What do we do with model simulation crashes? Recommendations for global sensitivity analysis of earth and environmental systems models

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Abstract. Complex, software-intensive, technically advanced, and computationally demanding models, presumably with ever-growing realism and fidelity, have been widely used to simulate and predict the dynamics of the Earth and environmental systems. The parameter-induced simulation crash (failure) problem is typical across most of these models, despite considerable efforts that modellers have directed at model development and implementation over the last few decades. A simulation failure mainly occurs due to the violation of the numerical stability conditions, non-robust numerical implementations, or errors in programming. However, the existing sampling-based analysis techniques such as global sensitivity analysis (GSA) methods, which require running these models under many configurations of parameter values, are ill-equipped to effectively deal with model failures. To tackle this problem, we propose a novel approach that allows users to cope with failed designs (samples) during the GSA, without knowing where they took place and without re-running the entire experiment. This approach deems model crashes as missing data and uses strategies such as median substitution, single nearest neighbour, or response surface modelling to fill in for model crashes. We test the proposed approach on a 10-parameter HBV-SASK rainfall-runoff model and a 111-parameter MESH land surface-hydrology model. Our results show that response surface modelling is a superior strategy, out of the data filling strategies tested, and can scale well to the dimensionality of the model, sample size, and the ratio of number of failures to the sample size. Further, we conduct a “failure analysis” and discuss some possible causes of the MESH model failure.

1 Introduction

1.1 Background and motivation

Since the start of the digital revolution and subsequent increase in computers’ processing power, the advancement of information technology has led to significant development of the modern software programs for Dynamical Earth Systems Models (DESMs). The current-generation DESMs typically span upwards of several thousand lines of code and require huge
amounts of data and computer memory. The flip side of the growing complexity of DESMs is that running these models will pose many types of software development and implementation issues such as simulation crashes/failures. The simulation crash problem happens mainly due to violation of the numerical stability conditions needed in DESMs. Certain combinations of model parameter values, improper integration time step, inconsistent grid resolution, or lack of iterative convergence as well as model thresholds and sharp discontinuities in model response surfaces, all associated with imperfect parameterizations, can cause numerical artefacts and stop DESMs from properly functioning.

When model crashes occur, the accomplishment of automated sampling-based model analyses such as sensitivity analysis, uncertainty analysis, and optimization (e.g., Raj et al., 2018; Williamson et al., 2017; Metzger et al., 2016; Safa et al., 2015) becomes challenging. These analyses are often carried out by running DESMs for a large number of parameter configurations randomly sampled from a domain (parameter space). In such situations, for example, the model’s solver may break down because of the implausible combinations of parameters (“unlucky parameter set” as termed by Kavetski et al., 2006), failing to complete the simulation. It is also possible that a model may be stable against perturbation of one parameter, while it may crash when several parameters are perturbed simultaneously. “Failure analysis” is a process that is performed to determine the cause(s) that have led to such crashes while running DESMs. Before achieving a conclusion on the most important causes of crashes, it is necessary to check the software code used in the DESMs and make sure if it is error-free; for example, a proper numerical scheme was adopted and correctly coded in the software. This often requires investigating both the software documentation and a series of nested modules. However, the existence of numerous nested programming modules in a typical DESMs can make the identification and removal of all software defects so tedious. In addition, as argued by Clark and Kavetski (2010), the numerical solution schemes implemented in DESMs are sometimes not presented in detail. This is one important reason why detecting the causes of simulation crashes in DESMs is usually troublesome. For example, Singh and Frevert (2002) and Burnash (1995) described the governing equations of their models without explaining the numerical solvers that were implemented in their codes.

Importantly, the impact of simulation crashes on the validity of global sensitivity analysis (GSA) results has often been overlooked in the literature, where simulation crashes are commonly classified as ignorable (see section 1.2). As such, a surprisingly limited number of studies have reported simulation crashes (examples related to uncertainty analysis include Annan et al., 2005; Edwards and Marsh, 2005; Lucas et al., 2013). This is despite the fact that these crashes can be very computationally costly for GSA algorithms because they can waste the rest of the model runs, prevent completion of GSA, or inevitably introduce ambiguity into the inferences drawn from GSA. For example, Kavetski and Clark (2010) demonstrated that how numerical artefacts can contaminate the assessment of parameter sensitivities in six hydrological models. Therefore, it is important to devise solutions that minimize the effect of crashes on GSA results. In the next subsection, we critically review the very few strategies for handling simulation crashes that have been proposed in the literature and identify their shortcomings.
1.2 Existing approaches to handling simulation crashes in DESMs

We have identified four types of approaches in the modelling community to handle simulation crashes, outlined below. The first two are perhaps the most common approaches (based on our personal communications with several modellers), however, we could not identify any publication that formally reports their application:

1. After the occurrence of a crash, modellers commonly adopt a conservative strategy to address this problem by altering/reducing the feasible ranges of parameters and re-starting the experiment in a hope to prevent recurrence of crashes in the new analyses.

2. Instead of GSA that runs many configurations of parameter values, analysts may choose to employ local methods such as local sensitivity analysis (LSA) through running the model in the vicinity of the known plausible parameter configurations.

3. Some modellers may adopt an ignorance-based approach by using only a set of ‘‘good’’ (or behavioural) outcomes/responses in sampling-based analyses and ignoring unreasonable (or non-behavioural) outcomes such as simulation crashes. This can be done via defining a performance metric to determine which simulations should be excluded from the analysis (see, e.g., Pappenberger et al., 2008; Kelleher et al., 2013).

4. The most rigorous approach seems to be a non-substitution approach that tries to predict whether or not a set of parameter values will lead to a simulation crash. Webster et al. (2004), Edwards et al. (2011), Lucas et al. (2013), Paja et al. (2016), and Treglown (2018) are among few studies that aimed at developing statistical methods to predict if a given combination of parameters can cause a simulation failure. For example, Lucas et al. (2013) adopted a machine learning method to estimate the probability of crash occurrence as a function of model parameters. They further applied this approach to investigate the impact of various model parameters on simulation failures. A similar approach is model pre-emption strategies where the simulation performance is monitored, while running, and terminated early if it is predicted that the simulation will not be informative (Razavi et al. 2010; Asadzadeh et al., 2014).

