot 1 \cdot 0.6 = 0.45$
3. $m_{r,i} = 1.52$, $m_{i,j+1} = 0.0005375$, $\epsilon_{m,k} = 1.8$ with $w = 0.75 \cdot 0 \cdot 0.4 = 0.00$
4. $m_{r,i} = 1.52$, $m_{i,j+1} = 0.0005375$, $\epsilon_{m,k+1} = 2.0$ with $w = 0.75 \cdot 0 \cdot 0.6 = 0.00$
5. $m_{r,i+1} = 1.56$, $m_{i,j} = 0$, $\epsilon_{m,k} = 1.8$ with $w = 0.25 \cdot 1 \cdot 0.4 = 0.10$
6. $m_{r,i+1} = 1.56$, $m_{i,j} = 0$, $\epsilon_{m,k+1} = 2.0$ with $w = 0.25 \cdot 1 \cdot 0.6 = 0.15$
7. $m_{r,i+1} = 1.56$, $m_{i,j+1} = 0.0005375$, $\epsilon_{m,k} = 1.8$ with $w = 0.25 \cdot 0 \cdot 0.4 = 0.00$
8. $m_{r,i+1} = 1.56$, $m_{i,j+1} = 0.0005375$, $\epsilon_{m,k+1} = 2.0$ with $w = 0.25 \cdot 0 \cdot 0.6 = 0.00$

The weighted sum of these 8 contributions (with 4 of them being zero) gives the optical properties of the mode.