Supplementary material for: Jena Soil Model: a microbial soil organic carbon model integrated with nitrogen and phosphorus processes

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This appendix includes a detailed description of the model with equations and tables of parameter values.

S1 The SOM cycle

The model represents different soil organic pools (woody (wl), polymeric (poly), and soluble (sol) litter, as well as dissolved organic matter (DOM, dom), microbial biomass (mic), microbial residue (res), mineral-associated DOM (aDom), and mineral-associated microbial residue (aRes)), and the dynamics of them (X = C, N, P, ¹³C, ¹⁴C, ¹⁵N) are described in general as:

\[
\frac{\partial}{\partial t}X_{wl} = \sum (f_{vp \rightarrow wl} F_{L_{vp}}) - \frac{X_{wl}}{\tau_{wl}} - \frac{\partial}{\partial z} \left( D_b \frac{\partial X_{wl}}{\partial z} \right) - \frac{\partial (\omega \cdot X_{wl,>1})}{\partial z} \tag{S1a}
\]

\[
\frac{\partial}{\partial t}X_{sol} = \sum (f_{vp \rightarrow sol} F_{L_{vp}}) - \frac{X_{sol}}{\tau_{sol}} - \frac{\partial}{\partial z} \left( D_b \frac{\partial X_{sol}}{\partial z} \right) - \frac{\partial (\omega \cdot X_{sol})}{\partial z} \tag{S1b}
\]

\[
\frac{\partial}{\partial t}X_{poly} = \sum (f_{vp \rightarrow poly} F_{L_{vp}}) + \eta_{wl \rightarrow poly} \frac{X_{wl}}{\tau_{wl}} - F_{depoly}^{poly \rightarrow dom} \tag{S1c}
\]

\[
\frac{\partial}{\partial t}X_{dom} = \eta_{sol \rightarrow dom} \frac{X_{sol}}{\tau_{sol}} + F_{depoly}^{poly \rightarrow dom} + F_{depoly}^{res \rightarrow dom} - F_{sorp}^{dom \rightarrow aDom} - \frac{\partial}{\partial z} \left( D_b \frac{\partial X_{dom}}{\partial z} \right) - \frac{\partial (\omega \cdot X_{dom})}{\partial z} \tag{S1d}
\]

\[
\frac{\partial}{\partial t}X_{mic} = F_{growth}^{mic} - \eta_{mic \rightarrow dom} \frac{X_{mic}}{\tau_{mic}} + \Phi_{immobilisation} - \frac{\partial}{\partial z} \left( D_b \frac{\partial X_{mic}}{\partial z} \right) - \frac{\partial (\omega \cdot X_{mic})}{\partial z} \tag{S1e}
\]

\[
\frac{\partial}{\partial t}X_{res} = \eta_{mic \rightarrow res} \frac{X_{mic}}{\tau_{mic}} - F_{depoly}^{res \rightarrow dom} - F_{sorp}^{res \rightarrow aRes} - \sigma_{recycle} - \frac{\partial}{\partial z} \left( D_b \frac{\partial X_{res}}{\partial z} \right) - \frac{\partial (\omega \cdot X_{res})}{\partial z} \tag{S1f}
\]

\[
\frac{\partial}{\partial t}X_{aDom} = F_{sorp}^{res \rightarrow aRes} - \frac{\partial}{\partial z} \left( D_b \frac{\partial X_{aDom}}{\partial z} \right) - \frac{\partial (\omega \cdot X_{aDom})}{\partial z} \tag{S1g}
\]

\[
\frac{\partial}{\partial t}X_{aRes} = F_{sorp}^{dom \rightarrow aDom} - \frac{\partial}{\partial z} \left( D_b \frac{\partial X_{aRes}}{\partial z} \right) - \frac{\partial (\omega \cdot X_{aRes})}{\partial z} \tag{S1h}
\]
where $F_{L_{vp}}$ is the litterfall of the various plant tissue types, $f_{vp\rightarrow i}$ are the coefficients determining the partitioning of this litterfall to the litter pools (see Section S1.1), $\tau_i$ are temperature and moisture adjusted turnover times of the respective pools ($X; i = \text{sol, wl, mic}$), $\eta_{i\rightarrow j}$ are the fractions of mass transfer from pool $i$ to $j$ (see Section S1.2), $F_{Y_{i\rightarrow j}}$ is the flux rates of processes $Y$ (depoly: depolymerisation; upt: microbial uptake of DOM; sorp: sorption to mineral surface) from pool $i$ to $j$ (see Section S1.4, S1.3 and S1.5), $F_{\text{growth}_{mic}}$ the microbial growth rate, $\sigma_{\text{recycle}}$ is the additional nutrients recycled to DOM when microbes decay, $\Phi_{\text{immobilisation}}$ is the immobilisation terms for N and P, required to balance the microbial C:N:P stoichiometry (see Sect. S1.5). The $D_b$ is a prescribed diffusion constant for transfer of soil organic matter through bioturbation, and $\omega$ is the flux rate representing the advective transport of soil organic matter due to SOM accumulation/diminishing, where the above-ground woody litter is not subjective to this transport (see Section S3), and $\frac{\partial v_{\text{dom}}}{\partial z}$ is the percolation loss term given by the dom concentration and water mass flow between soil layers.

### S1.1 Partitioning of litterfall to litter pools

Non-woody litterfall is partitioned to the soluble and polymeric litter according to the CENTURY approach (Parton et al., 1993). Litter from labile and reserve pools is assumed to enter the soluble pools, litter from sap- and heartwood enters the woody pool. The soluble fraction of litterfall from each vegetation pool ($vp$, i.e. leaves, fine and coarse roots, fruits and seed-bed) is determined as:

$$f_{vp\rightarrow sol,C} = f_{sol,\text{max},C} - k_{sol,C} \cdot LC_{vp} \frac{C_{vp}}{N_{vp}} \quad (S2)$$

where $f_{vp\rightarrow sol,C}$ is constrained to positive solutions, $f_{sol,\text{max},C}$ is the maximum fraction allocated to the soluble pool, $k_{sol,C}$ a factor relating the soluble litter fraction to the lignin to nitrogen ratio, $LC_{vp}$ the tissue-specific fraction of the lignin content of that tissue type, and $\frac{C_{vp}}{N_{vp}}$ the C:N ratio of litterfall from that tissue. The lignin content is assumed constant for all but the leaf tissues. For the latter, an empirical dependency between lignin content and specific leaf-area ($sla$) is used (White et al., 2000).

$$LC_{leaf} = LC_{\text{leaf, max}} + k_{\text{leaf, 2sla}} \cdot sla \quad (S3)$$

The remainder of litterfall is allocated to the polymeric pool. For N and P, the partitioning assumes that the relative proportions of C:N and N:P are preserved in the partitioning according to:

$$f_{vp\rightarrow sol,X} = 1 \left( 1 - f_{vp\rightarrow sol,C} \right) \cdot \frac{C_{vp}}{N_{vp}} \quad (S4)$$

### S1.2 Litter turnover

Woody decomposition is assumed to be a two-stage process. The first step implies physical destabilisation and a first level of biochemical processing, which releases a constant fraction of carbon ($1 - \eta_{C,wl \rightarrow poly}$) to heterotrophic respiration. The remainder is assumed to enter the polymeric litter that further depolymerises into DOM. Soluble litter decomposes with a similar two-stage process, where during the first step a fraction of carbon ($1 - \eta_{C,sol \rightarrow dom}$) is respired but the remainder directly enters the DOM pool, which is taken up by microbes.

The turnover times ($\tau_{i}^{\text{base}}$) of the woody and soluble litter respond to soil temperature ($T_{soil}$) and soil water content ($\Theta$) as follows:

$$\tau_i = \tau_i^{\text{base}} \cdot f(T_{soil}) \cdot g(\Theta_{soil})$$

$$f(T_{soil}) = e^{-\frac{k_{e,depoly}}{R} \left( \frac{1}{T_{soil}} - \frac{1}{T_{ref}} \right)} \quad (S5a)$$

$$g(\Theta) = \frac{a_{fps} \cdot k_{1a, fps}}{k_{a, fps} + a_{fps} \cdot k_{1a, fps}}$$

$$a_{fps} = \frac{\Theta_{fc} - \Theta_{soil}}{\Theta_{fc}} \quad (S5c)$$

$$\Theta_{soil} = \Theta_{fc} - \Theta_{soil}$$

$$\Theta_{fc} = \Theta_{fc}$$

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where $E_{a,depoly}$ is the activation energy of depolymerisation, $T^*_{ref}$ is the reference temperature of the turnover rate, $k_{1,afps}$ and $k_{a,afps}$ are parameters, $afps$ is the air filled pore space, and $\Theta_{soil}$ and $\Theta_{fc}$ are the absolute soil water content and soil water field capacity, respectively.