The above approaches have major limitations in handling simulation crashes in the GSA context, because:

1. Locating regions of parameter space responsible for crashes (i.e., “implausible regions”) is difficult and requires analysing the behaviour of DESMs throughout the often high-dimensional parameter space. Implausible regions usually have irregular, discontinuous, and complex shapes, and thus are too effortful to identify. Also, changing/reducing the parameter space changes the original problem at hand.

2. It is well-known that local methods (e.g., LSA) can provide inadequate assessments that can often be misleading (see e.g., Saltelli and Annoni, 2010, Razavi and Gupta, 2015).

3. When applying a sampling-based technique that uses an ad-hoc sampling strategy with particular spatial structure (e.g., the variance-based GSA proposed by Saltelli et al. (2010) or STAR-VARS of Razavi and Gupta (2016b)),...
ignorance-based procedures become impractical. In this case, excluding sample points associated with simulation crashes will distort the structure of the sample set, causing the failure of the entire GSA experiment. As a result, a new sample set (or a succession of sample sets) must be generated to resume the experiment, leading to a waste of previous model runs.

4. The implementation of the non-substitution procedures necessitates significant prior efforts to identify many model crashes based on which a statistical model can be built to predict and avoid simulation failures in the subsequent model runs. Such procedures can easily become infeasible in high-dimensional models, as then they would require an extremely large sample size to ensure an adequate coverage of the parameter space for characterizing implausible regions and building a reliable statistical model. These strategies can be more challenging when a model is computationally intensive. For example, to determine which parameters or combinations of parameters in a 16-dimensional climate model were predictors of failure, Edwards et al. (2011) used 1,000 evaluations (training samples) for constructing a statistical model to identify parameter configurations with high probability of failure in the next 1,087 evaluations (2,087 model runs in total). As pointed out by Edwards et al. (2011), although 2,087 evaluations might impose high computational burdens, a much larger sample size spreading out over the parameter space is required to guarantee reasonable exploration of the 16-dimensional space.

These shortcomings and gaps motivated our investigation to develop effective and efficient crash handling strategies suitable for GSA of DESMs, as introduced in section 2.

1.3 Scope and outline

The primary goal of this study is to identify and test practical “substitution” strategies to handle the parameter-induced crash problem in GSA of DESMs. Here, we treat model crashes as missing data and investigate the effectiveness of three efficient strategies to replace them using available information rather than discarding them. Our approach allows the user to cope with failed simulations in GSA without knowing where they will take place and without re-running the entire experiment. The overall procedure can be used in conjunction with any GSA technique. In this paper, we assess the performance of the proposed substitution approach on two hydrological models, by coupling it with the variogram-based GSA technique (VARS; Razavi and Gupta (2016a,b)).

The rest of the paper is structured as follows. We begin in the next section by introducing our proposed solution methodology for dealing with simulation crashes. In section 3, two real-world hydrological modelling case studies are presented. Next, in section 4, we evaluate the performance of the proposed methods across these real-world problems. The discussion is presented in section 5, before drawing conclusions and summarizing major findings in section 6.
2 Methodology

2.1 Problem statement

We denote the output of each model run (realization) \( y(\mathbf{X}) \), which corresponds to a \( d \)-dimensional input vector \( \mathbf{X} = \{x_1, x_2, \ldots, x_d\} \), where \( x_i (i = 1, 2, \ldots, d) \) is a factor that may be perturbed for the purpose of GSA (e.g., model parameters, initial conditions, or boundary conditions). Running a GSA algorithm usually requires generating \( n \) realizations of a simulation model using an experimental design \( \mathbf{X}_s = \{\mathbf{X}_1, \mathbf{X}_2, \ldots, \mathbf{X}_n\}^T \). Then, the model responses will form an output space as \( \mathbf{Y} = \{y(\mathbf{X}_1), y(\mathbf{X}_2), \ldots, y(\mathbf{X}_n)\}^T \). Here we deem simulation crashes as missing data and consider the model mapping of \( \mathbf{X}_s \rightarrow \mathbf{Y} \) as an incomplete data matrix. For a given \( \mathbf{Y} \in \mathbb{R}^{1 \times n} \) with missing values, let the vector \( \mathbf{Y}_a \) consist of the \( n_a \) locations in the input space for which, in the given \( \mathbf{Y} \), the model responses are available, and let the vector \( \mathbf{Y}_m \) consist of the remaining \( n_m \) locations (\( n_m = n - n_a \)) for which, in the given \( \mathbf{Y} \), the model responses are missing due to simulation crashes.

For convenience of expression and computation, we use the “\( \text{NAN}_j \)” symbol to represent the \( j \)th missing value in vector \( \mathbf{Y} \). The main goal now here is to develop and test data recovery methods that can be used to substitute model crashes \( \mathbf{Y}_m \) using available information (i.e., \( \mathbf{Y}_a \) and \( \mathbf{X}_s \)).

2.2 Proposed strategy for crash handling in GSA

We propose and test three techniques adopted from the “incomplete data analysis” for missing data replacement; the process known as imputation (Little and Rubin, 1987). We use imputation techniques to fill in missing values and ignore the mechanisms leading to missingness because identifying such mechanisms can be very challenging (Liu and Gopalakrishnan, 2017). Therefore, only the non-missing responses and the associated sample points are included in our analysis to infill model crashes during GSA, as described in the next sub-sections.

2.2.1 Median substitution

Perhaps replacing each simulation crash with some “central” value is the easiest and computationally simple method for imputation. Depending on the distribution of the model response variables \( \mathbf{Y} \), the central value can be median or mean. For example, if the distribution of model responses is not highly skewed, the crashes may be imputed with the mean of the non-missing values. Otherwise, if the distribution exhibits skewness, then the median may be a better replacement, because the mean is sensitive to the outliers (it is pulled in the direction of the outlying data values). This strategy treats each model response as a realization of a random function while ignoring the covariance structure of model responses, and thus considers the mean/median as a reasonable estimate for missing data. Although mean substitution preserves the mean of \( \mathbf{Y} \), a major shortcoming of this technique is that, depending on the number of crashes, it can distort other statistical characteristics of \( \mathbf{Y} \) through reducing its variance. In this paper, we used the median substitution technique.
2.2.2 Nearest neighbour substitution

The Nearest Neighbour (NN) technique (also known as hot deck imputation, see e.g., Beretta and Santaniello, (2016)) uses observations in the neighbourhood to fill in missing data. Let $X_j \in X_s$ be an input vector for which a simulation model fails to return an outcome. Basically, in the NN-based techniques, $NAN_j$ is replaced by either a response value corresponding to a single nearest neighbour (single NN) or a weighted average of the response variables corresponding to $k$ nearest neighbours ($k$-NN) where $k > 1$. The underlying rationale behind the NN-based techniques is that the sample points closer to $X_j$ may provide better information for imputing $NAN_j$.

