Replys to Anonymous Referee #2

We would like to thank you for the positive comments and constructive advices, which help us to make the manuscript more clearly and more persuasive. The responses for the comments are in following text.

Response for specific comment #1:

The necessity to conserve the resolved energy in numerical solutions to an energy conservation system is actually the same as that to conserve the resolved mass. To highlight the significance of constructing an energy conservation scheme for the TRiSK dynamic core, a clear explanation on the necessity should be provided in Section 1.

Reply:

This is a good advice, energy conservation is an important property for the closed physical system, the shallow water system without any energy sink or source is one of the closed system, and the numerical model such as TRiSK shallow water dynamic core is a kind of approximation to the closed system, therefore, the basic integral invariants should be conserved, as we cited from (Arakawa, 1977), the maintenance of integral make the physics of the discrete model more analogous to the physics of the continuous atmosphere, and on the other hand make the errors less systematic. Another interesting example could be found in (Wang, 1996), the numerical test of the linear ODE

$$\begin{cases} \frac{dx}{dt} = -ay \\ \frac{dy}{dt} = bx \end{cases}$$

the true solution of the equation is an ellipse conform to $bx^2 + ay^2 = c$ (c is a constant), but after long term numerical simulation (after 10^8 steps) with original Runge-Kutta, the generalized energy tends to zero, and the solution tends to a single point(Fig.2, Wang, 1996). I think it's clearly to see the importance of keeping energy conservation. The references are packed in the zip file.

Response for specific comment #2:

Line 19/Page 2: CRK is improperly used as the abbreviation of "a new class of Runger-Kutta scheme", because the word "class" does not describe the main characteristics of this scheme. NRK is better.

Reply:

CRK stands for Conservative Runge-Kutta in my opinion, which means this kind of Runge-Kutta helps make the square conservation, it's just an abbreviation, but of course, the naming right belongs to the proposer of the scheme, Bin Wang. I use this abbreviation just to make the article concise. The NRK is now in new version of the manuscript.

Response for specific comment #3:

I wonder why the title of Section 2 is exactly the same as that of Section 1 (Lin 22/Page 3).

Reply:

Thank you for finding out the problem. The right title of Section 2 is “Introduction of TRiSK”.

Response for specific comment #4:

The equality (3) (Line 4/Page 4) is not true, which missed the integration sign after the second equal mark.

Reply:

Indeed, the total energy should be defined as follow

$$\oiint_{\Omega} \epsilon \, ds = \oiint_{\Omega} g\epsilon_{R10} \, ds = \oiint_{\Omega} \phi K + \frac{1}{2}\phi^2 + \phi\phi_s \, ds = \|\phi K\| + \left\|\frac{1}{2}\phi^2\right\| + \|\phi\phi_s\|$$

Response for specific comment #5:

The semi-discrete form of the shallow water equation set [Equations (4)-(5) on Lines 4-5/Page 5] should no longer be a partial differential equation set, but an ordinary differential equation set.

Reply:

We are trying to express the same discrete system as which in (Ringler, 2010) Eqs.(19)-(20), you’re right, “semi-discrete form” should be modified to “discrete system”.

Response for specific comment #6:

Line 6/Page 5: u and v are not the variables of Eqs.(4)-(5).

Reply:

You are right, u is the evolution variable for the equation, v does not appear in Eqs.(4) and (5).

Response for specific comment #7:

Line 20/Page 7: The equality is not true, because a negative sign is missed before (not sure, but there is only one equation)

Reply:

Indeed, the derivation should be

$$\begin{cases} \frac{\partial \boldsymbol{u}}{\partial t} + \mathcal{M}(\phi, u) = 0 \\ \frac{\partial \phi}{\partial t} + \mathcal{N}(\phi, u) = 0 \end{cases},$$

For simplify expression, we write $\mathcal{M} = \mathcal{M}(\phi, u), \mathcal{N} = \mathcal{N}(\phi, u)$

$$\frac{\partial \boldsymbol{U}}{\partial t} = \sqrt{\phi}\frac{\partial \boldsymbol{u}}{\partial t} + \frac{\boldsymbol{u}}{2\sqrt{\phi}}\frac{\partial \phi}{\partial t} = -\sqrt{\phi}\mathcal{M} - \frac{\boldsymbol{u}}{2\sqrt{\phi}}\mathcal{N}$$

$$(\mathcal{L}(\boldsymbol{F}), \boldsymbol{F}) = -\left(\frac{\partial \boldsymbol{U}}{\partial t}, \boldsymbol{U}\right) - \left(\frac{\partial \phi}{\partial t}, \phi\right)$$

$$= \oiint_{\Omega} -\boldsymbol{U}\frac{\partial \boldsymbol{U}}{\partial t} - \phi\frac{\partial \phi}{\partial t} \, ds$$

$$= \oiint_{\Omega} -\boldsymbol{U}\left(-\sqrt{\phi}\mathcal{M} - \frac{\boldsymbol{u}}{2\sqrt{\phi}}\mathcal{N}\right) + \phi\mathcal{N}\ ds$$

$$= \oiint_{\Omega} \phi u \cdot \mathcal{M} + \frac{|\boldsymbol{u}|^2}{2}\mathcal{N} + \phi\mathcal{N}\ ds$$

$$= (\mathcal{M}, \phi u) + (\mathcal{N}, E)$$

$$= 0$$

This problem does not influence the conclusion, thank you for checking the derivation meticulously.

Response for minor comment #8:

Line 10/Page 1: “The square conservation theory is widely used on latitude-longitude grids” –> “The square conservation law is maintained in the dynamic cores on latitude-longitude grids”.

Reply:

The square conservation scheme is implemented in The Grid-point Atmospheric Model of IAP LASG(GAMIL), and the result of GAMIL was published in CMILP5, but your advice is good.

Response for minor comment #9 and #10:

9) Line 4/Page 2: “which is” –> “which are”.

10) Line 26/Page 2: “polar problem” –> “polar instability” or “polar singularity”.

Reply:

Thank you for finding out those mistakes, they will be fixed.

References

Akio Arakawa and Vivian R. Lam: Computational Design of the Basic Dynamical Processes of the UCLA General Circulation Model. Methods in Computational Physics, 17, 173–265, 1977.

Wang, B., Ji, Z., and Zeng, Q.: A Class of New Explicit Runge-Kutta Schemes, Progress In Nature Science, 6, 195-205, 1996.