### S1.3 Depolymerisation

The depolymerisation from polymeric litter or microbial residue to DOM (dom) are assumed to be enzyme-limited and described with "reverse Michaelis-Menten" kinetics ((Ahrens et al., 2015), (Schimel and Weintraub, 2003)).

\[
F_{\text{poly} \rightarrow \text{dom}}^{\text{depoly}} = v_{\text{max},\text{poly}}^{\text{depoly}}(T_{\text{soil}}, \Theta) \frac{X_{\text{mic}} E_{\text{frac}}^{\text{poly}}}{K_{m,\text{poly}}(T_{\text{soil}}, \Theta) + X_{\text{mic}} E_{\text{frac}}^{\text{poly}} X_{\text{poly}}} 
\]

\[
F_{\text{res} \rightarrow \text{dom}}^{\text{depoly}} = v_{\text{max},\text{res}}^{\text{depoly}}(T_{\text{soil}}, \Theta) \frac{X_{\text{mic}} E_{\text{frac}}^{\text{res}}}{K_{m,\text{res}}(T_{\text{soil}}, \Theta) + X_{\text{mic}} E_{\text{frac}}^{\text{res}} X_{\text{res}}, \text{where}} 
\]

\[
K_{m,\text{poly}}(T_{\text{soil}}, \Theta) = K_{m,\text{poly}} e^{-\frac{E_{a,\text{poly}}}{R} \left( \frac{1}{T_{\text{soil}}} - \frac{1}{T_{\text{ref}}} \right) - \left( \frac{\Theta}{\Theta_{fc}} \right)^{k_{h,\text{poly}}}} 
\]

where $v_{\text{max},\text{poly}}^{X}$ is the maximum depolymerisation rate of pool $X$ (poly or res) and its temperature and moisture responses are the same as those of litter turnover (Eq.S5), $E_{\text{frac}}^{X}$ is the fraction of enzyme (see Sect.S1.5.2) which targets to depolymerise pool $X$, and $K_{m,\text{poly}}(T_{\text{soil}}, \Theta)$ is the temperature and moisture corrected, microbial biomass scaled, half-saturation enzyme concentration of depolymerisation.

### S1.4 SOC sorption stabilisation

The sorption of DOM and microbial residue to mineral soil surface is represented with dynamic Langmuir isotherm, modified from the COMISSION model (Ahrens et al., 2015).

\[
F_{\text{dom} \rightarrow \text{ADom}}^{\text{sorp}} = k^{\text{ads}}_{\text{dom}} X_{\text{dom}} f(T_{\text{soil}}, \Theta) Q_{\text{av}}^{\text{org}} + k^{\text{des}}_{\text{dom}} f(T_{\text{soil}}, \Theta) X_{\text{ADom}} 
\]

\[
F_{\text{res} \rightarrow \text{Ares}}^{\text{sorp}} = k^{\text{ads}}_{\text{res}} X_{\text{res}} f(T_{\text{soil}}, \Theta) Q_{\text{av}}^{\text{org}} + k^{\text{des}}_{\text{res}} f(T_{\text{soil}}, \Theta) X_{\text{Ares}}, \text{where} 
\]

\[
f(T_{\text{soil}}, \Theta) = e^{-\frac{E_{a,\text{sorption}}}{R} \left( \frac{1}{T_{\text{soil}}} - \frac{1}{T_{\text{ref}}} \right) - \left( \frac{\Theta}{\Theta_{fc}} \right)^{k_{h,\text{ads}}}} 
\]

\[
Q_{\text{av}}^{\text{org}} = Q_{\text{max}}^{\text{org}} - X_{\text{ADom}} - X_{\text{Ares}} 
\]

\[
Q_{\text{max}}^{\text{org}} = q_{\text{max,mineral}}^{\text{org}} \cdot (\text{Silt} + \text{Clay}) \rho_{\text{soil}} V_{\text{mineral}}^{\text{frac}} 
\]

where $k^{\text{ads}}_{X}$ and $k^{\text{des}}_{X}$ are the adsorption rate and desorption rate between $X$ (dom or res) and its mineral-associated form (ADom or Ares), respectively, $dz$ is the soil depth, and Silt and Clay are the silt and clay content in weight fraction. It is assumed that the two substrates share the same sorption sites, which is represented by the maximum sorption capacity of organic matter $Q_{av}$ and it is calculated based on the weight and sorption capacity of fine soil, $q_{\text{max,mineral}}^{\text{org}}$, following COMISSION (Ahrens et al., In prep.).

### S1.5 Microbial growth and decay

In JSM, the growth of microbial biomass ($F^{\text{growth}}_{\text{mic}}$) depends on three factors: i) the uptake rate of DOM ($F^{\text{upt}}_{\text{dom} \rightarrow \text{mic}}$), ii) the microbial carbon use efficiency (CUE), $m_{\text{cave}}^{\text{mavg}}$, and iii) nutrient (N and P) limitation, $scal_{C,N,P \text{ limit}}^{\text{Cavg}}$.

\[
F^{\text{growth}}_{\text{mic}} = m_{\text{cave}}^{\text{mavg}} F^{\text{upt}}_{\text{dom} \rightarrow \text{mic}} 
\]

\[
F^{\text{upt}}_{\text{dom} \rightarrow \text{mic}} = MIN(F^{\text{upt}}_{\text{dom} \rightarrow \text{mic}}, scal_{C,N,P \text{ limit}}^{\text{Cavg}} \cdot C_{\text{dom}}) 
\]
The potential uptake of DOM, \( F_{\text{dom} \rightarrow \text{mic}}^{\text{upt}} \), is constrained by the DOM concentration, meaning at high DOM concentration the uptake is limited by ability of microbes to assimilate DOM (Ahrens et al., 2015). It is described with "Michaelis-Menten" kinetics,

\[
F_{\text{dom} \rightarrow \text{mic}}^{\text{upt}} = v_{\text{max} \cdot \text{upt}}^{\text{dom}} (T_{\text{soil}}, \Theta) C_{\text{mic}}^\text{dom} \frac{C_{\text{dom}}}{K_{\text{m,upt}}(T_{\text{soil}}, \Theta) + C_{\text{dom}}} \tag{S9a}
\]

where \( v_{\text{max} \cdot \text{upt}}^{\text{dom}} \) is the maximum DOM uptake rate and has the same temperature and moisture responses as those of litter turnover (Eq.S5), \( K_{\text{m,upt}}(T_{\text{soil}}, \Theta) \) is the temperature and moisture corrected half-saturation coefficient for DOM uptake.

Microbes respond to the instantaneous conditions and associated fluxes to adapt their CUE. It is assumed in JSM that all the microbial adaptation has a process-specific lag time. Therefore, the microbial growth is calculated using the time-averaging microbial CUE, \( m_{\text{mic}}^{\text{new}, \text{avg}} \), which is a moving average of the current effective microbial CUE (\( m_{\text{mic}}^{\text{eff}} \)) over a certain lag time.

\[
m_{\text{mic}}^{\text{new}, \text{avg}} = m_{\text{mic}}^{\text{old}, \text{avg}} \cdot (1 - \frac{dt}{\tau_{\text{mic}}^{\text{avg}}} ) + m_{\text{mic}}^{\text{eff}} \cdot \frac{dt}{\tau_{\text{mic}}^{\text{avg}}} \tag{S10a}
\]

where \( \tau_{\text{mic}}^{\text{avg}} \) is the time span of the microbial CUE acclimation, \( m_{\text{mic}}^{\text{min}} \) is the theoretical minimal microbial CUE, \( m_{\text{mic}}^{\text{max}} \) is the maximum potential microbial growth rate only considering the availability of element \( X \), which takes account of the microbial nitrogen and phosphorus use efficiencies, \( m_{\text{mic}}^{\text{new}} \) and \( m_{\text{pue}}^{\text{new}} \), and the potential microbial uptake rate of nutrient, \( U_{X, \text{mic}}^{*} \) (\( X = \text{NO}_3, \text{NH}_4, \text{PO}_4 \), see Sect.S2.2). The smallest growth potential determines the current effective CUE.