In the $k$-NN techniques weights are typically assigned based on the degree of similarity between $X_j$ and the $k$th nearest neighbour $X_k$ where $y(X_k) \in Y_a$ (Tutz and Ramazan, 2015). Note that kernel functions can be used to compute the corresponding weights. However, the choice of the kernel functions can also be subjective. Moreover, the $k$-NN techniques require a careful selection of the number of neighbours. The single NN substitution does not extrapolate outside the range of the sampled output space and, instead, fill-in values are determined from the pool of non-missing values. An important feature of the NN-based techniques is that the variance and covariance of the $Y$ variables tend to be preserved for $k = 1$ but not for $k > 1$ (McRoberts, 2009). In this paper, we used the single NN technique with Euclidean distance measure.

2.2.3 Model emulation-based substitution

Model emulation is a strategy that develops statistical, cheap-to-run surrogates of response surfaces of complex, often computationally intensive models (Razavi et al., 2012a). Here we develop an emulator $\hat{y}(.)$ which is a statistical approximation of the simulation model based on response surface modelling concept. This strategy consists in finding an approximate/surrogate model with low computational cost that fits the non-missing response values $Y_a$ to predict the fill-in values for the missing responses $Y_m$. In the literature various types of response surface surrogates exist and are extensively discussed (see e.g. Razavi et al., 2012a). Examples are polynomial regression, radial basis functions (RBF), neural networks, kriging, support vector machines, and regression splines. In this paper, we employed RBF approximation as a well-established surrogate model. It has been shown that RBF can provide an accurate model for high-dimensional problems (Jin et al., 2001; Herrera et al., 2011), particularly when the computational budget is limited (Razavi et al., 2012b). The predictive response $\hat{y}(X)$ at a sample point $X$ can be approximated by an RBF model as a weighted summation of $n_a$ basis functions (and a polynomial or constant value) as follows:

$$\hat{y}(X) = \sum_{i=1}^{n_a} \omega_i f(||X - X_i||) = f(X)\omega$$

where $f = \{f_1, f_2, ..., f_{n_a}\}$ is the vector of the basis functions, $\omega_i$ is the $i$th component of the radial basis coefficient vector $\omega = \{\omega_1, \omega_2, ..., \omega_{n_a}\}^T$, and $||X - X_i||$ is the Euclidian distance between two sample points.

There are various choices for the basis function, such as Gaussian, thin-plate spline, multi-quadratic, and inverse multi-quadratic (Jones, 2001). In the present study, we choose the widely-used Gaussian kernel function for RBF as...
\[ f(\|\mathbf{X} - \mathbf{X}_i\|) = \exp\left(\frac{\|\mathbf{X} - \mathbf{X}_i\|^2}{c_i^2}\right) \]  

(2)

where \( c_i \) is the shape parameter which determines the spread of the \( i \)th kernel function \( f_i \).

After choosing the form of the basis function, the coefficient vector \( \mathbf{\omega} \) can be obtained by enforcing the accurate interpolation condition, i.e.,

\[
\begin{bmatrix}
y(\mathbf{X}_1) \\
y(\mathbf{X}_2) \\
\vdots \\
y(\mathbf{X}_{n_a})
\end{bmatrix} =
\begin{bmatrix}
f_{11} & f_{12} & \ldots & f_{1n_a} \\
f_{21} & f_{22} & \ldots & f_{2n_a} \\
\vdots & \vdots & \ddots & \vdots \\
f_{n_{a1}} & f_{n_{a2}} & \ldots & f_{n_{an_a}}
\end{bmatrix}
\begin{bmatrix}
\omega_1 \\
\omega_2 \\
\vdots \\
\omega_{n_{a}}
\end{bmatrix},
\]  

(3)

where \( f_{uv} = f(\|\mathbf{X}_u - \mathbf{X}_v\|) \). In a matrix form, Eq. (3) can be simply rewritten as \( \mathbf{Y}_a = \mathbf{F} \mathbf{\omega} \). This equation has a unique solution \( \mathbf{\omega} = \mathbf{F}^{-1} \mathbf{Y}_a \) if and only if all the sample points are different from each other. Therefore, the fill-in values for remaining \( n_m \) locations, for which the model responses are missing due to simulation crashes, can be approximated by

\[ \hat{y}(\mathbf{X}_j) = f(\mathbf{X}_j)\mathbf{F}^{-1} \mathbf{Y}_a \quad (j = 1, 2, \ldots, n_m) \]  

(4)

To reduce the computational cost and avoid overfitting when building RBF, for each failed simulation at \( \mathbf{X}_j \) we only chose \( k \) non-missing nearest neighbours of that missing value (here we arbitrarily set \( k \) to 100). Then a function approximation can be built using these \( k \) sample points to fill in that missing value, i.e., in Eq. (3), we set \( n_a \) to 100. Moreover, the shape parameter \( c \) in the Gaussian kernel function, which is an important factor in the accuracy of the RBF, can be determined using an optimization approach. We used the Nelder-Mead simplex direct search optimization algorithm (Lagarias et al., 1998) to find an optimal value for \( c \) by minimizing the RBF fitting error (for more details see Forrester and Keane (2009) and Kitayama and Yamazaki (2011)).