Given the time-averaging microbial CUE, the potential microbial growth only considering C is recalculated as \( F_{\text{mic}}^{\text{growth}, C^{**}} \). The uptake rate of DOM will be reduced if \( F_{\text{mic}}^{\text{growth}, C^{**}} \) is bigger than \( F_{\text{mic}}^{\text{growth}, X^{*}} \), and the scaling factor \( \text{scal}_{C,N,P \text{ limit}} \) is calculated as,

\[
\text{scal}_{C,N,P \text{ limit}} = \begin{cases} 
\text{MIN} \left( \frac{F_{\text{mic}}^{\text{upt}}}{C_{\text{dom}}}, \frac{(U_{\text{NO}_3, \text{mic}}^{*} + U_{\text{NH}_4, \text{mic}}^{*}) C_{\text{mic}}^{N}}{m_{\text{mic}}^{\text{new}} C_{\text{mic}}^{N} + m_{\text{mic}}^{\text{pue}} C_{\text{mic}}^{N}} \right), & \text{if } F_{\text{mic}}^{\text{growth}, N^{*}} < F_{\text{mic}}^{\text{growth}, C^{**}} \& F_{\text{mic}}^{\text{growth}, P^{*}} \\
\text{MIN} \left( \frac{F_{\text{mic}}^{\text{upt}}}{C_{\text{dom}}}, \frac{U_{\text{PO}_4, \text{mic}}^{*} C_{\text{mic}}^{P}}{m_{\text{mic}}^{\text{pue}} C_{\text{mic}}^{P} + m_{\text{mic}}^{\text{new}} F_{\text{mic}}^{\text{upt}} C_{\text{mic}}^{P}} \right), & \text{if } F_{\text{mic}}^{\text{growth}, P^{*}} < F_{\text{mic}}^{\text{growth}, C^{**}} \& F_{\text{mic}}^{\text{growth}, N^{*}} 
\end{cases} \tag{S11a}
\]

where, \( F_{\text{mic}}^{\text{growth}, C^{**}} = m_{\text{mic}}^{\text{new}} F_{\text{dom} \rightarrow \text{mic}}^{\text{upt}} \tag{S11b} \)

\[\text{In Eq.S10f and all the following cases, the microbial } \chi_{\text{mic}}^{C:P} = \chi_{\text{mic}}^{C:N} \cdot \chi_{\text{mic}}^{N:P} \]
S1.5.1 Microbial nutrient uptake, mineralisation, and microbial recycle

Given the microbial growth and the C:N:P stoichiometry, the uptake of inorganic N and P, $U_{X,mic}$, is calculated as,

$$U_{NH_4,mic} = U^*_{NH_4,mic} \text{scal}_N$$  \hspace{1cm} (S12a)
$$U_{NO_3,mic} = U^*_{NO_3,mic} \text{scal}_N$$  \hspace{1cm} (S12b)
$$U_{PO_4,mic} = U^*_{PO_4,mic} \text{scal}_P,$$  \hspace{1cm} (S12c)

where

$$\text{scal}_N = MAX\left(\frac{F_{\text{growth},C}}{\chi_{mic}^N} - \text{mic}_\text{nue} F_{mic}^{upt,N}, 0.0\right),$$

$$\text{scal}_P = MAX\left(\frac{F_{\text{growth},C}}{\chi_{mic}^P} - \text{mic}_\text{pue} F_{mic}^{upt,P}, 0.0\right),$$

If $F_{\text{growth},C}^{mic} \leq F_{\text{growth},N^{**}}^{mic}$

$$F_{\text{upt},N}^{mic} = \text{mic}^{\text{mavg}} \frac{F_{\text{dom} \rightarrow \text{mic}}^{upt}}{\chi_{\text{dom}}}$$  \hspace{1cm} (S12d)

$$F_{\text{upt},P}^{mic} = \text{mic}^{\text{mavg}} \frac{F_{\text{dom} \rightarrow \text{mic}}^{upt}}{\chi_{\text{dom}}}$$  \hspace{1cm} (S12e)

If $F_{\text{growth},C}^{mic} \leq F_{\text{growth},P^{**}}^{mic}$

$$F_{\text{growth},N^{**}}^{mic} = \left(F_{\text{growth},N^{**}}^{mic} + U^*_{NO_3,mic} + U^*_{NH_4,mic}\right)\chi_{mic}^C : N$$  \hspace{1cm} (S12f)

$$F_{\text{growth},P^{**}}^{mic} = \left(F_{\text{growth},P^{**}}^{mic} + U^*_{PO_4,mic}\right)\chi_{mic}^C : P$$  \hspace{1cm} (S12g)

where $F_{\text{growth},N^{**}}^{mic}$ and $F_{\text{growth},P^{**}}^{mic}$ are the potential microbial growth only considering N or P availability given the time-averaging CUE.

The effective microbial nutrient use efficiency ($\text{mic}_\text{nue}^{\text{eff}}$ and $\text{mic}_\text{pue}^{\text{eff}}$) and the net mineralisation of N and P ($\Phi^\text{net}_{\text{NH}_4}$ and $\Phi^\text{net}_{\text{NO}_3}$) are thus calculated as,

$$\text{mic}_\text{nue}^{\text{eff}} = \min(\text{mic}_\text{nue},\frac{F_{\text{growth},C}}{\chi_{mic}^N} - U_{NH_4,mic} - U_{NO_3,mic})$$  \hspace{1cm} (S13a)

$$\text{mic}_\text{pue}^{\text{eff}} = \min(\text{mic}_\text{pue},\frac{F_{\text{growth},C}}{\chi_{mic}^P} - U_{PO_4,mic})$$  \hspace{1cm} (S13b)

$$\Phi^\text{net}_{\text{NH}_4} = (1 - \text{mic}_\text{nue}^{\text{eff}}) F_{\text{mic}}^{\text{upt},N} - U_{NH_4,mic}$$  \hspace{1cm} (S13c)

$$\Phi^\text{net}_{\text{NO}_3} = -U_{NO_3,mic}$$  \hspace{1cm} (S13d)

$$\Phi^\text{net}_{\text{PO}_4} = (1 - \text{mic}_\text{pue}^{\text{eff}}) F_{\text{mic}}^{\text{upt},P} - U_{PO_4,mic}$$  \hspace{1cm} (S13e)

When microbes decay in JSM, there is a faction of the dead microbes ($\eta_{\text{mic} \rightarrow \text{dom}}$) which directly recycles into the DOM pool and the rest ($\eta_{\text{mic} \rightarrow \text{res}}$) become microbial residues. Due to the fact that microbial cell wall has a lower nutrient content than plasma, it is assumed in JSM that N and P are more prone to be recycled into DOM when microbes decay, which is represented by $\sigma_{\text{recycle}}^X$ in Eq.S1.

$$\sigma_X^{\text{recycle}} = \eta_{\text{mic} \rightarrow \text{res}} X_{\text{mic}} \text{res} \left(1 - \frac{X_{\text{res}}}{\tau_{\text{mic}}}ight) \eta_{\text{res} \rightarrow \text{dom}}$$  \hspace{1cm} (S14a)
where $X$ represents N or P, and $\eta^X_{\text{res} \rightarrow \text{dom}}$ is the fraction of $X$ that is recycled from newly formed $\text{res}$ to $\text{dom}$ during microbial decay.

### S1.5.2 Enzyme allocation in depolymerisation

The enzyme allocation to polymeric litter and microbial residue is presented with the steady state of the Revenue strategy in the SEAM model (Wutzler et al., 2017), assuming that the microbial community adapts in a way that the fraction of enzyme allocation is proportional to its revenue (return-investment rate) of the limiting elements (C, N, or P).

\[
\alpha^X_{\text{poly}} = \frac{\text{Rev}^X_{\text{poly}}}{\text{Rev}^X_{\text{poly}} + \text{Rev}^X_{\text{res}}}, \quad \text{where}
\]

\[
\text{Rev}^X_{\text{poly}} = \frac{\text{return investment}}{v_{\text{max, depoly}} X_{\text{poly}}} = \frac{v_{\text{max, depoly}} \cdot \alpha^X_{\text{poly}} C_{\text{mic}}}{K_{m, \text{depoly}} + \alpha^X_{\text{poly}} C_{\text{mic}}} \cdot X_{\text{poly}}
\]

\[
\alpha^X_{\text{res}} = \frac{\text{Rev}^X_{\text{res}}}{\text{Rev}^X_{\text{poly}} + \text{Rev}^X_{\text{res}}}, \quad \text{and}
\]

\[
\alpha^X_{\text{poly}} + \alpha^X_{\text{res}} = 1
\]

where $\alpha^X_{Y}$ is the potential enzyme allocation fraction to $Y$ ($\text{poly}$ or $\text{res}$) based on the revenues of element $X$ (C, N, or P), and $\text{Rev}^X_{Y}$ is the revenue of element $X$ from source $y$ and is defined as the production of $X$ divided by the potential allocation of enzyme. In JSM the enzyme levels are not explicitly represented, thus a steady state assumption is made: the production and turnover of the enzyme is in equilibrium therefore the enzyme is always linear with the microbial biomass. We could get the analytical solution of the potential enzyme allocation fractions by rearranging Eq.S15,

\[
\frac{\alpha^X_{\text{poly}}}{\alpha^X_{\text{res}}} = \frac{\text{Rev}^X_{\text{poly}}}{\text{Rev}^X_{\text{res}}}
\]

\[
\frac{v_{\text{poly}}^{\text{max, depoly}} X_{\text{poly}}}{v_{\text{res}}^{\text{max, depoly}} X_{\text{res}}} = \frac{\alpha^X_{\text{poly}}}{\alpha^X_{\text{res}}} \frac{K_{m, \text{depoly}} + \alpha^X_{\text{poly}} C_{\text{mic}}}{1 - \alpha^X_{\text{poly}} (K_{m, \text{depoly}} + (1 - \alpha^X_{\text{poly}}) C_{\text{mic}})}
\]

\[
\alpha^X_{\text{poly}} = \frac{d_{\text{poly}} K_{m, \text{depoly}} + 2d_{\text{poly}} C_{\text{mic}} + d_{\text{res}} K_{m, \text{depoly}} - \sqrt{D}}{2C_{\text{mic}} (d_{\text{poly}} - d_{\text{res}})}, \text{where}
\]

\[
d_{\text{poly}} = v_{\text{poly}}^{\text{max, depoly}} X_{\text{poly}}
\]

\[
d_{\text{res}} = v_{\text{res}}^{\text{max, depoly}} X_{\text{res}}
\]

\[
D = 4d_{\text{poly}} d_{\text{res}} C_{\text{mic}}^2 + 8d_{\text{poly}} d_{\text{res}} C_{\text{mic}} K_{m, \text{depoly}} + K_{m, \text{depoly}}^2 d_{\text{poly}}^2 + 2d_{\text{poly}} d_{\text{res}} K_{m, \text{depoly}}^2 + K_{m, \text{depoly}}^2 d_{\text{res}}^2
\]