Note that in general depending on the complexity and dimensionality of a model response surface, other types of emulations can be incorporated into our proposed framework. However, for the crash handling problem, it is beneficial to utilize the function approximation techniques that exactly fit to the all sample points (i.e., the response surface surrogates categorized as “Exact Emulators” in Razavi et al. (2012a)) such as kriging and RBF. This is mainly because that DESMs are deterministic, and therefore generate identical outputs/responses given the same set of input factors. In other words, an exact emulator at any successful sample point \( \mathbf{X}_k \) (not crashed) reflects our knowledge about the true value of the model’s output at that point, i.e., it returns \( \hat{y}(\mathbf{X}_k) \) without uncertainty. Thus, exact emulators can be appropriate surrogates to adequately characterize the shape of the response surfaces in deterministic DESMs for handling simulation crashes.

### 2.3 The utilized GSA framework

We illustrate the incorporation of the proposed crash handling methodology into a variogram-based GSA approach called Variogram Analysis of Response Surfaces (VARS; Razavi and Gupta (2016a,b)). The VARS framework has successfully been applied to several real-world problems of varying dimensionality and complexity (see e.g., Sheikholeslami et al., 2017;
Yassin et al., 2017; Krogh et al., 2017; Leroux and Pomeroy, 2019). VARS is a general GSA framework that utilizes directional variograms and covariograms to quantify the full spectrum of sensitivity-related information, thereby providing a comprehensive set of the sensitivity measures called IVARS (Integrated Variogram Across a Range of Scales) at a range of different “perturbation scales” (Haghnegahdar and Razavi, 2017). Here, we used IVARS-50, referred to as “total-variogram effect”, as a comprehensive sensitivity measure since it contains sensitivity analysis information across a full range of perturbation scales.

Here, the STAR-VARS implementation of the VARS framework has been used. STAR-VARS is highly efficient and statistically robust algorithm that provides stable results with minimum number of model runs compared with other GSA techniques, and thus is suitable for high-dimensional problems (Razavi and Gupta, 2016b). This algorithm employs a star-based sampling scheme, which consists of two steps: (1) randomly selecting star centres in the parameter space, and (2) using a structured sampling technique to identify sample points revolved around the star centres. Due to the structured nature of the generated samples in STAR-VARS, ignorance-based procedures (see section 1.2) cannot be useful in dealing with simulation crashes because deleting sample points associated with crashed simulations will demolish the structure of the entire sample set. In this study, to achieve a well-designed computer experiment, we used PLHS algorithm in the first step of the STAR-VARS to sequentially locate samples in the parameter space. It has been shown that PLHS can grasp maximum amount of information from output space with minimum sample size, while outperforming traditional sampling algorithms (for more details see Sheikholeslami and Razavi, 2017).

3 Case studies

3.1 A conceptual rainfall-runoff model

As an illustrative example, we use the HBV-SASK conceptual hydrologic model to assess the performance of the proposed crash handling strategies in a real-world problem. HBV-SASK is based on Hydrologiska Byråns Vattenbalansavdelning model (Lindström et al., 1997) and was developed by the second author for educational purposes (Razavi et al., 2019; Gupta and Razavi, 2018). We applied HBV-SASK to simulate daily streamflows in the Oldman river basin in Western Canada (Fig. 1) with watershed area of 1434.73 km2. Historical data is available for periods 1979-2008, from which we estimate average annual precipitation to be 611 mm, and average annual streamflow to be 11.7 m3/s with a runoff ratio of approximately 0.42. HBV-SASK has 12 parameters, 10 of which are perturbed in this study (Table 1).

3.2 A land surface-hydrology model

In the second case study, we demonstrate the utility of the imputation-based methods in crash handling via their application to the GSA of a high-dimensional problem. The model used is Modélisation Environmentale– Surface et Hydrologie (MESH; Pietroniro et al., 2007)) which is a semi-distributed, highly-parameterized land surface-hydrology modelling framework developed by Environment and Climate Change Canada (ECCC) mainly for large-scale watershed modelling.
with consideration of cold region processes in Canada. MESH combines the vertical energy and water balance of the Canadian Land Surface Scheme (CLASS, Verseghy, 1991; Verseghy et al., 1993) with the horizontal routing scheme of the WATFLOOD (Kouwen et al., 1993). We encountered a series of simulation failures while assessing the impact of uncertainties in 111 model parameters (see Table A in Appendix) on simulated daily streamflows in Nottawasaga river basin in Ontario, Canada (Fig. 3). Here, the drainage basin of nearly 2700 km² is discretized into 20 grid cells with a spatial resolution of 0.1667 degrees (~15 km). The dominant land cover in the area is cropland followed by deciduous forest and grassland. The dominant soil type in the area is sand followed by silt and clay loam (for more details see Haghnegahdar et al., 2015).

3.3 Experimental setup

For the first case study, we ran the HBV-SASK model with 9,100 randomly selected parameter sets from the feasible ranges of Table 1 generated by the STAR-VARS (100 star centres with a resolution of 0.1). The Nash-Sutcliffe efficiency criterion on streamflows (NS) was used as the model output for sensitivity analysis. After calculating the NS values, we ran a series of experiments each with a different assumed “ratio of failure” (from 1% to 20%), defined as the percentage of failed parameter sets to the total number of parameter sets. In each experiment, we randomly chose a number of sampled points based the associated ratio of failure and assume them as simulation failures. Then, we evaluated the performance of the crash handling strategies to replace simulation failures during GSA of the HBV-SASK model and compared the results with the case when there are no failures. Also, we accounted for the randomness in the comparisons by carrying out 50 replicates of each experiment with different random seeds. This allowed us to see a range of possible performances for each strategy and to assess their robustness when crashes occurred at different locations in the parameter space.

In the second case study with 111 parameters, 100 star centres were randomly generated using STAR-VARS algorithm with a resolution of 0.1, resulting in a total of 100,000 MESH runs. The NS performance metric was used to measure daily model streamflow performance, calculated for a period of three years (October 2003-September 2007) following a one-year model warmup period. Due to various physical and/or numerical constraints inside MESH (or more precisely in CLASS), some combinations of the 111 parameters caused model crashes. Here, approximately 3% of our simulations failed (3,084 out of 100,000 runs). We applied the proposed crash handling strategies to infill the missing model outcomes in GSA of the MESH model.