It is assumed that the microbial community would acclimate gradually to allocate the enzyme to optimize the utilisation of the most limiting element of depolymerisation, which is determined similarly as that in microbial growth (Eq.S10) except that the uptake of inorganic nutrients are not considered.
\[ \text{Enz}_{\text{poly,new}}^{\text{frac}} = \text{Enz}_{\text{poly,old}}^{\text{frac}} \cdot (1 - \frac{dt}{\tau_{\text{enzyme}}}) + \alpha_{\text{poly, mavg}}^{X_{\text{new}}} \cdot \frac{dt}{\tau_{\text{mavg}}} , \] where

\[ \alpha_{\text{poly, mavg}}^{X_{\text{new}}} = \alpha_{\text{poly, mavg}}^{X_{\text{old}}} \cdot (1 - \frac{dt}{\tau_{\text{enzyme}}}) + \alpha_{\text{poly}}^{X} \cdot \frac{dt}{\tau_{\text{mavg}}} \] (S17a)

and the most limiting element X is determined as (S17c)

\[
\text{MIN} \left( \frac{F_{\text{depoly}}^{\text{poly \rightarrow dom}} + F_{\text{depoly}}^{\text{res \rightarrow dom}}}{\lambda_{\text{poly}}} \cdot \text{mic}_{\text{mavg}} \cdot \text{MIC} \right),
\]

\[
\left( \frac{F_{\text{depoly}}^{\text{poly \rightarrow dom}}}{\lambda_{\text{poly}}} + \frac{F_{\text{depoly}}^{\text{res \rightarrow dom}}}{\lambda_{\text{res}}} \right) \cdot \text{mic}_{\text{mavg}} \cdot \text{MIC} \]

\[
\left( \frac{F_{\text{depoly}}^{\text{poly \rightarrow dom}}}{\lambda_{\text{poly}}} + \frac{F_{\text{depoly}}^{\text{res \rightarrow dom}}}{\lambda_{\text{res}}} \right) \cdot \text{mic}_{\text{mavg}} \cdot \text{MIC} \]

\[
\frac{\partial}{\partial z} \left( D_{b} \frac{\partial X}{\partial z} \right) - \frac{\partial (\omega \cdot X)}{\partial z} \right) - \frac{\partial v_{X} X}{\partial z}
\]

(S17b)

where \( \tau_{\text{mavg}}^{\text{enzyme}} \) is the time span of enzyme allocation acclimation, and \( \text{mic}_{\text{mavg}} \) and \( \text{mic}_{\text{mavg}} \) are the time averaging microbial N and P use efficiency, which is calculated similarly as \( \text{mic}_{\text{mavg}}^{\text{NH}_{4}} \) in Eq.S10.

### S2 Inorganic nutrient cycles

In JSM, the net mineralisation and plant uptake of \( \text{NH}_{4} \) and \( \text{NO}_{3} \) are represented next to transport process. The dynamics of inorganic nitrogen are described as:

\[
\frac{\partial}{\partial t} \text{NH}_{4} = F_{\text{dep}, \text{NH}_{4}} - U_{\text{NH}_{4}, \text{plant}} + \Phi_{\text{NH}_{4}}^{\text{net}}
\]

\[
- \frac{\partial}{\partial z} \left( D_{b} \frac{\partial \text{NH}_{4}}{\partial z} \right) - \frac{\partial (\omega \cdot \text{NH}_{4})}{\partial z} \right) - \frac{\partial v_{\text{NH}_{4}} \text{NH}_{4}}{\partial z}
\]

(S18a)

\[
\frac{\partial}{\partial t} \text{NO}_{3} = F_{\text{dep}, \text{NO}_{3}} - U_{\text{NO}_{3}, \text{plant}} + \Phi_{\text{NO}_{3}}^{\text{net}}
\]

\[
- \frac{\partial}{\partial z} \left( D_{b} \frac{\partial \text{NO}_{3}}{\partial z} \right) - \frac{\partial (\omega \cdot \text{NO}_{3})}{\partial z} \right) - \frac{\partial v_{\text{NO}_{3}} \text{NO}_{3}}{\partial z}
\]

(S18b)

where \( U \) are the uptake rates of plants and microbes (see Section S2.2); the \( F_{\text{dep}, X} \) are the atmospheric deposition fluxes;

\[
\frac{\partial v_{X} X}{\partial z}, \frac{\partial}{\partial z} \left( D_{b} \frac{\partial X}{\partial z} \right), \text{ and } \frac{\partial (\omega \cdot X)}{\partial z}
\]

are vertical transport terms due to percolation loss, bioturbation, and SOM accumulation/diminishing, respectively (see Sect.S3).
The inorganic phosphorus cycle is mostly based on that of the QUINCY model (Thum et al., 2019) with modifications due to microbial interactions. The dynamics of inorganic phosphorus are described as:

\[
\frac{\partial}{\partial t} PO_4 = F_{dep, PO_4} + F_{weath, PO_4} + F_{biomin, PO_4} - U_{plant, PO_4} - F_{adsorp, PO_4} + \Phi^{net}_{PO_4} - \frac{\partial}{\partial z} \left( D_b \frac{\partial PO_4}{\partial z} \right) - \frac{\partial (\omega \cdot PO_4)}{\partial z} - \frac{\partial v_{PO_4}}{\partial z}\]

(S19a)

\[
\frac{\partial}{\partial t} P_{lab} = F_{adsorp, PO_4} - F_{absorb, PO_4} - \frac{\partial (\omega \cdot P_{lab})}{\partial z}\]

(S19b)

\[
\frac{\partial}{\partial t} P_{sorb} = F_{absorb, PO_4} - F_{occlusion, PO_4} - \frac{\partial (\omega \cdot P_{sorb})}{\partial z}\]

(S19c)

\[
\frac{\partial}{\partial t} P_{ocl} = F_{occlusion, PO_4} - \frac{\partial (\omega \cdot P_{ocl})}{\partial z}\]

(S19d)

\[
\frac{\partial}{\partial t} P_{primary} = -F_{weath, PO_4} - \frac{\partial (\omega \cdot P_{primary})}{\partial z}\]

(S19e)

where \( P_{lab}, P_{sorb}, P_{ocl}, \) and \( P_{primary} \) are adsorbed, absorbed, occluded, and primary P, respectively; the \( F_{dep, PO_4}, F_{weath, PO_4}, F_{biomin, PO_4}, F_{adsorp, PO_4}, \) and \( F_{absorb, PO_4} \) are the atmospheric deposition, weathering, fast adsorption, and absorption fluxes, respectively (see Section S2.1).

### S2.1 Phosphorus weathering, biomineralisation and absorption

Weathering is assumed to be driven by root and microbial exudation, and modified from Wang et al. (2010) as:

\[
F_{weath, PO_4} = f(T_{soil}, \Theta) \frac{C_{exu}}{K_{m,weath} + C_{enz}} k_{weath}^\rho_{soil}^{cor}, \text{ where}
\]

\[
f(T_{soil}, \Theta) = e^{-\frac{E_{a,hsc}}{R} \left( \frac{1}{T_{soil}} - \frac{1}{T_{ref}} \right) \left( \frac{\Theta}{\Theta_{fc}} \right)^3},
\]

(S20a)

\[
C_{exu} = C_{fine-root} k_{enz,root} + C_{mic} k_{enz,mic}
\]

(S20b)

where \( k_{weath} \) is the rate constant for weathering, \( \rho_{soil}^{cor} \) is the soil bulk density corrected by SOM content, \( C_{exu} \) represents an implicit general assemble of all exudation, analogous to the enzymatic abundance of fine roots and microbes (\( k_{enz,root} \) and \( k_{enz,mic} \)), and \( K_{m,weath} \) is the half-saturation coefficient for weathering. The weathering rate decreases with soil depth as the fine root C and microbial biomass decreases and is modified by soil temperature and moisture.