4. Numerical results

4.1 Results for the HBV-SASK model

According to the IVARS-50 sensitivity index, the VARS algorithm ranks (after 9,100 function evaluations) the parameters of the HBV-SASK in order of importance as follows FRAC, FC, C0, TT, alpha, K1, LP, ETF, beta, and K2, when there are no crashes (we consider the corresponding assessments to be “true”). Based on the dendrogram (Fig. 3) generated by the factor
grouping algorithm introduced by Sheikholeslami et al., (2019), we categorized these parameters into three groups with respect to their importance, i.e., \{FRAC, FC, C0\} are the strongly influential parameters, \{TT, alpha, K1\} are moderately influential parameters, and \{LP, ETF, beta, K2\} are weakly influential parameters.

Fig. 4 and 5 show cumulative distribution functions (CDFs) for the 50 independent estimates of IVARS-50, obtained when 1%, 3%, 5%, 8%, 10%, 12%, 15%, and 20% of model runs were deemed to be simulation failures. Overall, the RBF and single NN techniques outperformed the median substitution in terms of closeness to the true GSA results and robustness when crashes happened at different locations of parameter space. As can be seen, by increasing the ration of failure, the performance of the crash handling strategies, particularly the median substitution became progressively worse. Note that the median substitution technique resulted in a significant bias manifested through over-estimation of the sensitivity indices for all the parameters. From the results, we see that using the RBF technique the sensitivity indices of the most important parameters (FRAC, FC) (Fig. 4(a)) and less important parameters (LP, ETF, beta, K2) (Fig. 5) were estimated with high degree of accuracy and robustness. However, for moderately influential parameters (Fig. 4(b)) its performance was reduced (i.e., the CDFs are wider in Fig. 4(b)).

More importantly, as the number of crashes increases, ranking of the parameters in terms of their importance may change. For example, Fig. 6 shows the number of times out of 50 independent runs that the rankings of the parameters were equal to the “true” ranking. In all 50 runs, regardless of the number of model crashes, the rankings obtained by VARS algorithm using the RBF technique were the same as the “true” ranking which is an indication of high degree of robustness in terms of parameter ranking. The performance of the single NN slightly reduced when the crash percentage were more than 15%, while the VARS algorithm wrongly determined the rankings in more than 50% percent of the replicates using median substitution technique (see Fig. 6c and d). This highlights that the rankings can be estimated much more accurately than the sensitivity indices in the presence of simulation crashes. Also, it can be seen that while the RBF-based strategy performed perfectly in this example, the performance of the single NN technique was comparably well.

Finally, Fig. 7 presents the performance of the single NN (Fig. 7a) and RBF (Fig. 7b) strategies in approximating the fill-in values for the missing responses when 20% of HBV-SASK simulations were deemed to be failures. As shown, the RBF outperformed single NN technique in terms of closeness to the true NS values. The linear regression has an R2 value of 0.834 when single NN was used, while the RBF strategy achieved a linear regression with an R2 value of 0.996. Also, the result of the RBF strategy is almost unbiased as the linear regression plotted on Fig. 7b is very close to the ideal (1:1) line.

4.2 Results for the MESH model

Here we demonstrate the GSA results by categorizing the 111 MESH model parameters into three groups as shown in Fig. 8 (for more details on grouping see Sheikholeslami et al. (2019)). Fig. 9-11 present the sensitivity analysis results obtained by the VARS algorithm for the MESH model, when different crash handling techniques were applied. These groups were labeled according to their importance, i.e., Group 1 (Fig. 9) contains the strongly influential parameters, while parameters in Group 2 (Fig. 10) are moderately influential, and Group 3 (Fig. 11) is the group of weakly influential parameters.
Four most influential parameters in Group 1 are SDEPC and DRNC (“C” stands for crops) controlling water storage and movement in the soil, WFR22 (river channel routing), and ZSNL (snow cover fraction). As shown in Fig. 9 (upper panel), the sensitivity indices associated with these parameters are almost similar regardless of the employed crash handling technique. It is worth mentioning that, as discussed in our failure analysis (see Section 5.1), we also identified three of these parameters (i.e., SDEPC, DRNC, and ZSNL) responsible for some of the model crashes. In other words, the parameters which strongly contribute to the variability of the MESH model output can also be convicted of model crashes. To enhance future development and application of the MESH model, more efforts should be directed at better understanding the functioning of these parameters and their effects acting individually or in combination with other parameters over their entire range of variations.

For the other 15 influential parameters in Group 1 (Fig. 9, bottom panel), there is general agreement with three crash handling techniques about the sensitivity indices calculated by VARS except for the parameter ROOTC which defines the annual maximum rooting depth of vegetation category. The RBF and median substitution methods give more importance to ROOTC compared to the single NN technique. It is noteworthy that the oversaturation of soil layer, which can cause many model runs to fail, is subject to the interaction between ROOTC and SDEPC.

Fig. 10 illustrates the sensitivity indices for the moderately influential parameters (i.e., Group 2). Note that for all these 78 parameters the sensitivity analysis results were highly dependent on the chosen crash handling strategy. As can be seen, the sensitivity indices associated with the median substitution and RBF techniques are higher than those obtained by the single NN technique (this difference is considerable for the parameters in the upper and lower subplots than those in the middle subplot).

Finally, the results of the sensitivity analysis for the weakly or non-influential (Group 3) parameters of the MESH model are plotted in Fig. 11. As shown, although the VARS algorithm identified these parameters as weakly influential (very low IVARS-50 values) using the proposed crash handling techniques, the associated sensitivity indices obtained by the RBF imputation method are about two order of magnitude larger for the parameters in the left panel (Fig. 11 (a, c)) and about four order of magnitude larger for the parameters in the left panel (Fig. 11 (b, d)) than compared to those obtained by the single NN and median substitution methods.

However, it is important to note that in high-dimensional DESMs, when the number of parameters is very large, the estimation of sensitivity indices is likely not robust to sampling variability. On the other hand, parameter ranking (order of relative sensitivity) is often more robust to sampling variability and converges more quickly than factor sensitivity indices (see e.g., Vanrolleghem et al., 7 2015; Razavi and Gupta, 2016b; Sheikholeslami et al., 2019). To investigate how different crash handling strategies can affect the ranking of the model parameters in terms of their importance, Fig. 12 compares the rankings obtained by the RBF, single NN, and median substitution techniques.

As shown in Fig. 12a, the single NN and median substitution techniques resulted in almost similar parameter rankings for the strongly influential (Group 1) and weakly influential (Group 3) parameters, while for moderately influential parameters (Group 2) the rankings are significantly different. Meanwhile, the RBF and median substitution techniques yielded very
distinctive rankings except for the strongly influential parameters (Fig. 12b). Furthermore, Fig. 12c indicates that the single NN and RBF method give similar rankings for influential parameters.

A closer examination, however, reveals that rankings can be very contradictory for some of the parameters, when using different crash handling strategies (see Fig. 12d-f). For example, consider the soil moisture suction coefficient for crops (PSGAC) which is used in the calculation of the stomatal resistance in the evapotranspiration process of the MESH (for more details see Fisher et al., 1981; Choudhury and Idso 1985; Verseghy, 2012). As can be seen, according to the RBF method, PSGAC is one of the weakly influential parameters (ranked 5th), while using the single NN it is determined to be one of the moderately influential parameters (ranked 43rd). In contrast, it is one of the strongly influential parameters based on the median substitution (ranked 83rd). However, in a comprehensive study of the MESH model using various model configurations and different hydroclimatic regions in Eastern and Western Canada, Haghnegahdar et al. (2017) found that PSGAC is one of the least influential parameters considering three model performance criteria with respect to high flows, low flows, and total flow volume of the daily hydrograph. As another example, consider ZPLS7 (maximum water ponding depth for snow-covered areas) and ZPLG7 (maximum water ponding depth for snow-free areas) which are used in the surface runoff algorithm of the MESH (i.e., PDMROF). The single NN and median substitution methods both ranked ZPLS7 as 2nd and ZPLG7 as 3rd least influential parameters, whereas the RBF ranked them as 61 and 45 (i.e., moderately influential) which is in accordance with results reported by Haghnegahdar et al. (2017).

5. Discussion

5.1 Potential causes of failure in the MESH

Considering the existing difficulties in failure analysis, however, our further investigations of the MESH model revealed at least two possible causes responsible for many of the simulation failures. First, we observed that the threshold behaviour of a parameter called ZSNL, which represents the snow depth threshold below which snow coverage is considered less than 100%, can cause many model crashes. When ZSNL was relatively large, it resulted in calculation of overly thick snow columns inside the model violating the snow energy balance constraints there and triggering a simulation abort. This situation became more severe when the calculated snow depth was invariantly larger than the maximum vegetation height(s) depending on their assigned values via parameter perturbations. Fig. 13 (left column) shows the scatterplots of ZSNL values sampled from the feasible ranges for all model simulations used for GSA of MESH, with failed designs marked by red dots. One possible solution to alleviate this issue is to reduce the upper bound of the ZSNL parameter to lower values as used by Haghnegahdar et al. (2017) or to fix ZSNL at a lower value of, for example, 0.1 m as suggested by CLASS manual (Verseghy, 2012).

We also found that the second reason responsible for the MESH failure was oversaturation of the soil layers. Our investigations revealed that this oversaturation can happen especially at lower values of the soil permeable depth (SDEP) and when it becomes less than the maximum vegetation rooting depth (ROOT). The situation is more severe when the soil
drainage index (DRN) is also reduced (all these three parameters are part of the 111 perturbed parameters in here). These interactions can collectively cause a thinner soil column for water storage and movement that now has a lower chance for transpiration and drainage. This will result in over accumulation of the water beyond the physical limits set for the soil in the model, thus leading to simulation failures. Fig. 13 (right column) displays the scatterplots of these three parameters for the crop vegetation type. To avoid model crashes, it is necessary to ensure that SDEP and ROOT values are not unrealistically low and that their values and/or their ranges are assigned as accurately as possible using available data as discussed in Haghnegahdar et al. (2017). Also, fixing DRN to 1, may allow for the maximum physically-meaningful drainage from the soil column and reduces the risk of oversaturation.

As can be seen from Fig. 13, very high values of parameters DRNC and SDEPC can also cause simulation crashes, while these crashes were happened at lower values of ZSNL7. Note that from these 2-dimensional projections of the 111-dimensional parameter space of the MESH no general conclusions can be drawn. This even becomes more complicated when noticing some isolated crashes in regions where most of the simulations were successful. Furthermore, as shown in Fig. 13, there are considerable overlaps between successful simulations and crashed ones in the feasible ranges of parameters. For example, there are many crashed simulations when DRNC was sampled from [3.5-4], at the same time a high density of successful simulations can also be observed in the same range. This indicates that locating regions of parameter space responsible for crashes is difficult, if not impossible, and necessitates analysing the MESH’s response surface throughout a high-dimensional parameter space.

5.2 The role of sampling strategies in handling model crashes

Due to the extremely large parameter space ($X$) of high-dimensional DESMs, it may require many properly distributed sample points ($X_s$) to generate/explore a full spectrum of model behaviors such as simulation crashes, discontinuities, stable regions, optima, etc. Together with the computationally intensive nature of DESMs, this issue can make both non-substitution procedures and imputation-based methods (those proposed in the present study) very costly in dealing with crashes, if not impractical.

Because the non-substitution procedures rely on constructing a statistical model based on observed crashes to predict and avoid them in the follow-up experiments, they need a good coverage of the domain to attain a reliable statistical model. This issue also challenges the use of imputation-based methods. For example, in the NN techniques one major concern is that the sparseness of sample points may affect the quality of the results. In regions of the parameter space where sample points are sparsely distributed, distances to nearest neighbours can be relatively large, leading to choosing physically incompatible neighbours. Moreover, in response surface modelling-based techniques, building an accurate and robust function approximation is directly depends on the utilized sampling strategy and how dense mappings between parameter and output spaces are (see, e.g., Jin et al., 2001; Mullur and Messac, 2006; Zhaou and Xue, 2010).

A crucial consideration in the use of any sampling strategy is the exploration ability of that strategy (i.e., space-fillingness), which significantly influences the effectiveness of the utilized crash handling approach. When having this
feature enabled, the non-substitution procedures can reliably identify implausible regions in the entire parameter space, meaning that the sample set is not confined to only a limited number of regions. Furthermore, it can notably improve the predictive accuracy of the response surface modelling-based methods (Crombecq et al., 2011). Exploration requires sample points to be evenly spread across the entire parameter space to ensure that all regions of the domain are equally explored, and thus sample points should be located almost equally apart. This feature rectifies the problem relating to the distances between sample points when using NN techniques since in space-filling designs these distances are as even as possible.