The biomineralisation of \( PO_4 \) is determined as an additional enzyme-catalysed cleavage of the P contained in the solid SOM pools (\( X = res, aRes, aDom \)), modified by temperature and moisture modifiers, affected by the concentration of \( PO_4 \) and enzyme abundance, and constrained by the C:P ratio of the organic pools:

\[
F_{biomin, PO_4} = v_{max,biomin} f(T_{soil}, \Theta) f(C_{enz}) f(PO_4) f(X_{C:P}), \text{ where}
\]

(S21a)

\[
f(C_{enz}) = \frac{C_{enz}}{K_{m,biomin} + C_{enz}}
\]

(S21b)

\[
f(PO_4) = \frac{K_{PO_4}^{biomin}}{K_{m,biomin}^{PO_4} + PO_4}
\]

(S21c)

\[
f(X_{C:P}) = \frac{1}{1 + X_{C:P} K_{CP}^{biomin}}
\]

(S21d)
where \( K_{m,\text{biomin}} \), \( K_{m,\text{biomin}}^{PO_4} \) and \( K_{m,P:C} \) are constants constraining the biomineralisation rate under low enzyme, high PO\(_4\) concentration, and high SOM C:P ratio, respectively, \( \chi_{C:P}^{X} \) is the C:P ratio of the organic pools, and the temperature and moisture responses are calculated as those in Eq. S5.

\( \text{PO}_4 \) absorption and occlusion are modelled as:

\[
F_{\text{occlusion,PO}_4} = k_{ocl} P_{sorb} \tag{S22a}
\]

\[
F_{\text{desorp,PO}_4} = f(T_{\text{soil}}, E_{a,\text{abs}}) k_{abs} P_{lab} - f(T_{\text{soil}}, E_{a,\text{des}}) k_{s,\text{des}} P_{sorb}, \text{ where}
\]

\[
f(T_{\text{soil}}, E_{a}) = e^{-\frac{E_{a}}{R} \left( \frac{1}{T_{\text{soil}}} - \frac{1}{T_{\text{ref}}} \right)} \tag{S22c}
\]

where \( k_{ocl}, k_{abs} \) and \( k_{s,\text{des}} \) are the rate constants of occlusion, absorption and slow desorption, and \( E_{a,\text{abs}} \) and \( E_{a,\text{des}} \) are the respective activation energies which equal to that of OM sorption (Eq.S7).

### S2.2 Nutrient acquisition

It is assumed in the JSM model that the soluble forms of inorganic NH\(_4\), NO\(_3\), and PO\(_4\) are the only bio-available nutrients for plants and microbes, and specifically for PO\(_4\), the soluble inorganic form is assumed to be the only form that could be adsorbed by the mineral surfaces. The uptake of plants and microbes as well as the PO\(_4\) adsorption are all represented by the formations of consumer-substrate network using their full equilibrium chemistry approximations (ECA), following Tang and Riley (2013):

\[
U_{X,y}^* = f(T_{\text{soil}}, \Theta)_{yi} v_{max,yi}^{X} \frac{[X]}{K_{X,m,yi}^{X} + [X] + Enz_{yi}^{X} + K_{X,m,yi}^{X} Enz_{yi}^{X} + K_{X,m,yi}^{X} Enz_{yi}^{X}}, \text{ where}
\]

\[
Enz_{\text{plant}}^{X} = C_{\text{fine,root}}^{X} k_{\text{enz,root}}^{X} \tag{S23b}
\]

\[
Enz_{\text{mic}}^{X} = C_{\text{mic}}^{X} k_{\text{enz,mic}}^{X} \tag{S23c}
\]

Specifically, for PO\(_4\) adsorption flux we assumed

\[
Enz_{\text{PO}_4}^{PO_4} = S_{\text{max}}^{PO_4} - P_{lab} \tag{S23e}
\]

\[
v_{\text{max,adsorp}}^{PO_4} = k_{ads} \cdot Enz_{\text{PO}_4}^{PO_4} \cdot PO_4 \tag{S23f}
\]

\[
k_{ads} = \frac{k_{des,f}}{K_{S}} \tag{S23g}
\]

\[
[PO_4] = P_{lab} + PO_4 \tag{S23h}
\]

where \( U_{X,y}^* \) is the potential acquisition rate of substrate \( X \) (NH\(_4\), NO\(_3\), and PO\(_4\)) through process \( y \) (\text{plant}: plant uptake, \text{mic}: microbial uptake, or \text{adsorp}: adsorption). The maximum uptake rates \( (v_{max,y}^{X}) \) of plant and microbes are adopted from literature (see Table S1), while the maximum PO\(_4\) adsorption rate is calculated following Van der Zee et al. (1989); \( Enz_{yi}^{X} \) represents the enzymatic capacity of \( y \) to consume the substrate \( X \) and it is assumed to be linear with the root biomass of plant, microbial biomass for microbe, and equals with the available sorption sites for mineral soil, and \( k_{\text{enz,y}}^{X} \) is the coefficient representing the transporters’ abundance and capacity; the \([X]\) represents the total substrate concentration for all the relevant acquisition processes, and it equals the soluble inorganic concentration for NH\(_4\) and NO\(_3\), while for PO\(_4\) it is the sum of soluble and labile inorganic P. The uptake rate of plants and microbes are influenced by temperature and moisture as that in Eq.S6.
The potential acquisition rate will be down scaled if the sum of them exceed the size of the substrate pool. The actual microbial uptake rate also depends on the uptake demand of microbes (see Sect.S1.5). The actual adsorption rate are further calculated based on the assumption that the soluble inorganic P tend to equilibrate with the adsorbed P (see Sect.S2.2.1).

\[ U_{X,y}^* = U_{X,y}^* \cdot \frac{X}{\sum (U_{X,y}^*)} \text{ when } X < \sum (U_{X,y}^*) \]  

(S24a)

5 S2.2.1 PO\textsubscript{4} adsorption equilibrium

The adsorption \( (F_{\text{adsorp,PO}_4}) \) flux from soil solution to the soil adsorption sites is calculated assuming constant Langmuir equilibrium (Barrow, 1978) between soluble and adsorbed P:

\[ PO_4 = \frac{S_{\text{PO}_4}^{\text{max}} \cdot PO_4}{K_S + PO_4}, \text{ where} \]

(S25a)

\[ S_{\text{PO}_4}^{\text{max}} = \Theta_{\text{soil}} \cdot (S_{\text{PO}_4}^{\text{max,om}} V_{\text{frac,org}} \rho_{\text{bulk}} + S_{\text{PO}_4}^{\text{max,mineral}} V_{\text{frac,mineral}} \rho_{\text{soil}}) \]  

(S25b)

\[ K_S = K_{S, \text{om}} V_{\text{frac,org}} \rho_{\text{bulk}} + K_{S, \text{mineral}} V_{\text{frac,mineral}} \rho_{\text{soil}} \]  

(S25c)

where \( S_{\text{PO}_4}^{\text{max}} \) and \( K_S \) are the maximum sorption capacity, and the half-saturation concentration coefficient of the soil, and are both modified by soil moisture and SOM content; \( V_{\text{frac,org}} \) and \( V_{\text{frac,mineral}} \) are volumetric fraction of organic matter and fine soil minerals, respectively. \( S_{\text{PO}_4}^{\text{max,om}} \) and \( S_{\text{PO}_4}^{\text{max,mineral}} \) are the maximum PO\textsubscript{4} sorption capacity of pure organic matter and pure fine soil, respectively. \( K_{S, \text{om}} \) and \( K_{S, \text{mineral}} \) are the half-saturation concentration coefficient of pure organic matter and pure fine soil, respectively.

The Eq.S25 is solved analytically since \( S_{\text{PO}_4}^{\text{max}} \) and \( K_S \) are also changing with time.

\[ P_{\text{lab}}' = \frac{[PO_4]' + S_{\text{PO}_4}^{\text{max}} + K_S - \sqrt{([PO_4]' + S_{\text{PO}_4}^{\text{max}} + K_S)^2 - 4 \cdot [PO_4]' S_{\text{PO}_4}^{\text{max}}}}{2} \]  

(S26a)

\[ PO_4' = \frac{[PO_4]' - P_{\text{lab}}}{2}, \text{ where} \]

(S26b)

\[ [PO_4]' = P_{\text{lab}} + PO_4 + \frac{\partial (P_{\text{lab}} + PO_4)}{\partial t} \]  

(S26c)

\[ \frac{\partial (P_{\text{lab}} + PO_4)}{\partial t} = F_{\text{dep,PO}_4} + F_{\text{weath,PO}_4} + F_{\text{biomin,PO}_4} - U_{\text{plant,PO}_4} - U_{\text{mic,PO}_4} + \Phi_{\text{PO}_4} \]

\[ - \frac{\partial}{\partial z} (D_b \frac{\partial PO_4}{\partial z}) - \frac{\partial (\omega \cdot PO_4)}{\partial z} - \frac{\partial v_{\text{PO}_4} \cdot PO_4}{\partial z} - F_{\text{absorb,PO}_4} - \frac{\partial (\omega \cdot P_{\text{lab}})}{\partial z} \]  

(S26d)

where the pools with apostrophe as superscript denote the size at the end of time step.
S3 Transport and bulk density correction

In JSM the soil profile always starts from the top of the organic layer, therefore changes of SOM content would affect the layering of the soil profile, which is represented by an advective transport rate $\omega$, following COMISSION model Ahrens et al. (2015)

\[ \omega_{sl} = \frac{1}{n} \sum_{s=1}^{n} \frac{\Delta W_{OM,sl} \cdot dz_{sl}}{\rho_{bulk}}, \text{ where} \]

\[ \Delta W_{OM,sl} = \sum (C'_{x,sl} - C_{x,sl}) \frac{Mol_{C}}{1000 \cdot frac_{C_{OM}}} \]  

where subscript $sl$ denotes the soil layer, $dz$ the layer depth, $\rho_{bulk}$ the bulk density of organic material; $\Delta W_{OM}$ is the change of total SOM weight which includes all the organic pools (NOTE: for the first layer woody litter is excluded), and is calculated based on the change of C content ($C'_{x,sl} - C_{x,sl}$), carbon molecular weight ($Mol_{C}$) and weight fraction of C in OM ($frac_{C_{OM}}$).