Given this, regardless of the chosen method for solving simulation crash problem in GSA, it is advisable to spend some time up front to find an optimal sample set before submitting it for evaluation to the computationally expensive DESMs. It is, therefore, necessary to prudently use improved sampling algorithms such as Progressive Latin Hypercube Sampling (PLHS; Sheikholeslami and Razavi, 2017), K-extended Latin Hypercubes (k-extended LHCs; Williamson, 2015), or Sequential Exploratory Experimental Design (SEED; Li, 2004). Generally, these sampling techniques optimize some characteristics of the sample points such as sample size, space-fillingness, projective properties, and so on.

6 Conclusion

Understanding complex physical processes in Earth and environmental systems, prediction and scenario analysis regarding the Earth’s future resources rely routinely on high-dimensional, computationally expensive models, typically comprising model calibration, and/or uncertainty and sensitivity analysis. If a simulation failure/crash occurs at any of these stages, these models will stop functioning, and thus need user intervention. Generally, there are many reasons for failure of a simulation in models, including those that come from an inconsistent integration time step or grid resolution, lack of convergence, and existing of model thresholds. Determining whether these “defects” exist in the utilized numerical schemes or they are programming bugs can only be done through analysing a high-dimensional parameter space and characterizing implausible regions responsible for crashes. This imposes a heavier computational burden on analysts. More importantly, every “crashed” simulation can be very demanding in terms of computational cost for global sensitivity analysis (GSA) algorithms because they can prevent completion of the analysis and introduce ambiguity into the GSA results.

These challenges motivated us to implement three missing data imputation-based strategies for handling simulation crashes, which involves substituting plausible values for failed simulations without a priori knowledge regarding the nature of the failures. Here, our focus was to find simple yet computationally frugal techniques to palliate the effect of model crashes on the GSA of dynamical Earth systems models (DESMs). Thus, we utilized three techniques, including median substitution, single nearest neighbour, and emulation-based substitution (here we used radial basis functions as a surrogate model) to fill in a value for a failed simulation using available information and other non-missing model responses.

Compared to other crash handling strategies (ignorance-based and non-substitution procedures), the efficiency of our proposed substitution-based strategy were shown to be remarkable, particularly when dealing with GSA of the computationally expensive models since this strategy does not need repeating the entire experiment again. We compared the
performance of the proposed strategy in GSA of the two modelling case studies in Canada, including a 10-parameter HBV-SASK conceptual hydrologic model and a 111-parameter MESH land surface-hydrology model. Our analyses revealed that:

- Overall, the emulation-based substitution can effectively handle the simulation crashes and produce promising sensitivity analysis results compared to the single nearest neighbour and median substitution techniques.
- As expected, the performance of the proposed methods degrades as the ratio of failures increases. The rate of degradation is dependent on the number of model parameters (dimensionality of parameter space).
- We observed in our experiments that the rankings of strongly and weakly influential parameters identified by the utilized GSA algorithm (i.e., VARS) are not affected by the chosen crash handling technique, whereas for the moderately influential parameters, different techniques yielded different rankings.

Furthermore, we conducted a failure analysis of the MESH model and identified the parameters that seem to be frequently causing model failures. Such analyses are helpful and much needed to improve the fidelity and numerical stability of DESMs and may constitute a promising avenue of research. In doing so, applying other advanced methods (see e.g., Lucas et al. (2013)) can be beneficial to diagnose existing defects of the complex models.

Future work should include extending the proposed crash handling strategy to time-varying sensitivity analysis of the DESMs because a comprehensive GSA requires a full consideration of the dynamical nature of the DESMs. Our proposed approach for handling simulation crashes can be integrated with any time-varying sensitivity analysis algorithm, for example, with the recently developed Generalized Global Sensitivity Matrix (GGSM) method (Gupta and Razavi, 2018; Razavi and Gupta, 2019). This further helps understand the temporal variation of the parameter importance and model behaviour.

Code Availability.

The MATLAB codes for the proposed crash handling techniques and the HBV-SASK model are included in the VARS-TOOL software package, which is a comprehensive, multi-algorithm toolbox for sensitivity and uncertainty analysis (Razavi et al., 2019). VARS-TOOL is freely available for non-commercial use and can be downloaded from http://vars-tool.com/

The most recent version of the MESH model can be downloaded from https://wiki.usask.ca/display/MESH/Releases

Additional data and information are available upon request from the authors.

Appendix

Parameters of the MESH model and their corresponding groups are listed in Table A. The description of parameters and their feasible ranges can be found in Haghneghahdar et al. (2017).
Author contributions.

All authors contributed to conceiving the idea of the study. RS and SR designed the method and experiments. The simulations for the first case study were carried out by RS. AH developed the second case study and performed the MESH simulations. RS developed the MATLAB codes for the proposed crash handling strategy and conducted all the experiments. RS wrote the manuscript with contributions from SR and AH. All authors contributed to the interpretation of the results, structuring and editing of the paper at all stages.

Competing interests.

The authors declare that they have no conflict of interest.

References


Figure 1: Oldman river basin located in the Rocky Mountains in Alberta, Canada, flows into the Saskatchewan River Basin.
Figure 2: Nottawasaga river basin in Southern Ontario, Canada.

Figure 3: Grouping of the 10 parameters of the HBV-SASK model when applied on the Oldman River Basin. The parameters are sorted from the most influential (to the left) to the least influential (to the right).
Figure 4: Comparison of the proposed crash handling strategies in sensitivity analysis of the HBV-SASK model using the STAR-VARS algorithm for different ratios of failures. The CDFs of the sensitivity indices for (a) strongly influential parameters \{FRAC, FC, C0\} (upper panel) and (b) moderately influential parameters \{C0, TT, alpha, K1\} (lower panel) are compared in this plot. The vertical line (solid black) on each subplot represents the corresponding “true” sensitivity index obtained when there were no failures.
Figure 5: Comparison of the proposed crash handling strategies in sensitivity analysis of the HBV-SASK model using the STAR-VARS algorithm for different ratios of failures. The CDFs of the sensitivity indices for weakly influential parameters (LP, ETF, beta, K2) are shown in this plot. The vertical line (solid black) on each subplot represents the corresponding “true” sensitivity index obtained when there were no failures.

Figure 6: Comparison of the crash handling strategies in estimating the parameter rankings for HBV-SASK model when the ratio of failure was (a) 5%, (b) 10%, (c) 15%, and (d) 20%. The y-axis in each subplot shows the number of times out of 50 replicates that the rankings of the parameters were equal to the true ranking.
Figure 7: Scatter plots of the true NS values versus the imputed NS values when the ratio of failure was 20% for the HBV-SASK model. The accuracy of crash handling techniques is demonstrated in subplot (a) for the single NN method and in subplot (b) for the RBF method. These results belong to one replicate (arbitrarily chosen) out of 50 independent runs.

Figure 8: Grouping of the 111 parameters of the MESH model. The parameters are sorted from the most influential (to the left) to the least influential (to the right). This grouping is based on the results of the RBF method.
Figure 9: Sensitivity analysis results of the MESH model using different crash handling strategies for the most influential parameters. To better illustrate the results, the highly influential parameters in Group 1 are separately shown in two subplots.
Figure 10: Sensitivity analysis results of the MESH model for moderately influential parameters using different crash handling strategies. To better illustrate the results, the moderately influential parameters in Group 2 are separately shown in three subplots.
Figure 11: Sensitivity analysis results of the MESH model using different crash handling strategies for weakly/non-influential parameters in Group 3. The bottom panel (c and d) shows a zoom-in of the top subplots for very small values on the vertical axis.
Figure 12. Plots comparing rankings of the MESH model parameters obtained by different crash handling strategies. Subplots (d), (e), and (f) (right column) show a zoom-in of the subplots (a), (b), and (c) (left column), respectively. The red line is the ideal (1:1) line. Note that a ranking of 1 represents the least influential and a ranking of 111 represents the most influential parameter.
Figure 13: A 2-D projections of the MESH parameters for successful (blue dots) and crashed (red dots) simulations. Left column shows the threshold snow depth parameters $ZSNL$ and right column shows soil permeable depth ($SDEP$), maximum rooting depth ($ROOT$), and drainage index ($DRN$) for crop vegetation type.
Table 1. HBV-SASK model parameters and their feasible ranges, used in this study. For information on the full parameter set, refer to Razavi et al. (2019).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Range</th>
<th>Description</th>
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</thead>
<tbody>
<tr>
<td>TT</td>
<td>[-4,4]</td>
<td>Air temperature threshold in °C for melting/freezing and separating rain and snow</td>
</tr>
<tr>
<td>C0</td>
<td>[0,10]</td>
<td>Base melt factor, in mm/°C per day</td>
</tr>
<tr>
<td>ETF</td>
<td>[0,1]</td>
<td>Temperature anomaly correction in 1/°C of potential evapotranspiration</td>
</tr>
<tr>
<td>LP</td>
<td>[0,1]</td>
<td>Limit for PET as a multiplier to FC, i.e., soil moisture below which evaporation becomes supply limited</td>
</tr>
<tr>
<td>FC</td>
<td>[50,500]</td>
<td>Field capacity of soil, in mm. The maximum amount of water that the soil can retain</td>
</tr>
<tr>
<td>beta</td>
<td>[1,3]</td>
<td>Shape parameter (exponent) for soil release equation (unitless)</td>
</tr>
<tr>
<td>FRAC</td>
<td>[0.1,0.9]</td>
<td>Fraction of soil release entering fast reservoir (unitless)</td>
</tr>
<tr>
<td>K1</td>
<td>[0.05,1]</td>
<td>Fast reservoir coefficient, which determines what proportion of the storage is released per day (unitless)</td>
</tr>
<tr>
<td>alpha</td>
<td>[1,3]</td>
<td>Shape parameter (exponent) for fast reservoir equation (unitless)</td>
</tr>
<tr>
<td>K2</td>
<td>[0,0.05]</td>
<td>Slow reservoir coefficient which determines what proportion of the storage is released per day (unitless)</td>
</tr>
</tbody>
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Table A. Grouping of 111 MESH model parameters. These groups are numbered in order of importance.

<table>
<thead>
<tr>
<th>Group number</th>
<th>Parameters</th>
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</thead>
<tbody>
<tr>
<td>1</td>
<td>SDEPC, WFR22, ZSNL3, DRNC, VPDAC, ZPLS4, SDEPD, ROOTC, SDEPG, XSLPC, RATIOs, ZSNL4, ZSNL1, ZPLG4, DDENC, VPDAD, LAMIND, VPDAG, LNZ0D</td>
</tr>
<tr>
<td>2</td>
<td>CLAYSa3, SANDSa2, LAMAXC, XSLPD, SANDSa1, RSMNC, ROOTG, ZSNL11, ZSNL7, XSLPG, ZPLG3, ZPLS3, ZPLS1, ZPLG1, DDEND, CLAYSi3, SANDSi3, LNZ0G, SANDSi3, CLAYSa2, CLAYSa1, QA50C, DRNG, VPDBC, DRND, DDENG, LAMAXG, THLQ3, CLAYSi1, SANDSi2, SANDCL3, QA50D, GRKFC, LNZ0C, ALICCC, ALVCC, CLAYSi2, ALICG, SANDCL2, SANDCL1, TBAR2, PSGAC, THLQ1, ORGSI3, ORGSI1, PSGBG, THLQ2, TBAR3, TPOND, TBAR1, CMASC, MANNNC, ZPOND, RATIOCl, QA50G, RSMNG, RSMND, ORGSI2, RATIQCL, CLAYCL3, GRKFD, CMASD, ORGSA3, ORGSA2, ORGSA1, ORGCL1, ORGCL2, CLAYCL2, ORGCL3, CLAYCL1, ALICD, LAMAXD, ALVCG, GRKFG, ALVCD, VPDBG, CMASG</td>
</tr>
<tr>
<td>3</td>
<td>ZPLS11, VPDBD, ZPLG11, PSGAG, PSGBG, LAMING, PSGAD, PSGBD, MANNNG, ROOTD, ZPLG7, ZPLS7, TCANO, MANNND</td>
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