Not only all the organic pools and inorganic pools shift with the advective transport rate $\omega$ in JSM, but also the physical soil properties, such as soil texture, mineral soil density ($\rho_{soil}$) and mineral soil volumetric fraction ($V_{frac_{mineral}}$), shift with $\omega$ to ensure that the soil bulk density is properly corrected by the SOM content.

\[ \rho_{soil}^{cor} = V_{frac_{mineral}} \rho_{soil} + (1 - V_{frac_{mineral}}) \rho_{bulk} \] (S28a)
Table S1. JSM parameters

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Value</th>
<th>Unit</th>
<th>Equation</th>
<th>Citation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_{\text{sol}, \text{max}, \text{C}}$</td>
<td>maximum fraction of soluble litter formation</td>
<td>0.85</td>
<td>-</td>
<td>S2</td>
<td>Parton et al. (1993)</td>
</tr>
<tr>
<td>$k_{\text{sol}, \text{C}}$</td>
<td>slope of soluble fraction with lignin to N ratio</td>
<td>0.018</td>
<td>-</td>
<td>S2</td>
<td>Parton et al. (1993)</td>
</tr>
<tr>
<td>$LC_{\text{fine _root}}$</td>
<td>lignin content of fine root</td>
<td>0.2565592</td>
<td>mol$^{-1}$</td>
<td>S2</td>
<td>White et al. (2000)</td>
</tr>
<tr>
<td>$LC_{\text{coarse _root}}$</td>
<td>lignin content of coarse roots</td>
<td>0.8163248</td>
<td>mol$^{-1}$</td>
<td>S2</td>
<td>Thum et al. (2019)</td>
</tr>
<tr>
<td>$LC_{\text{woody _litter}}$</td>
<td>lignin content of woody litter</td>
<td>0.8163248</td>
<td>mol$^{-1}$</td>
<td>S2</td>
<td>White et al. (2000)</td>
</tr>
<tr>
<td>$LC_{\text{fruit}}$</td>
<td>lignin content of fruit bed</td>
<td>0.2565592</td>
<td>mol$^{-1}$</td>
<td>S2</td>
<td>Thum et al. (2019)</td>
</tr>
<tr>
<td>$LC_{\text{seed _bed}}$</td>
<td>lignin content of fine seed bed</td>
<td>0.2565592</td>
<td>mol$^{-1}$</td>
<td>S2</td>
<td>Thum et al. (2019)</td>
</tr>
<tr>
<td>$LC_{\text{leaf _max}}$</td>
<td>maximum lignin content of leaves</td>
<td>0.3440226</td>
<td>mol$^{-1}$</td>
<td>S3</td>
<td>White et al. (2000)</td>
</tr>
<tr>
<td>$k_{\text{leaf}, \text{sla}}$</td>
<td>slope of lignin to $\text{sla}$ relationship</td>
<td>-0.4328854</td>
<td>m$^{-2}$</td>
<td>S3</td>
<td>Parton et al. (1993)</td>
</tr>
<tr>
<td>$k_{\text{sol}, \text{vp}, \text{N}}$</td>
<td>proportionality factor controlling C:N of soluble vs. polymeric pool</td>
<td>5.0</td>
<td>-</td>
<td>S4</td>
<td>Parton et al. (1993)</td>
</tr>
<tr>
<td>$k_{\text{sol}, \text{vp}, \text{P}}$</td>
<td>proportionality factor controlling C:P of soluble vs. polymeric pool</td>
<td>5.0</td>
<td>-</td>
<td>S4</td>
<td>Parton et al. (1993)</td>
</tr>
<tr>
<td>$\eta_{\text{C, _wl _poly}}$</td>
<td>fraction of woody litter C transformed into polymeric litter</td>
<td>0.3</td>
<td>-</td>
<td>Sect.S1.2</td>
<td>This study</td>
</tr>
<tr>
<td>$\eta_{\text{C, _sol _dom}}$</td>
<td>fraction of soluble litter C transformed into DOM</td>
<td>0.7</td>
<td>-</td>
<td>Sect.S1.2</td>
<td>This study</td>
</tr>
<tr>
<td>$\tau_{\text{sol}}$</td>
<td>turnover time of soluble litter</td>
<td>0.033</td>
<td>years</td>
<td>S5</td>
<td>Parton et al. (1993)</td>
</tr>
<tr>
<td>$\tau_{\text{base}}$</td>
<td>turnover time of woody litter</td>
<td>2.5</td>
<td>years</td>
<td>S5</td>
<td>Thum et al. (2019)</td>
</tr>
<tr>
<td>$T_{\text{ref}}$</td>
<td>reference temperature for depolymerisation</td>
<td>293.15</td>
<td>K</td>
<td>S5</td>
<td>Wang et al. (2012)</td>
</tr>
<tr>
<td>$E_{a, \text{depoly}}$</td>
<td>activation energy for depolymerisation</td>
<td>53000.0</td>
<td>Jmol$^{-1}$</td>
<td>S5</td>
<td>Ahrens et al. (In prep.)</td>
</tr>
<tr>
<td>$k_{1, \text{fps}}$</td>
<td>parameter for moisture response of litter turnover</td>
<td>1.33</td>
<td>-</td>
<td>S5</td>
<td>This study</td>
</tr>
<tr>
<td>$k_{2, \text{fps}}$</td>
<td>parameter for moisture response of litter turnover</td>
<td>0.001</td>
<td>-</td>
<td>S5</td>
<td>This study</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
<td>Value</td>
<td>Unit</td>
<td>Equation</td>
<td>Citation</td>
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<td>----------</td>
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<tr>
<td>$v_{\text{poly}}^{\text{max, depoly}}$</td>
<td>maximum depolymerisation rate of polymeric litter</td>
<td>0.1849</td>
<td>yr$^{-1}$</td>
<td>S6</td>
<td>This study</td>
</tr>
<tr>
<td>$v_{\text{res}}^{\text{max, depoly}}$</td>
<td>maximum depolymerisation rate of microbial residue</td>
<td>0.2317</td>
<td>yr$^{-1}$</td>
<td>S6</td>
<td>This study</td>
</tr>
<tr>
<td>$K_{m,\text{depoly}}$</td>
<td>half-saturation microbial biomass for depolymerisation</td>
<td>3.70</td>
<td>mol/m$^3$</td>
<td>S6</td>
<td>This study</td>
</tr>
<tr>
<td>$E_{a,hsc}$</td>
<td>activation energy of half-saturation point</td>
<td>30000.0</td>
<td>J/mol</td>
<td>S6</td>
<td>Wang et al. (2013)</td>
</tr>
<tr>
<td>$k_{hsc}$</td>
<td>scaling factor for the sensitivity of half-saturation constant to moisture limitation</td>
<td>0.001</td>
<td>-</td>
<td>S6</td>
<td>Davidson et al. (2012)</td>
</tr>
<tr>
<td>$k_{\text{ads}}^{\text{dom}}$</td>
<td>adsorption rate of DOM</td>
<td>0.720</td>
<td>m$^3$/mol yr</td>
<td>S7</td>
<td>This study</td>
</tr>
<tr>
<td>$k_{\text{des}}^{\text{dom}}$</td>
<td>desorption rate of mineral-associated DOM</td>
<td>0.508</td>
<td>mol yr$^{-1}$</td>
<td>S7</td>
<td>This study</td>
</tr>
<tr>
<td>$k_{\text{ads}}^{\text{res}}$</td>
<td>adsorption rate of microbial residue</td>
<td>0.00372</td>
<td>m$^3$/mol yr</td>
<td>S7</td>
<td>This study</td>
</tr>
<tr>
<td>$k_{\text{des}}^{\text{res}}$</td>
<td>desorption rate of mineral-associated residue</td>
<td>0.154</td>
<td>yr$^{-1}$</td>
<td>S7</td>
<td>This study</td>
</tr>
<tr>
<td>$E_{a,\text{sorption}}$</td>
<td>activation energy for sorption</td>
<td>5000.0</td>
<td>J/mol$^{-1}$</td>
<td>S7</td>
<td>Ahrens et al. (In prep.)</td>
</tr>
<tr>
<td>$\rho_{\text{soil}}$</td>
<td>bulk density of mineral soil</td>
<td>1000 - 1600</td>
<td>kg/m$^3$</td>
<td>S7</td>
<td>Lang et al. (2017)</td>
</tr>
<tr>
<td>$\rho_{\text{org}}^{\text{bulk}}$</td>
<td>bulk density of organic material</td>
<td>248.84</td>
<td>kg/m$^3$</td>
<td>S28</td>
<td>Lang et al. (2017)</td>
</tr>
<tr>
<td>$D_b$</td>
<td>diffusion velocity due to bioturbation</td>
<td>0.15</td>
<td>m$^2$/kg yr</td>
<td>S1</td>
<td>Koven et al. (2013)</td>
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<tr>
<td>$v_{\text{dom}}^{\text{max, upt}}$</td>
<td>maximum microbial uptake rate of DOM</td>
<td>95.76</td>
<td>day$^{-1}$</td>
<td>S8</td>
<td>This study</td>
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<tr>
<td>$K_{m,\text{upt}}$</td>
<td>half-saturation DOM density for microbial DOM uptake</td>
<td>85.34</td>
<td>mol C/m$^3$</td>
<td>S9</td>
<td>This study</td>
</tr>
<tr>
<td>$\text{mic}^{\text{min}}_{\text{CUE}}$</td>
<td>minimal microbial CUE</td>
<td>0.3</td>
<td>-</td>
<td>S10</td>
<td>Manzoni et al. (2008)</td>
</tr>
<tr>
<td>$\text{mic}^{\text{max}}_{\text{CUE}}$</td>
<td>maximum microbial CUE</td>
<td>0.6</td>
<td>-</td>
<td>S10</td>
<td>Manzoni et al. (2008)</td>
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<tr>
<td>$\text{mic}_{\text{nue}}$</td>
<td>microbial nitrogen use efficiency</td>
<td>0.8</td>
<td>-</td>
<td>S10</td>
<td>Sinsabaugh et al. (2016)</td>
</tr>
<tr>
<td>$\text{mic}_{\text{pue}}$</td>
<td>microbial phosphorus use efficiency</td>
<td>0.89</td>
<td>-</td>
<td>S10</td>
<td>Sinsabaugh et al. (2016)</td>
</tr>
<tr>
<td>$\chi_{\text{C}}^{\text{N}}$</td>
<td>microbial CN ratio</td>
<td>13</td>
<td>mol mol$^{-1}$</td>
<td>S10</td>
<td>Lang et al. (2017)</td>
</tr>
<tr>
<td>$\chi_{\text{N}}^{\text{P}}$</td>
<td>microbial NP ratio</td>
<td>0.8</td>
<td>mol mol$^{-1}$</td>
<td>S10</td>
<td>Lang et al. (2017)</td>
</tr>
<tr>
<td>$\tau_{\text{mic}}$</td>
<td>microbial turnover time</td>
<td>154.7</td>
<td>days</td>
<td>S14</td>
<td>Ahrens et al. (2015)</td>
</tr>
<tr>
<td>$\eta_{\text{mic} \rightarrow \text{res}}$</td>
<td>fraction of microbial biomass that become residue during decay</td>
<td>0.828</td>
<td>-</td>
<td>S14</td>
<td>Ahrens et al. (2015)</td>
</tr>
<tr>
<td>$N_{\text{res} \rightarrow \text{dom}}$</td>
<td>fraction of N recycled from res to dom during microbial decay</td>
<td>0.4</td>
<td>-</td>
<td>S14</td>
<td>This study</td>
</tr>
<tr>
<td>$P_{\text{res} \rightarrow \text{dom}}$</td>
<td>fraction of P recycled from res to dom during microbial decay</td>
<td>0.8</td>
<td>-</td>
<td>S14</td>
<td>This study</td>
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<td>$\tau_{\text{mic avg}}^{\text{enzyme}}$</td>
<td>memory time scale for microbial enzyme allocation</td>
<td>7</td>
<td>days</td>
<td>S10</td>
<td>This study</td>
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<tr>
<td>$\tau_{\text{mic avg}}^{\text{en}}$</td>
<td>memory time scale for microbial enzyme allocation</td>
<td>7</td>
<td>days</td>
<td>S17</td>
<td>This study</td>
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Table S1. JSM parameters (ctnd.)

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Value</th>
<th>Unit</th>
<th>Equation</th>
<th>Citation</th>
</tr>
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<tbody>
<tr>
<td>$v_{\text{NH}_4}^{\text{max, mic}}$</td>
<td>maximum microbial uptake rate of $\text{NH}_4$</td>
<td>1278.7</td>
<td>$\mu\text{mol N mol C}_n^3$</td>
<td>S23</td>
<td>Kuzyakov and Xu (2013)</td>
</tr>
<tr>
<td>$v_{\text{NO}_3}^{\text{max, mic}}$</td>
<td>maximum microbial uptake rate of $\text{NO}_3$</td>
<td>1039.0</td>
<td>$\mu\text{mol N mol C}_n^3$</td>
<td>S23</td>
<td>Kuzyakov and Xu (2013)</td>
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<tr>
<td>$K_{\text{m, mic}}^{\text{NH}_4}$</td>
<td>half-saturation concentration for microbial $\text{NH}_4$ uptake</td>
<td>0.0129</td>
<td>$\mu\text{mol N mol C}_n^3$</td>
<td>S23</td>
<td>Kuzyakov and Xu (2013)</td>
</tr>
<tr>
<td>$K_{\text{m, mic}}^{\text{NO}_3}$</td>
<td>half-saturation concentration for microbial $\text{NO}_3$ uptake</td>
<td>0.0293</td>
<td>$\mu\text{mol N mol C}_n^3$</td>
<td>S23</td>
<td>Kuzyakov and Xu (2013)</td>
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<tr>
<td>$k_N^{\text{enz, mic}}$</td>
<td>coefficient of microbial transporter for N uptake</td>
<td>0.00005</td>
<td>$\mu\text{mol N} \mu\text{mol C}_n^3$</td>
<td>S23</td>
<td>Zhu et al. (2016)</td>
</tr>
<tr>
<td>$v_{\text{NH}_4}^{\text{max, plant}}$</td>
<td>maximum plant uptake rate of $\text{NH}_4$</td>
<td>1305.4</td>
<td>$\mu\text{mol N mol C}_n^3$</td>
<td>S23</td>
<td>Kuzyakov and Xu (2013)</td>
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<tr>
<td>$v_{\text{NO}_3}^{\text{max, plant}}$</td>
<td>maximum plant uptake rate of $\text{NO}_3$</td>
<td>218.4</td>
<td>$\mu\text{mol N mol C}_n^3$</td>
<td>S23</td>
<td>Kuzyakov and Xu (2013)</td>
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<td>$K_{\text{m, plant}}^{\text{NH}_4}$</td>
<td>half-saturation concentration for microbial $\text{NH}_4$ uptake</td>
<td>0.0857</td>
<td>$\mu\text{mol N mol C}_n^3$</td>
<td>S23</td>
<td>Kuzyakov and Xu (2013)</td>
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<tr>
<td>$K_{\text{m, plant}}^{\text{NO}_3}$</td>
<td>half-saturation concentration for microbial $\text{NO}_3$ uptake</td>
<td>0.129</td>
<td>$\mu\text{mol N mol C}_n^3$</td>
<td>S23</td>
<td>Kuzyakov and Xu (2013)</td>
</tr>
<tr>
<td>$k_p^{\text{enz, root}}$</td>
<td>coefficient of root transporter for N uptake</td>
<td>0.000125</td>
<td>$\mu\text{mol N} \mu\text{mol P}$</td>
<td>S23</td>
<td>Zhu et al. (2016)</td>
</tr>
<tr>
<td>$v_{\text{PO}_4}^{\text{max, mic}}$</td>
<td>maximum microbial uptake rate of $\text{PO}_4$</td>
<td>188.6</td>
<td>$\mu\text{mol P mol C}_n^3$</td>
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<td>Zhu et al. (2016)</td>
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<td>$K_{\text{PO}_4}^{\text{m, mic}}$</td>
<td>half-saturation concentration for microbial $\text{PO}_4$ uptake</td>
<td>0.000645</td>
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<td>Zhu et al. (2016)</td>
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<tr>
<td>$v_{\text{PO}_4}^{\text{max, plant}}$</td>
<td>maximum microbial uptake rate of $\text{PO}_4$</td>
<td>15.84</td>
<td>$\mu\text{mol P mol C}_n^3$</td>
<td>S23</td>
<td>Kavka and Polle (2016)</td>
</tr>
<tr>
<td>$K_{\text{PO}_4}^{\text{m, plant}}$</td>
<td>half-saturation concentration for root $\text{PO}_4$ uptake</td>
<td>0.00216</td>
<td>$\mu\text{mol P mol C}_n^3$</td>
<td>S23</td>
<td>Kavka and Polle (2016)</td>
</tr>
<tr>
<td>$k_p^{\text{enz, mic}}$</td>
<td>coefficient of microbial transporter for P uptake</td>
<td>0.0005</td>
<td>$\mu\text{mol P mol C}_n^3$</td>
<td>S23</td>
<td>This study</td>
</tr>
<tr>
<td>$k_p^{\text{enz, plant}}$</td>
<td>coefficient of root transporter to take up P</td>
<td>0.000125</td>
<td>$\mu\text{mol P mol C}_n^3$</td>
<td>S23</td>
<td>This study</td>
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**Soil Pi fluxes**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Value</th>
<th>Unit</th>
<th>Equation</th>
<th>Citation</th>
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<tbody>
<tr>
<td>$k_{\text{weather}}$</td>
<td>Weathering rate constant of mineral soil</td>
<td>8.16208</td>
<td>$10^{-14} \text{mol P mol C}_n^3$</td>
<td>S20</td>
<td>Wang et al. (2010)</td>
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<td>$K_{\text{m, weather}}$</td>
<td>half-saturation C exudation level for $\text{PO}_4$ weathering</td>
<td>0.083</td>
<td>$\text{mol C m}^3$</td>
<td>S20</td>
<td>This study</td>
</tr>
<tr>
<td>$v_{\text{m, biomin}}$</td>
<td>maximum biomineralisation rate of $\text{PO}_4$</td>
<td>0.005</td>
<td>$\text{mol P mol C}_n^3$</td>
<td>S21</td>
<td>Bünemann et al. (2016)</td>
</tr>
<tr>
<td>$K_{\text{m, biomin}}$</td>
<td>half-saturation C exudation level for $\text{PO}_4$ biomineralization</td>
<td>0.417</td>
<td>$\text{mol C m}^3$</td>
<td>S21</td>
<td>This study</td>
</tr>
<tr>
<td>$K_{\text{P, biomin}}$</td>
<td>half-saturation solute P concentration for $\text{PO}_4$ biomineralization</td>
<td>0.001</td>
<td>$\text{mol P m}^3$</td>
<td>S21</td>
<td>This study</td>
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<td>$K_{\text{m, biomin}}$</td>
<td>half-saturation substrate P:C ratio for $\text{PO}_4$ biomineralization</td>
<td>0.0002</td>
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<td>S21</td>
<td>This study</td>
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<tr>
<td>$k_{\text{ocl}}$</td>
<td>Occlusion coefficient of sorbed $\text{PO}_4$</td>
<td>3.86</td>
<td>$10^{-13} \text{mol C}_n^3$</td>
<td>S22</td>
<td>Yang et al. (2014)</td>
</tr>
<tr>
<td>$k_{\text{des}, f}$</td>
<td>$\text{PO}<em>4$ fast desorption rate from $P</em>{\text{lab}}$ to $\text{PO}_4$</td>
<td>0.014</td>
<td>$\text{mol C}_n^3$</td>
<td>S23</td>
<td>Van der Zee et al. (1989)</td>
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<tr>
<td>$k_{\text{abs}}$</td>
<td>$\text{PO}<em>4$ (a)sorption rate from $P</em>{\text{lab}}$ to $P_{\text{sorb}}$</td>
<td>651.85</td>
<td>$\text{mol C}_n^3$</td>
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<td>Yang et al. (2014)</td>
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<tr>
<td>$S_{\text{m, sorb}}$</td>
<td>$\text{PO}_4$ sorption capacity of organic matter</td>
<td>0.4</td>
<td>$\text{kg P kg OM}^{-1}$</td>
<td>S25</td>
<td>Thum et al. (2019)</td>
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<td>$S_{\text{m, mineral}}$</td>
<td>$\text{PO}_4$ sorption capacity of mineral soil</td>
<td>0.387</td>
<td>$\text{mmol P kg soil}^{-1}$</td>
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<td>Thum et al. (2019)</td>
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<td>$K_{\text{S, m, sorb}}$</td>
<td>half-saturation concentration for $\text{PO}_4$ adsorption to OM</td>
<td>0.045</td>
<td>$\text{mmol P kg OM}^{-1}$</td>
<td>S25</td>
<td>Thum et al. (2019)</td>
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<td>$K_{\text{S, mineral}}$</td>
<td>half-saturation concentration for $\text{PO}_4$ adsorption to soil mineral</td>
<td>0.00225</td>
<td>$\text{mmol P kg soil}^{-1}$</td>
<td>S25</td>
<td>Thum et al. (2019)</td>
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Table S2. Parameters for sensitivity analysis

<table>
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<td>All the values vary between 80% and 120% of the default values in Table S1</td>
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<td>$k_{ads}$</td>
<td>OM sorption</td>
</tr>
<tr>
<td>$k_{des}$</td>
<td>OM sorption</td>
</tr>
<tr>
<td>$k_{ads}$</td>
<td>OM sorption</td>
</tr>
<tr>
<td>$k_{res}$</td>
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<td>$k_{des}$</td>
<td>Nutrient acquisition</td>
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<td>$k_{des,f}$</td>
<td>Microbial growth and decay</td>
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<tr>
<td>$C:N$</td>
<td>Microbial growth and decay</td>
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<td>$\chi_{mic}$</td>
<td>Microbial growth and decay</td>
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<tr>
<td>$\chi_{N:P}$</td>
<td>Microbial growth and decay</td>
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<tr>
<td>$\mu_{\text{caue}}$</td>
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<tr>
<td>$\mu_{\text{pue}}$</td>
<td>Microbial growth and decay</td>
</tr>
<tr>
<td>$\mu_{\text{cue}}$</td>
<td>Microbial growth and decay</td>
</tr>
<tr>
<td>$\eta_{\text{C,wl}} \rightarrow \text{poly}$</td>
<td>Litter partitioning and turnover</td>
</tr>
<tr>
<td>$\eta_{\text{C,sol}} \rightarrow \text{dom}$</td>
<td>Litter partitioning and turnover</td>
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<tr>
<td>$\tau_{\text{mic}}$</td>
<td>Microbial growth and decay</td>
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<tr>
<td>$v_{\text{max,upt}}$</td>
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<td>$v_{\text{poly}}$</td>
<td>Depolymerisation</td>
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<td>$v_{\text{po}}$</td>
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<td>$k_{\text{N,enz,root}}$</td>
<td>Nutrient acquisition</td>
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<tr>
<td>$k_{\text{P,enz,root}}$</td>
<td>Nutrient acquisition</td>
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<tr>
<td>$k_{\text{weath}}$</td>
<td>Soil Pi cycle</td>
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<tr>
<td>$k_{\text{oocl}}$</td>
<td>Soil Pi cycle</td>
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References


Figure S1. Simulated and observed (a) SOC content, (b) C:N ration in SOM, (c) C:P ratio in SOM, (d) organic P to inorganic P ratio in soil, microbial C, N, and P content ((e) to (g)), and (h) soil bulk density at the study site up to 1m soil depth. Black lines and dots: observations; Color lines and shades: simulated mean values and ranges of standard deviation by Base Scenario, the Global Microbial Stoichiometry (Glob. Mic. Stoi), and the extended base scenarios with simulation length of 1000 years (Base_1000) and 5000 years (Base_5000). The microbial C, N, and P are only measured in top 30cm soil. Simulated means and standard deviations are calculated using data of the last 10 years from the model experiments.
Figure S2. Simulated seasonal and vertical distribution of (a) respiration, (b) net N mineralisation, (c) biochemical P mineralisation, (d) net P mineralisation, (e) microbial inorganic P uptake, (f) plant P uptake, (g) microbial inorganic N uptake, and (h) plant N uptake at the study site up to 1m soil depth. Points represent the mean values and error bars represent the standard deviations, both calculated using data of the last 10 years from the model experiments.
Figure S3. Simulated SOC fractions (upper panel) and their respective radiocarbon profiles (bottom panel) for up to 1m soil depth. Column (a): mineral-associated carbon (MOC), including adsorbed DOM and adsorbed microbial residue; Column (b): litter, including woody, polymeric and soluble litter; Column (c): live and dead microbes. Data points are derived from the last 10 years’ data of the model experiments.
Figure S4. Simulated yearly budget of (A) nitrogen and (B) phosphorus in soil solution. In panel A, sourcing fluxes of N includes gross mineralisation of NH₄ and NO₃, N deposition; sinking fluxes of N includes plant and microbial uptake of NH₄, plant and microbial uptake of NO₃, N leaching (both inorganic and organic), and size change of soluble N (δsol_N). In panel B, sourcing fluxes of P includes weathering, gross mineralisation of PO₄, biochemical mineralisation of PO₄, P deposition; sinking fluxes of P includes adsorption (Exchange_fast), microbial and plant uptake, P leaching (both inorganic and organic), and size change of soluble P (δsol_P). The budget are calculated using data of the full simulation from the model experiments.
Figure S5. QUINCY simulated and observed (a) SOC content, (b) C:N ration in SOM, (c) C:P ratio in SOM, (d) organic P to inorganic P ratio in soil at the study site up to 1m soil depth. Black lines and dots: observations; Organe lines and shades: simulated mean values and ranges of standard deviation using QUINCY model(Thum et al., 2019). Simulated means and standard deviations are calculated using data of the last 10 years from the model experiments.